Radio and Optical Wave Propagation – Prof. Lorenzo Luini
Sample exam

1	2	3	4	5	6	7	8	9	10

do not write above

SURNAME AND NAME ______________________________

ID NUMBER ______________________________

SIGNATURE ______________________________

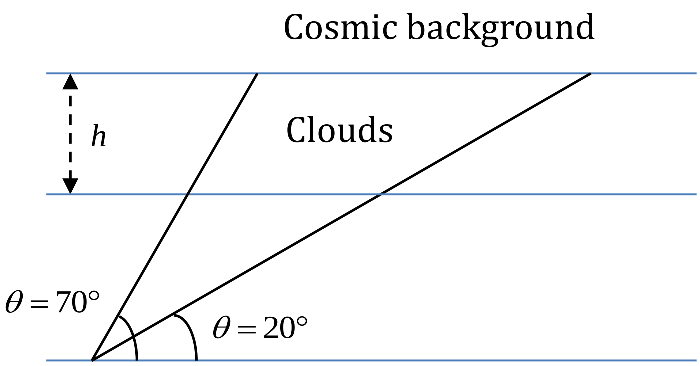
Exercise 1

In a cloudy day, a radiometer pointed at 70° elevation measures a brightness temperature T_{70} = 110 K. Calculate:

1) The corresponding attenuation, assuming a medium temperature T_{mr} = 275 K.
2) The brightness temperature measured and the attenuation estimated by the radiometer in the same day, if pointed at 20° elevation (assume same T_{mr}).
3) Assuming clouds have a thickness of 1.5 km, compute the equivalent specific attenuation in dB/km.

Consider the cosmic background noise, save for point 3), in which clouds are assumed to be the only source of noise.

Solution:



1) We know the brightness temperature measured at 70° elevation angle, from which we can derive the attenuation due to clouds (T_C = 2.73 K):

$T_{70} = T_{mr}(1-\alpha_{70}) + \alpha_{70} T_C = 110$ K → $\alpha_{70} = \dfrac{T_{70} - T_{mr}}{T_C - T_{mr}} = 0.606$ → $A_{70} = 10\log10\left(\dfrac{1}{\alpha_{70}}\right) = 2.18$ dB

2) Assuming horizontal homogeneity for clouds, the attenuation due to clouds along the zenith (θ = 90°) is:

$A_Z = A_{70} \sin 70° = 2.04 \text{ dB}$

In the same way, at 20°, the attenuation would be:

$A_{20} = \dfrac{A_z}{\sin 20°} = 5.96 \text{ dB}$ → $\alpha_{20} = 10^{-\frac{A_{20}}{10}} = 0.254$ → $T_{20} = T_{mr}\,(1-\alpha_{20}) + \alpha_{20}\, T_C = 206$ K

3) The zenithal attenuation is given as:

$A_Z = \gamma h$

Therefore the specific attenuation of clouds γ is:

$\gamma = \dfrac{A_z}{h} = 1.36 \text{ dB/km}$

Exercise 2

A satellite operating at 12 GHz in RHCP (right hand circular polarization) is seen by the ground station at elevation $\theta = 20°$.
Calculate:

1) The power transmitted by the satellite P_T in order counteract the attenuation caused by a slab of uniform rain intensity along the path of 50 mm/h (oblate drops, equi-oriented, minor axis in the vertical direction) and to have an SNR of 5 dB at the ground station.
   Assume:
   a) Rain extends up to h_R = 3.5 km in height
   b) Specific attenuation at 12 GHz and 20° elevation is $\gamma = aR^b$,where a_V = 0.01655, b_V = 1.175 for vertical polarization, a_H = 0.01835, b_H = 1.219 for horizontal polarization
   c) $\gamma = (\gamma_V + \gamma_H) / 2$ for design purposes
   d) On-board antenna G_s = 35 dB
   e) Ground receiver noise temperature T_R = 250 °K
   f) Signal bandwidth B = 300 MHz
   g) Ground antenna gain G_g = 50 dB
   h) Extra attenuation due to gases and clouds A_{cg} = 1.5 dB
   i) Ground-to-satellite distance L = 36000 km
   l) Antennas optimally pointed

2) The wave polarization in front of the ground receiving antenna.

Solution:
1) The link budget equation, including the attenuation due to rain, clouds and gases $A_{dB} = A_{rain} + A_{cg}$, is:

$$P_R = P_T G_S f_T \left(\frac{\lambda}{4\pi L}\right)^2 G_g f_R A$$

In this equation, both f_T and f_R are equal to 1 because the antennas are optimally pointed. We also have information on the required SNR = 5 dB = 3.162, which allows to calculate the minimum received power at the ground station (P_N is the noise power):

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{T_R B k} \quad \rightarrow \quad P_R = SNR\, T_R B k = 3.3 \text{ pW}$$

The attenuation due to rain (50 mm/h) can be calculated as:

$\gamma_H = 2.16$ dB/km, $\gamma_V = 1.64$ dB/km, $\gamma = (\gamma_V + \gamma_H) / 2$ = 1.9 dB/km $\rightarrow$ $A_{rain} = \gamma \dfrac{h_R}{\sin\theta} = 19.4$ dB

The total attenuation due to rain, clouds and gases is therefore $A_{dB} = A_{rain} + A_{cg}$ = 20.9 dB $\rightarrow$
$A = 10^{-A_{dB}/10} = 0.0081$

Thus we can invert the link budget equation to derive P_T:

$$P_T = \frac{P_R}{G_S \left(\dfrac{\lambda}{4\pi L}\right)^2 G_g A} = 420.4 \text{ W}$$

where $\lambda = c/f$ = 0.025 m and all the quantities have been converted to linear scale, i.e. G_S = 3162.3, G_g = 10000.

2) The wave gets depolarized by crossing the rain slab, as drops are oblate spheroids. Specifically, the vertical and horizontal components of the original RHCP wave undergo a different attenuation. The RHCP wave is given by the following components:

$E_V = E_0$ and $E_H = -jE_0$

The two components propagate through the rain slab and at the receiver we have:

$$E_V(L) = E_0 e^{-(\alpha_V + j\beta_V)L_R} = E_0 e^{-\alpha_V L_R} e^{-j\beta_V L_R} = E_0 0.1447 e^{-j\beta L_R}$$

$$E_H(L) = -jE_0 e^{-(\alpha_H + j\beta_H)L_R} = -jE_0 e^{-\alpha_H L_R} e^{-j\beta_H L_R} = -jE_0 0.0784 e^{-j\beta L_R}$$

where the phase constant is the same for both components ($\beta = 2\pi/\lambda = 251.3$ rad/m), $L_R = h_R/\sin(20°) = 10.2$ km is the path travelled through the rain slab and:

$$\alpha_V = \gamma_V \cdot 8.686 \cdot 1000 = 1.889 \cdot 10^{-4} \text{ Np/m}$$

$$\alpha_H = \gamma_H \cdot 8.686 \cdot 1000 = 2.388 \cdot 10^{-4} \text{ Np/m}$$

As is clear, the differential phase between the two components is the same, while they attenuate differently, which leads to a right-end elliptical polarization in front of the receiver: indeed the wave has been depolarized by the rain slab, as the elliptical polarization can be seen as the combination of a RHCP and LHCP waves.

Exercise 3

Calculate the additional attenuation of a wave travelling through the ionosphere for a zenithal pointing, assuming that the collision rate is $\nu = 10^4$ collisions/s and that the electrons are present and with a constant content N from 100 km up to 400 km. The critical frequency is 900 kHz and the wave frequency is 10 MHz.

Solution:

From the critical frequency, we can derive the electron content N:

$$f_C \approx 9\sqrt{N} \Rightarrow N = \frac{f_C^2}{81} \approx 10^{10} \text{ e/m}^3$$

The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{Ne^2\nu}{m(\nu^2+\omega^2)} = 7.2\cdot 10^{-10} \text{ S/m}$$

where $m = 9\cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6\cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{Ne^2}{m\varepsilon_0} = 3.2\cdot 10^{13} \text{ rad}^2/\text{m}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu^2+\omega^2} = 0.9919$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 1.36\cdot 10^{-7} + j0.21 \text{ 1/m}$$

The total path attenuation is:

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.0012 \text{ dB/km}$$

$$A = \alpha_{dB} L = \alpha_{dB}(400-100) = 0.355 \text{ dB}$$

Exercise 4

A uniform plane wave at f = 20 GHz with linear polarization propagates from a medium with ε_{r1} =2 to another medium with different electromagnetic features. The electric field of the incident wave, whose value is E_0 = 10 V/m, is parallel to the boundary between two media and the incident angle is 20°.

Calculate:

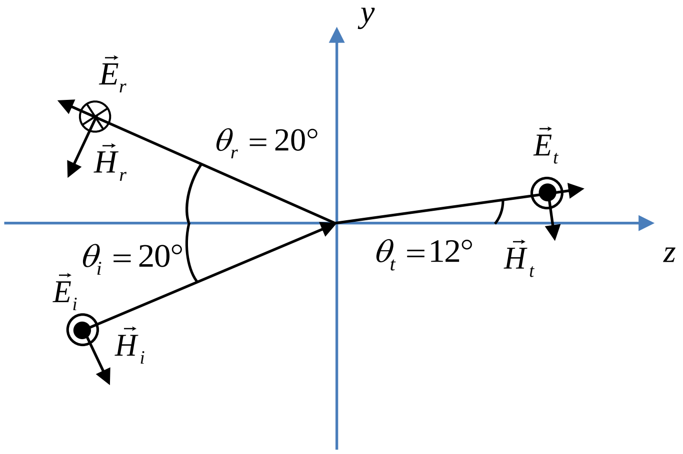
1) The electric field of the incident field

2) The density power carried by the wave travelling in the second medium with a refraction angle of 12°

3) The magnetic field of the reflected wave

Assume both media are lossless and non-magnetic.

Solution:



1) Knowing the incidence and refraction angles and exploiting Snells' law, we can derive the relative electric permittivity of the second medium:

$$\sqrt{\varepsilon_{r1}}\sin\theta_i \Rightarrow \sqrt{\varepsilon_{r2}}\sin\theta_t \quad \rightarrow \quad \varepsilon_{r2} = \left(\frac{\sqrt{\varepsilon_{r1}}\sin\theta_i}{\sin\theta_t}\right)^2 = 5.4$$

Therefore the expression for the incident electric field will be:

$$\vec{E}_i = (-10\vec{\mu}_x)e^{-j\beta_1(\cos\theta_i\vec{\mu}_z+\sin\theta_i\vec{\mu}_y)}$$

where $\beta_1 = \dfrac{2\pi f}{c}\sqrt{\varepsilon_{r1}} = 592.4 \ \text{ rad/m}$

2) To calculate the power density flowing in the second medium, we first need to calculate the reflection coefficient (TE case) and the intrinsic impedance of the two media:

$$\eta_1^{TE} = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}}\frac{1}{\cos\theta_i} = 283.7 \ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}}\frac{1}{\cos\theta_t} = 165.9 \ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.26$$

We can now calculate the power density flowing into medium as:

$$S_1 = \frac{1}{2}\frac{\left|\vec{E}_i\right|^2}{\eta_0/\sqrt{2}} = 0.188 \text{ W/m}^2$$

$$S_2^z = S_1^z\left(1-\left|\Gamma^{TE}\right|^2\right) \quad \Rightarrow \quad S_2 = \frac{S_1\cos\theta_i\left(1-\left|\Gamma^{TE}\right|^2\right)}{\cos\theta_t} = 0.168 \text{ W/m}^2$$

3) For the reflected magnetic field, we first need to calculate the reflected electric field as:

$$\vec{E}_r = \vec{E}_i \;\Gamma^{TE} = 2.6\vec{\mu}_x \text{ V/m}$$

The absolute value of the reflected magnetic field will be:

$$\left|\vec{H}_r\right| = \left|\vec{E}_r\right| / \eta_1 = 0.0098 \text{ A/m}$$

Finally, the full expression for the magnetic field will be:

$$\vec{H}_r = \left(-\left|\vec{H}_r\right|\cos\theta_i\vec{\mu}_y - \left|\vec{H}_r\right|\sin\theta_i\vec{\mu}_z\right)e^{-j\beta(-\cos\theta_r z+\sin\theta_r y)} \text{ A/m}$$

Exercise 5

Consider a transmitting antenna installed on top of a pole whose height is h = 70 m, with an output power P_T = 5 W (antenna gain G_{tx} = 40 dB).

Calculate:

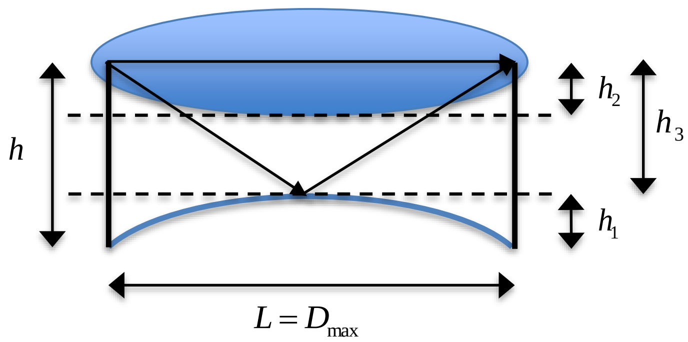
1) The maximum distance D_{max} covered by the antenna in standard atmospheric conditions.
2) Assuming now a receiver at distance D_{max} from the transmitter, at the same height h = 70 m (antenna with the same gain G_{tx} = 40 dB), calculate the frequency of the link by knowing that, in the propagation conditions at point 1), the distance ground-direct ray is 2.5 times the semi-minor axis of the first Fresnel's ellipsoid.
3) The power received considering only the direct ray (assume optimal antenna pointing).
4) Considering also the reflected ray (assume perfectly reflecting surface), is the combination of the two rays at the receiver constructive of destructive?

Solution:

1) The maximum distance covered can be calculated as (standard atmosphere → equivalent Earth radius factor = 4/3):

$$D_{\max} = \sqrt{2hR_{eq}} = \sqrt{2hkR_E} = \sqrt{2h\frac{4}{3}R_E} = 34.4 \text{ km}$$

2)



From the text we know that h_3 = 2.5 h_2, being h_2 the semi-minor axis of the first Fresenel's ellipsoid, i.e. $h_2 = \sqrt{\lambda L}/2$. At $L = D_{max}$ = 34.3 km, we have:

$$h_1 = \frac{1}{2}\frac{(L/2)^2}{\mathrm{R}_{eq}} = 17.4 \text{ m}$$

$$h_3 = 2.5h_2 = h - h_1 = 52.6 \text{ m} \rightarrow h_2 = \frac{h_3}{2.5} = 21 \text{ m} \rightarrow \lambda = \frac{(2h_2)^2}{L} = 0.0513 \text{ m}$$

Therefore:

$$f = \frac{c}{f} = 5.85 \text{ GHz}$$

3) Using the link budget:

$$P_R = P_T G_{tx}\left(\frac{\lambda}{4\pi L}\right)^2 G_{tx} = 7 \ \mu\text{w}$$

4) Defining E_0 as the electric field at the receiver and associated to the direct ray, the combination of the direct and reflected rays can be assessed as:

$$E = E_0(1 + e^{-j\beta\delta}) \text{ V/m}$$

where δ is the differential path, i.e. the difference between the path travelled along the reflected ray and the one travelled along the direct ray. This quantity can be well approximated by:

$$\delta = \frac{2h_3^2}{L} = 0.1609 \text{ m}$$

Therefore:

$$E = E_0(1 + e^{-j\beta\delta}) = E_0(1.6531 - j0.7573) \text{ V/m} \rightarrow |E| = |E_0| 1.818 \text{ V/m}$$

The combination of the two rays is therefore constructive.