Radio and Optical Wave Propagation – Prof. Lorenzo Luini
Sample exam

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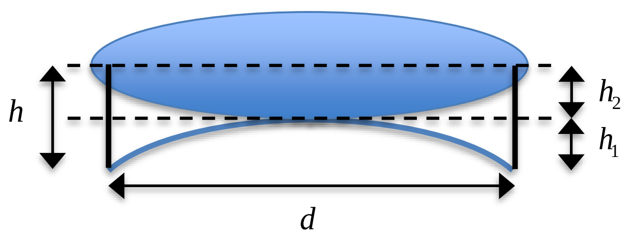
Exercise 1

Given a transmitter for TV broadcasting operating at frequency f = 12 GHz installed over a tower with height h = 50 m, evaluate:

1) A possible distance where to set up a test receiver (with the same height h) in order to have the first Fresnel zone free from ground interference, assuming dN/dh= -100 units/km.

2) What happens if the refractivity gradient changes to dN/dh= -50 units/km?

Solution:



1) Making reference to the picture above, the equivalent Earth concept has been applied to change the curvature of the planet. The equivalent Earth radius is:

$$R_{eq} = kR_E = R_E \frac{1}{1 + R_E\, dn/dh} = 17556 \text{ km}$$

This means that the rays tend to follow the Earth's surface, though not yet completely, which would happen with dN/dh ≈ -155 units/km.

In the picture above, h_2 corresponds to the semi-minor axis of the first Fresnell's elliposoid, whose value is (λ is the wavelength):

$$a = h_2 = \frac{\sqrt{\lambda d}}{2}$$

h_1 can be determined using the customary parabolic approximation for the Earth's surface:

$$h_1 = \frac{1}{2}\frac{(d/2)^2}{\mathrm{R}_{eq}}$$

Both h_1 and h_2 will increase as d increases and when their summation is higher than h, the tower height, the Earth's surface will interfere with the first Fresnell's elliposoid. Therefore, the condition to be imposed is:

$$h_1 + h_2 < h$$

The equation is complex to be solved analytically, so some d values can be tried to verify if the said condition holds. For example, for d = 64 km, the summation would yield h_1+h_2 = 49.16 m, which is slightly lower than h, therefore an acceptable solution. In this case h_1 = 29.16 m and h_2 = 20 m.

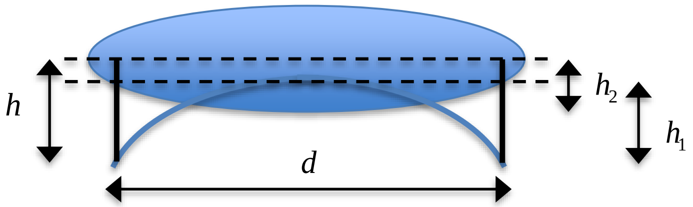
2) If the refractivity gradient changes to dN/dh ≈ -50 units/km, the ray bending is less strong and therefore, rays will tend to follow less the Earth's surface. This translates into a shorter equivalent Earth radius (higher curvature of the Earth):

$$R'_{eq} = kR_E = R_E \frac{1}{1 + R_E\, dn/dh} = 9349 \text{ km}$$

Assuming that the TX and RX are always set on the tower at height h = 50 m and at the same distance d = 64 km, the new value for h_1, due to the change in the propagation conditions, will be:

$$h_1 = \frac{1}{2}\frac{(d/2)^2}{\mathrm{R}'_{eq}} = 56.1 \text{ m}$$

The Fresnell's ellipsoid will not change, but it will be partially blocked by the Earth's surface (as shown in the figure below).



Exercise 2

Consider a radio link through the ionosphere connecting two stations.

The critical frequency is 5 MHz. Calculate:

1) The maximum value of free electron density N [e/m^3]

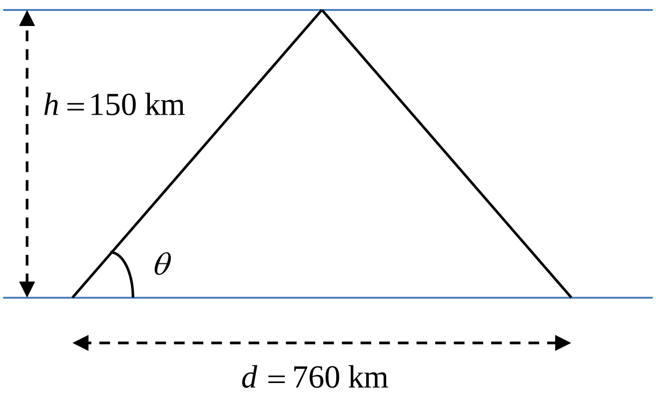
2) The maximum usable frequency to connect stations that are 760 km apart, assuming a reflection virtual height of 150 km.

Solution:

1) Given the critical frequency, it is possible to calculate the maximum value for N along the profile:

$$f_C \approx 9\sqrt{N_{\max}} \Rightarrow N_{\max} = \frac{f_C^2}{81} \approx 3.1\times10^{11} \text{ e/m}^3$$

2) Let's make reference to the following figure:



The elevation angle between the TX on the left and the RX on the right is determined easily:

$$\theta = \tan^{-1}\left(\frac{h}{d/2}\right) \approx 21.5°$$

The maximum usable frequency is therefore:

$$MUF = \frac{f_C}{\sin\theta} \approx 13.6 \text{ MHz}$$

Exercise 3

Given a terrestrial optical link operating at 1550 nm whose length is L = 800 m, calculate the link attenuation due to rain (considered to be constant along the whole link) in the following two situations:
1) mono dispersion of drops with diameter d = 1 mm and particle concentration C = 2500 drops/m^3;
2) mono dispersion of drops with diameter d = 0.5 mm and particle concentration C = 7500 drops/m^3.

Solution:
Given that the wavelength is far smaller than the dimension of the water particles, the asymptotic optical limit holds, according to which (S is the geometric cross section of the drop, which is considered to be spherical given its limited dimension):

$$C_{EXT} = 2S = 2\pi r^2 = 2\pi\left(D/2\right)^2$$

For cases 1) and 2), C_{EXT} is 1.57 mm^2 and 0.39 mm^2, respectively.
As for the specific attenuation, in case of mono dispersion (i.e. all drops of the same dimension):

$$\alpha = \frac{1}{2} N C_{EXT}$$

For case 1), we obtain:

$$\alpha = \frac{1}{2} N C_{EXT} = 1963.5\ \text{mm}^2/\text{m}^3 = 1963.5 \cdot 10^{-6}\ \text{m}^2/\text{m}^3 = 1.963 \cdot 10^{-3}\ \text{Np/m} = 17.05\ \text{dB/km}$$

In the last passage the equivalence 1 Np = 8.686 dB has been used.
Considering the second case, α turns out to be $1.472 \cdot 10^{-3}$ Np/m = 12.8 dB/km
Finally, the total rain attenuation along the link, assuming that rain is constant along the whole link, is:

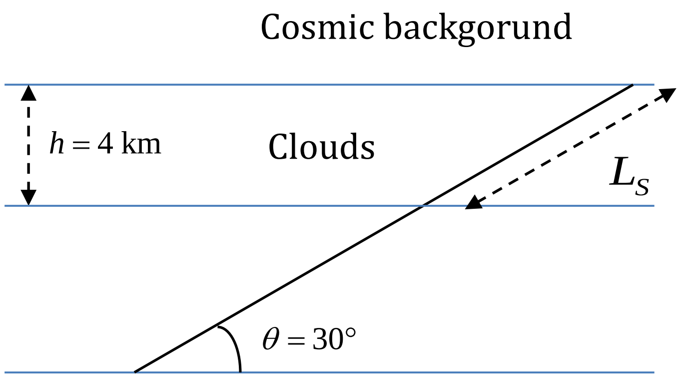
$$A = \alpha L$$

For cases 1) and 2), A is 13.64 dB and 10.24 dB.

Exercise 4

Evaluate the antenna noise temperature T_a perceived by a satellite receiver pointed at 30° elevation in cloudy weather conditions. Assume a cloud depth h = 4 Km with specific attenuation equal to γ = 0.2 dB/km and a cloud mean temperature T_{mr} = 2 °C at 30 GHz.
In the same meteorological conditions and assuming the same T_{mr}, evaluate to which elevation the antenna should be pointed in order for the received to have $T_n = 1.5T_a$.

Solution:



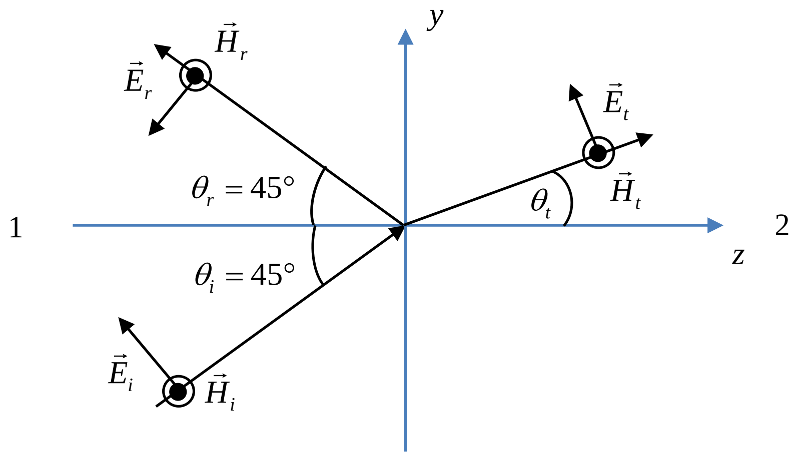
Let us consider the picture above. The attenuation due to clouds along the zenith (i.e. θ = 90°) is:
$A_Z = \gamma h = 0.8$ dB
The attenuation along the slant path (i.e. θ = 30°) is:
$A = A_Z / \sin\theta = 1.6$ dB
This gives an attenuation, in linear scale, equal to:
$\alpha = 10^{-\frac{A}{10}} = 0.6918$
Knowing this, the noise temperature captured by the receiver antenna, is:
$T_a = T_{mr}(1-\alpha) + \alpha T_C = 86.7$ K
where T_{mr} = 2° C + 273.15 = 275.15 K and the T_C = 2.73 K is the cosmic background noise temperature.

In the same meteorological conditions, lowering the elevation angle increases the antenna noise temperature; according to the data → $T_n = 1.5T_a = 130$ K
Inverting the equation above, we find:
$$\alpha = \frac{T_n - T_{mr}}{T_C - T_{mr}} = 0.5328 \rightarrow A = 10\log10\left(\frac{1}{\alpha}\right) = 2.73 \text{ dB}$$
This is the new slant path attenuation; therefore:
$\sin\theta = A_Z / A = 0.293 \rightarrow \theta = \sin^{-1}(A_Z/A) \approx 19.5°$

Exercise 5

A plane sinusoidal wave at f = 2.5 GHz propagating in a lossless medium characterized by ε_{r1}=1, μ_{r1} = 1, is impinging, with incident angle θ_i = 45°, on the boundary surface with a second lossless medium characterized by ε_{r2} = 3, μ_{r2} = 1. The magnetic field is parallel to the boundary surface. Write the equation of the total electric field in the first medium by assuming that the amplitude of the incident magnetic field at the boundary is H_0 = 0.1 A/m.

Solution:



This is a TM (Transverse Magnetic) case, as the magnetic field is parallel to the boundary between the two media. Let's suppose the incident fields are as in the figure above. We can easily calculate the absolute value of the incident electric field as:

$$\left|\vec{E}_i\right| = \left|\vec{H}_i\right| \eta_1 = 37.7 \text{ V/m}$$

being the first medium vacuum → $\eta_1 = \eta_0 = 377\ \Omega$

From Snell's law we can calculate the propagation angle of the transmitted wave:

$$\sqrt{\varepsilon_{r1}\mu_{r1}} \sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}} \sin\theta_t \rightarrow \theta_t \approx 24.1°$$

Now we can calculate the intrinsic impedances for the two media (for TM waves):

$$\eta_1^{TM} = \eta_1 \cos\theta_i = 266.6\ \Omega$$

$$\eta_2^{TM} = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} \cos\theta_i = 198.7\ \Omega$$

The reflection coefficient for the TM case is:

$$\Gamma^{TM} = \frac{\eta_2^{TM} - \eta_1^{TM}}{\eta_2^{TM} + \eta_1^{TM}} = -0.15$$

This coefficient relates the incident and reflected components of the electric field that are parallel to y:

$$E_r^y = E_i^y\ \Gamma^{TM} = \left|\vec{E}_i\right| \cos\theta_i\ \Gamma^{TM} = -4 \text{ V/m}$$

This is the reason why in the figure above the reflected electric field points towards –y and –z.
Therefore the total reflected electric field is:

$$\left|\vec{E}_r\right| = E_r^y / \cos\theta_r = 5.6 \text{ V/m}$$

Now all the elements are available to write the complete expression of the electric field in the first medium, given by the superposition of the incident and reflected fields:

$$\vec{E} = \left(\left| \vec{E}_i \right| \cos\theta_i \vec{\mu}_y - \left| \vec{E}_i \right| \sin\theta_i \vec{\mu}_z \right) e^{-j\beta(\cos\theta_i z + \sin\theta_i y)} + \left(- \left| \vec{E}_r \right| \cos\theta_r \vec{\mu}_y - \left| \vec{E}_r \right| \sin\theta_r \vec{\mu}_z \right) e^{-j\beta(-\cos\theta_r z + \sin\theta_r y)} \text{ V/m}$$

where:

$$\beta = \frac{2\pi c}{f} = 0.754 \text{ rad/m}$$