Radio and Optical Wave Propagation – Prof. L. Luini,
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SURNAME AND NAME ______________________________

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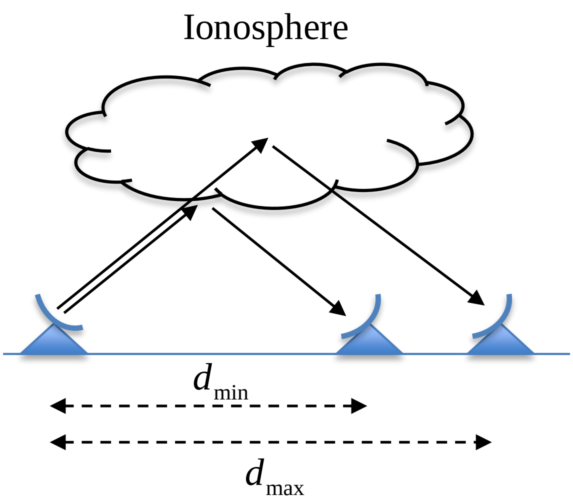
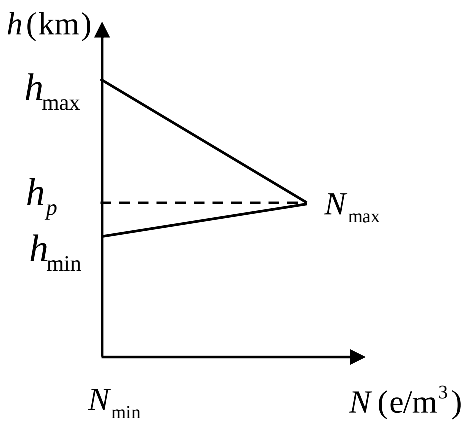
Problem 1

Making reference to the figure below, a bistatic ground-based pulsed radar system aims at measuring the profile of the ionospheric electron content (not the N values, but where the profile begins and ends), which is depicted in the figure below ($N_{\min}$ = 2×10^{11} e/m^3, $N_{\max}$ = 4×10^{12} e/m^3, $h_{\max}$ = 400 km, $h_{\min}$ = 100 km and h_p = 200 km, horizontally homogeneous). The transmitter elevation angle is θ = 40°. The transmitter can work in an extended frequency range, and the receiver side consists of an array of antennas at different distance from the transmitter (see figure below). In this context:

1) Find the minimum frequency range to be used to achieve, as much as possible, the measurement of the electron content profile.
2) According to the frequency range determined at point 1, calculate the distance $d_{\min}$ and $d_{\max}$ between the transmitter and the receiver.
3) (OPTIONAL) If the elevation angle increases to θ = 60°, assuming that also the frequency range needs to be changed accordingly to still achieve reflection, will the radar be more or less accurate if compared to θ = 40°? (discuss qualitatively)

Assume: flat Earth, virtual reflection height $h_V = 1.2h_R$ (where h_R is the real reflection height).





Solution

1) The operational frequency range of the radar can be obtained by exploiting the following expression:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N}}{f}\right)^2}$$

where f is the radar frequency. Therefore we can identify two frequencies associated to the minimum and maximum electron content:

$$f_{\min} = \sqrt{\frac{81N_{\min}}{1-\left[\cos(\theta)\right]^2}} = 6.26 \text{ MHz}$$

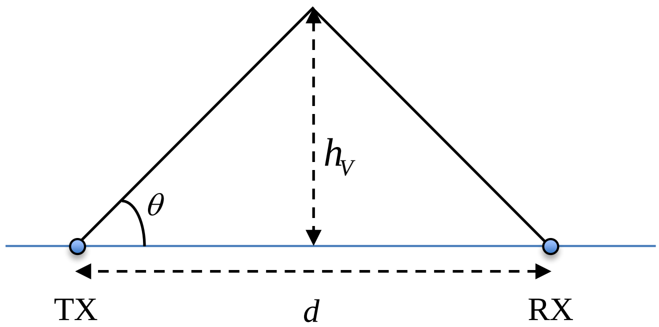
$$f_{\max} = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta)\right]^2}} = 28 \text{ MHz}$$

Using any frequency below $f_{\min}$ will guarantee total reflection at $h_{\min}$; increasing the frequency beyond $f_{\min}$ will allow progressively extending the reflection at altitudes higher than $h_{\min}$; for $f > f_{\max}$, the wave will cross the ionosphere. Therefore the minimum frequency range is $f_{\min} < f < f_{\max}$. As a result, only a portion of the profile will be measured.

2) Exploiting the concept of virtual reflection height:

$$d_{\min} = \frac{2(1.2h_{\min})}{\tan\theta} = 286\,\text{km}$$

$$d_{\max} = \frac{2(1.2h_p)}{\tan\theta} = 572\,\text{km}$$



3) If the elevation angle increases, in order to still obtain reflection, the frequency range will need to decrease. This will induce a higher ionospheric delay, which is given by the expression:

$$T_{iono} = \frac{1}{2c}\frac{81}{f^2}\text{TEC}$$

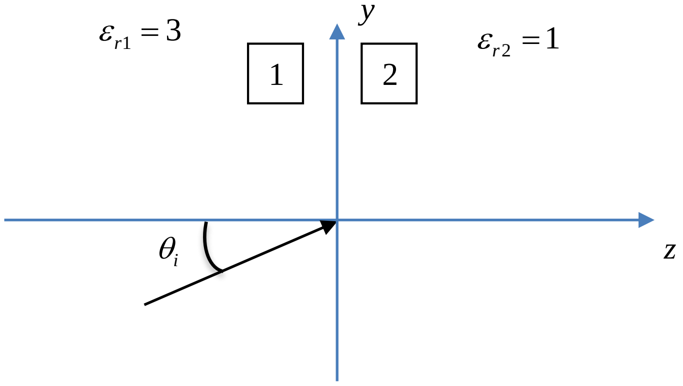
In turn, this will induce a higher estimation error, as the radar exploits T_{iono} to determine the value of $h_{\min}$ and $h_{\max}$.

Problem 2

A plane sinusoidal EM wave propagates from a medium with electric permittivity $\varepsilon_{r1} = 3$ into free space (assume $\mu_r = 1$ for both media). The expression for the incident electric field is:

$$\vec{E}(z,y) = \left[\vec{\mu}_x - j\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right)\right] e^{-j\beta\cos\theta z} e^{-j\beta\sin\theta y} \text{ V/m}$$

1) Determine the polarization of the incident wave.
2) Determine the polarization of the reflected wave when the incident angle is $\theta_i = 30°$.
3) (OPTIONAL) Determine the polarization of the reflected wave when the incident angle is $\theta_i = 50°$.



Solution

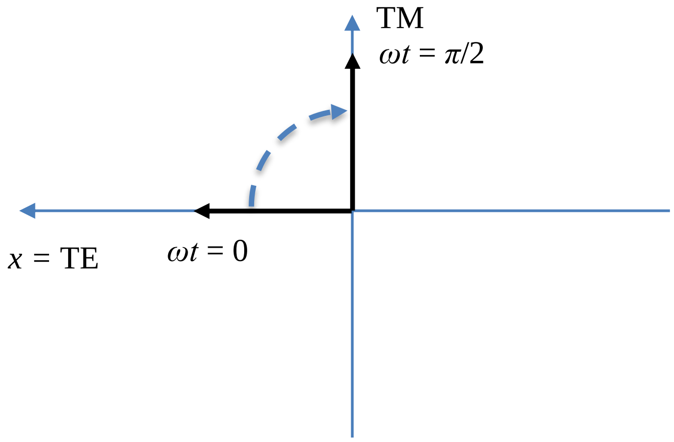
1) The polarization of the incident wave is right-hand circular, as the two TE and TM components have the same amplitude and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can easily understand the electric field rotation direction:

$$\vec{E}(0,0,t) = \mathrm{Re}\left\{\left[\vec{\mu}_x - j\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right)\right]e^{j\omega t}\right\} = \cos(\omega t)\,\vec{\mu}_{TE} - \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TM} \text{ V/m}$$

Thus, for $t = 0$ → $\vec{E}(0,0)\Big|_{\omega t=0} = \vec{\mu}_{TE}$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0,0)\Big|_{\omega t=\pi/2} = \vec{\mu}_{TM}$ V/m

Looking from behind the wave along its propagation direction, we can see the following:



2) As the wave has a TM component, it is worth checking the Brewster angle:

$$\theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}}\right) = 30°$$

As $\theta_i = \theta_B$, the TM component is totally transmitted, so the reflected wave will have a linear polarization (TE component)

3) Checking Snell's law:

$$\sin\theta_i\sqrt{\varepsilon_{r1}} = \sin\theta_t\sqrt{\varepsilon_{r2}} \quad \Rightarrow \quad \sin\theta_t = \sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = 1.3268$$

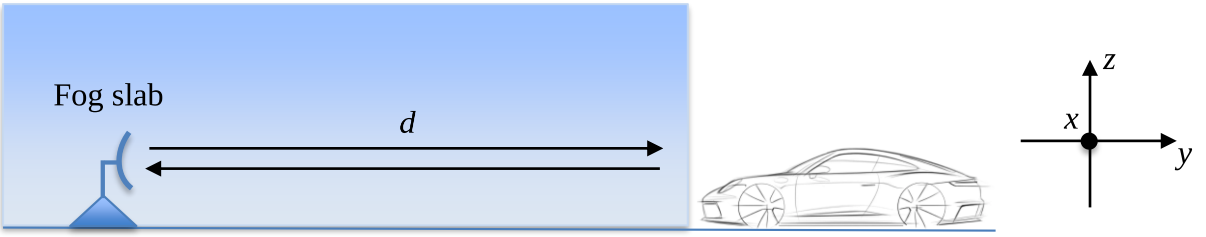
This is a hint of evanescent waves in the second medium; therefore, both the TM and TE components will undergo total reflection, but the reflection coefficients for both will be complex (the intrinsic impedance of the second medium will be imaginary) and different for TE and TM. This will introduce a different additional phase to the TE and the TM wave: as a result, though the absolute values of both TE and TM will remain the same, the phase shift between the two components will no longer be $\pi/2$ → the wave will have elliptical polarization.

Problem 3

Making reference to the figure below, a ground-based pulsed radar, operating with carrier frequency of 40 GHz and pointed horizontally, is used to identify cars in low-visibility foggy conditions at a distance d = 20 km. The fog slab consists of spherical droplets, whose associated specific attenuation α = 0.075 dB/km is constant both horizontally and vertically. The polarization of the wave transmitted by the radar is horizontal. In this context:

1) Determine the polarization of the wave in front of the car.
2) Calculate the minimum back scatter section of the car, σ, considering that the radar requires a minimum SNR_{min} = 8 dB to operate properly.

Consider the following data: radar transmit power P_T = 1 kW; radar antenna gain G = 40 dB; neglect the attenuation due to gases; LNA equivalent noise temperature T_R = 300 K; mean radiating temperature of fog T_{mr} = 200 K; LNA very close to the radar antenna feed; radar bandwidth B = 1 MHz.



Solution

1) As the droplets are spherical, the wave will not be depolarized: in front of the car, the wave will still be horizontally polarized.

2) First, let us calculate the power density reaching the car:

$$S_C = \frac{P_T}{4\pi d^2} G f A_F$$

where G = 10000, f = 1 (radar pointing to the car) A_F is the fog attenuation in linear scale. This is first calculated in dB as:

$A_F^{dB} = \alpha d = 1.5$ dB → $A_F = 0.7079$

Therefore:

$$S_A = 0.0014 \text{ W/m}^2$$

The power reirradiated by the car (with gain = 1 according to the definition of backscatter section), is:

$$P_A = S_A \sigma$$

The power density reaching the radar is:

$$S_R = \frac{P_A}{4\pi d^2} A_F \text{ W/m}^2$$

Finally, the power received by the radar is:

$$P_R = S_R A_E \text{ W}$$

where A_E is the equivalent area, expressed as:

$$A_E = G\frac{\lambda^2}{4\pi} = 0.0447\,\mathrm{m}^2$$

The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{SYS}B}$$

where T_{SYS} is the system equivalent noise temperature given by:

$$T_{SYS} = T_R + T_A = T_R + T_{mr}(1 - A_F) = 358.4\ \mathrm{K}$$

Therefore:

$$P_N = kT_{SYS}B = 4.95\times10^{-15}\ \mathrm{W}$$

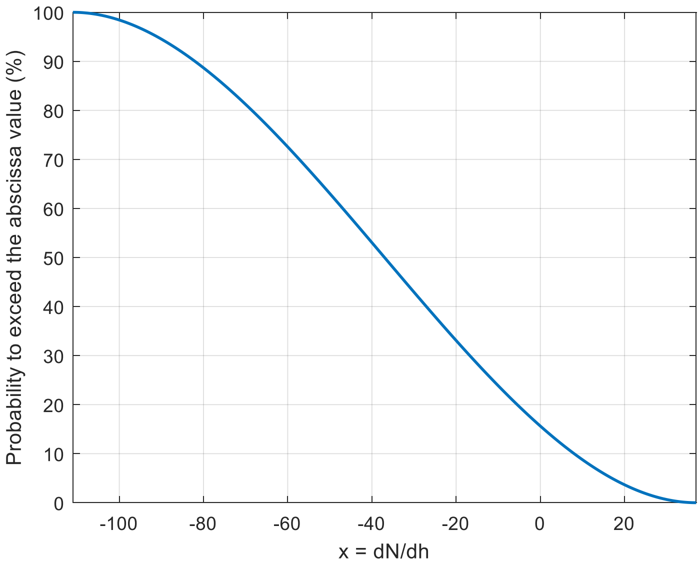
By imposing SNR > SNR_{min} and combining all the equations above:

$$SNR = \frac{P_R}{kT_{SYS}B} > SNR_{\min} \quad \rightarrow \quad \sigma > \frac{SNR_{\min}(4\pi d^2)P_N}{S_A A_F A_E} = 3.52\ \ \mathrm{m}^2$$

Problem 4

A TV broadcasting transmitter operates at frequency f = 20 GHz: it is installed on a tower whose height is h = 20 m and uses an antenna that can be considered isotropic. As shown in the figure below, the yearly Complementary Cumulative Distribution Function (CCDF) of the ground refractivity gradient $x = dN/dh$ for the site is given by the following expression (probability expressed in percentage values, dN/dh expressed in km^{-1}):

$$P(x) = 25\left(\frac{x+37}{74}\right)^3 - 75\left(\frac{x+37}{74}\right) + 50 \text{ valid for -111 km}^{-1} < x < 37 \text{ km}^{-1}$$



In this scenario, calculate the yearly probability to cover at least an area around the transmitter with radius r = 20 km.

Solution

The coverage area will depend on the propagation conditions, i.e. on the statistics of *dN/dh* reported above. We can derive the limit equivalent Earth radius from:

$$h = \frac{1}{2}\frac{r^2}{R_E} \rightarrow R_E = \frac{1}{2}\frac{r^2}{h} = 10000 \text{ km}$$

The limit dn/dh can be derived from:

$$R_E = \frac{R_{Earth}}{1 + R_{Earth}(dn/dh)} \rightarrow dn/dh = \frac{1}{R_{Earth}}\left(\frac{R_{Earth}}{R_E} - 1\right) \approx -57\times10^{-6} \text{ km}^{-1} \rightarrow dN/dh = -57 \text{ km}^{-1}$$

The more dN/dh becomes negative, the more the rays will bend: therefore, the target area is covered when $dN/dh \leq -57$ km^{-1}. Using this value as input to the analytical expression in the text, we obtain:

$$P_{NC} = P(dN/dh > -57\,\text{km}^{-1}) \approx 70\,\%$$

According to the definition of CCDF, this is the probability to exceed -57 km^{-1} though, i.e. the probability that the area is NOT covered (values of dN/dh increasing towards the positive, i.e.

towards sub-refraction, with rays bending upwards). Therefore the probability to cover the area is given by $P_C = P(dN/dh \le -57\,\mathrm{km}^{-1}) = 1 - P_{NC} = 30\,\%$.