Radio and Optical Wave Propagation – Prof. L. Luini,
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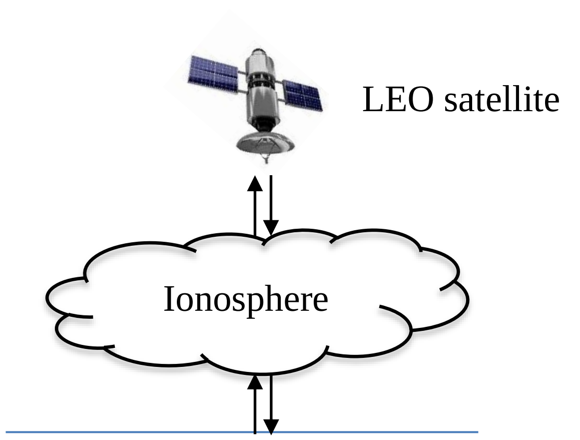
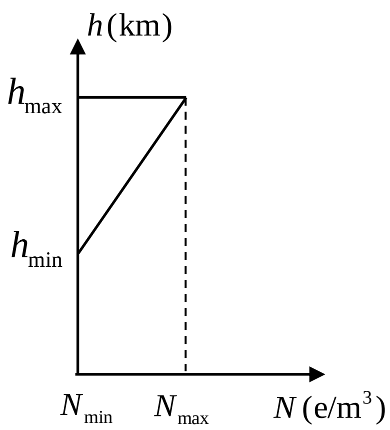
Exercise 1

Making reference to the figure below, a LEO satellite, whose altitude is $d = 900$ km, is used as a radar altimeter. To this aim, the satellite transmits an electromagnetic pulse (zenithal pointing) and the altitude of the ground is measured by knowing the time required for the pulse to reach back the satellite after the reflection on the Earth's surface. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where h_{max} = 600 km, h_{min} = 100 km, $N_{min} = 10^8$ e/m^3 and $N_{max} = 9\times10^9$ e/m^3. For this system:

1) Calculate the minimum operating frequency for the radar to work properly

2) Assuming that the operative frequency of the radar is f = 100 MHz, calculate the expected error in measuring the altitude (only due to the ionosphere)





Solution

1) For the radar to work properly, the pulse needs to cross the ionosphere, i.e., for zenithal pointing, the operating frequency needs to be higher than the critical frequency f_C, whose value is:

$$f_C \approx 9\sqrt{N_{max}} = 0.85 \text{ MHz}$$

2) The error in measuring the altitude is mainly due to the ionosphere, which introduces a delay in the propagation of the pulse. Such delay depends on the total electron content (TEC), which, in turn, is calculated as:

$$TEC = \int_{h_{\min}}^{h_{\max}} N(h)dh = N_{\min}\left(h_{\max} - h_{\min}\right) + \frac{\left(N_{\max} - N_{\min}\right)\left(h_{\max} - h_{\min}\right)}{2} = 2 \times 10^{15} \text{ e/m}^2$$

The time required for the pulse to cover d (one way distance) depends both on the physical distance and on the presence of the ionosphere. The total time is calculated as the summation of the free space travel time (i.e. as if ionosphere was not present) and of the ionospheric delay:

$$T = T_{FS} + T_{IONO} = \frac{d}{c} + \frac{1}{2c}\frac{81}{f_{sat}^2} TEC$$

where f_{sat} = 100 MHz.

We obtain:

$$T_{FS} = 3 \text{ ms}$$
$$T_{IONO} = 33.75 \text{ ns}$$

The error in measuring the altitude ε is only due to T_{IONO}; considering both the forward and return paths for the pulse:

$$\varepsilon = 2cT_{IONO} = 20.25 \text{ m}$$

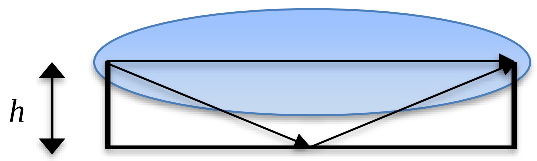
Exercise 2

Consider a terrestrial link operating at f = 30 GHz with transmitter TX and receiver RX at distance d = 1 km. Both TX and RX are set at h = 9.5 m from the ground, whose reflection coefficient is Γ = - 0.8. For this link:

1) Calculate the first Fresnel's ellipsoid semi-minor axis
2) Determine whether the reflected ray causes constructive or destructive interference at RX
3) Determine the minimum increase (or decrease) Δh to be applied to both antennas (same value) to maximize the electric field received at RX
4) If the antenna height h cannot be modified, mention an alternative method to maximize the electric field received at RX

Assumptions: flat Earth, no atmosphere.

Solution



1) The semi-minor axis of the first Fresnel's ellipsoid is given by:

$a = \sqrt{\lambda d}/2 = 1.58$ m being $\lambda = 0.01$ m at 30 GHz.

2) The two rays combine at the receiver with different phases. The total electric field is given by:

$$E = E_0\left(1 + \Gamma e^{-j\beta\delta}\right)$$

where E_0 is the field received from the direct wave and:

$$\beta = {}^{2\pi}/_{\lambda} = 628.32 \text{ 1/m}$$
$$\delta = \frac{2hh}{D} = 0.1805 \text{ m}$$

Therefore

$$|E| = |E_0| 0.344$$

Thus the rays combine destructively.

3) In order to maximize the power received, the two rays must combine constructively. Considering that Γ is negative, this happens, for example, when:

$e^{-j\beta\delta}$ = -1 → $\beta\delta = \pi$ → $\Delta h = \sqrt{\dfrac{\pi d}{2\beta}} - h = -7.92$ m

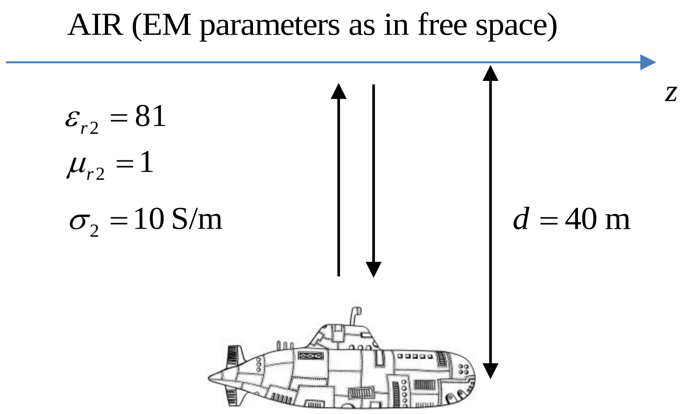
As expected, one of the solutions leads to $h_{new} = h + \Delta h = 1.58$ m $= a$.

4) If h cannot be modified, an alternative way to achieve the same result as in point 3), for example, is to change the operating frequency of the link, such that $e^{-j\beta\delta} = -1$.

Exercise 3

A submarine transmits electromagnetic pulses towards the sea surface to keep track of its depth. Assuming that the submarine emits plane waves with electric field $\vec{E}_{out}$ at frequency f = 1 kHz, and that its depth is d = 40 m, calculate:

1) The wavelength underwater
2) The time required for the pulse to reach the sea surface
3) The value of E_{out} such that the submarine can properly receive the pulse reflected by the sea surface (to this aim, consider a receiver sensitivity of S_{rx} = 1 pW)



Solution

1) First we need to characterize the electromagnetically the first medium (sea water). In this case, the loss tangent is $\frac{\sigma}{\omega\varepsilon}$ >> 1. Therefore the second medium can be well approximated as a good conductor. Therefore the attenuation and propagation constants are:

$$\alpha = \beta = \sqrt{\pi f \mu \sigma} = 0.1987 \text{ 1/m}$$

As for the intrinsic impedance, we obtain:

$$\eta_1 = \sqrt{\frac{\pi f \mu}{\sigma}}(1+j) = 0.0199(1+j) \ \Omega$$

The wavelength is:

$$\lambda = \frac{2\pi}{\beta} = 31.62 \text{ m}$$

2) The propagation velocity is:

$$v = \frac{\omega}{\beta} = 3.162 \times 10^4 \text{ m/s}$$

Therefore, the time required by the pulse to reach the sea surface is:

$$\tau = \frac{d}{v} = 1.3 \text{ ms}$$

3) For the first medium (air/free space), $\eta_2 = 377\ \Omega$. The reflection coefficient is therefore:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx 1$$

The power density emitted by the submarine is:

$$S_{out} = \frac{1}{2}\frac{\left|\vec{E}_{out}\right|^2}{|\eta_1|}\cos(\measuredangle\eta_1)$$

The power density reaching the submarine after reflection is:

$$S_{back} = S_{out}e^{-2\alpha d}\left|\Gamma\right|^2 e^{-2\alpha d} = \frac{1}{2}\frac{\left|\vec{E}_{out}\right|^2}{|\eta_1|}\cos(\measuredangle\eta_1)e^{-4\alpha d}\left|\Gamma\right|^2$$

The absolute value of $\vec{E}_{out}$ is found by imposing that:

$$S_{back} \geq S_{rx}$$

which yields:

$$\left|\vec{E}_{out}\right|^2 \geq \frac{2S_{rx}|\eta_1|}{\cos(\measuredangle\eta_1)e^{-4\alpha d}\left|\Gamma\right|^2} = 5.1 \text{ V/m}$$

Exercise 4

Consider a zenithal link (elevation angle θ_1 = 90°) from a LEO satellite to a ground station, operating at f = 30 GHz, for which the signal goes through a uniform liquid water cloud (of thickness h = 4 km). The specific attenuation of the cloud α = 0.1 dB/km is constant and uniform through the whole cloud.
Knowing that the satellite transmits the horizontal polarization:

1) Calculate the signal-to-noise ratio at the receiver.

2) Determine the polarization in front of the receiver.

3) Calculate the signal-to-noise ratio for the new elevation angle θ_2 = 30°, for which the distance between the LEO satellite and the ground station becomes H_2 = 1100

Assumptions:

- antennas optimally pointed

Additional data:

- cloud temperature T_{cloud} = -10 °C
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 25 dB
- power transmitted by the satellite: P_T = 100 W
- altitude of the LEO satellite: H = 900 km
- bandwidth of the receiver: B = 10 MHz
- internal noise temperature of the receiver: T_R = 350 K

Solution

1) The signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi H\right)^2 G_R f_R A}{kTB}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T is the total noise temperature (summation of T_R and the antenna noise T_A), $f_R = f_T = 1$ (antenna optimally pointed). In order to calculate the noise power, first let us calculate the attenuation due to the ice cloud:

$$A_{dB} = \alpha h = 0.4 \text{ dB} \rightarrow A = 10^{-\frac{A_{dB}}{10}} = 0.912$$

The antenna noise temperature is:

$$T_A = T_C A + T_{cloud}(1-A) = 25.6 \text{ K}$$

Therefore: SNR = 137.5 = 21.38 dB

2) The polarization is not affected by the presence of the cloud as suspender liquid water droplets are isotropic.

3) If the elevation angle changes, so does the attenuation due to the cloud, which can be scaled easily as follows:

$A_{dB,30} = A_{dB,30}/\sin(30°) = 0.8 \rightarrow A_{30} = 10^{-\frac{A_{dB,30}}{10}} = 0.8318$

The new antenna noise temperature is:

$$T_{A,30} = T_C A_{30} + T_{cloud}(1 - A_{30}) = 46.5 \text{ K}$$

Therefore:

$$SNR_{30} = \frac{P_{R,30}}{P_{N,30}} = \frac{P_T G_T f_T \left(\lambda/4\pi H_2\right)^2 G_R f_R A_{30}}{k(T_R + T_{A,30})B} = 79.55 = 19 \text{ dB}$$