Radio and Optical Wave Propagation – Prof. L. Luini,
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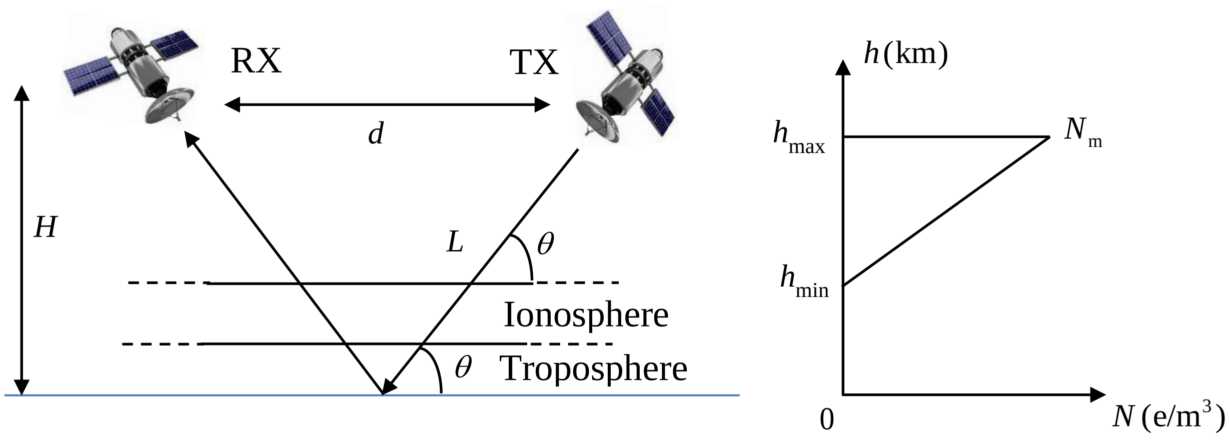
Problem 1

The figure below shows a bistatic radar system consisting of two LEO satellites flying along the same orbit (satellite height above the ground H = 600 km). The radar extracts information on the ground by measuring the power received at RX by reflection. Making reference to the electron content profile (right side, $h_{\min}$ = 100 km, $h_{\max}$ = 400 km, $N_m = 4\times10^{12}$ e/m^3, N homogeneous horizontally) and to the simplified geometry (left side) reported below:

1) Determine the maximum distance d between the two satellites along the orbit for the system to work properly, when the operational frequency is f_1 = 36 MHz.
2) Assuming the minimum distance at point 1) and that the radar operational frequencies changes to f_2 = 10 GHz, considering that the zenithal tropospheric attenuation is A_Z = 3 dB, determine the power received at RX.

Assumptions: the gain of both antennas is G = 50 dB, the backscatter section of the ground is σ = 1000 m^2, the transmit power is P_T = 100 W.



Solution

1) For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{m}}}{f_1}\right)^2} \quad \Rightarrow \quad \theta_{min} = 30°$$

Fixing $\theta = \theta_{min}$, the maximum distance is calculated as:
$d = 2\, H \tan(90° - \theta) = 2078.5$ km

2) Given the θ value determined at point 1), $L = H/\cos(90-\theta) = 1200$ km. Working at $f_2 = 10$ GHz, the ionospheric effects can be neglected, but not the atmospheric ones. The power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l = 1.39 \times 10^{-7}\ \mathrm{W}$$

where

$$A = \frac{A_Z}{\sin(\theta)} = 6\ \mathrm{dB} \;\rightarrow\; A_l = 10^{-A/10} = 0.2512$$

The power received by RX is:

$$P_R = \frac{S\sigma}{4\pi L^2} A_l A_E = 1.38 \times 10^{-17}\ \mathrm{W}$$

where:

$$A_E = \frac{\lambda^2}{4\pi} G = 7.16\ \mathrm{m}^2$$

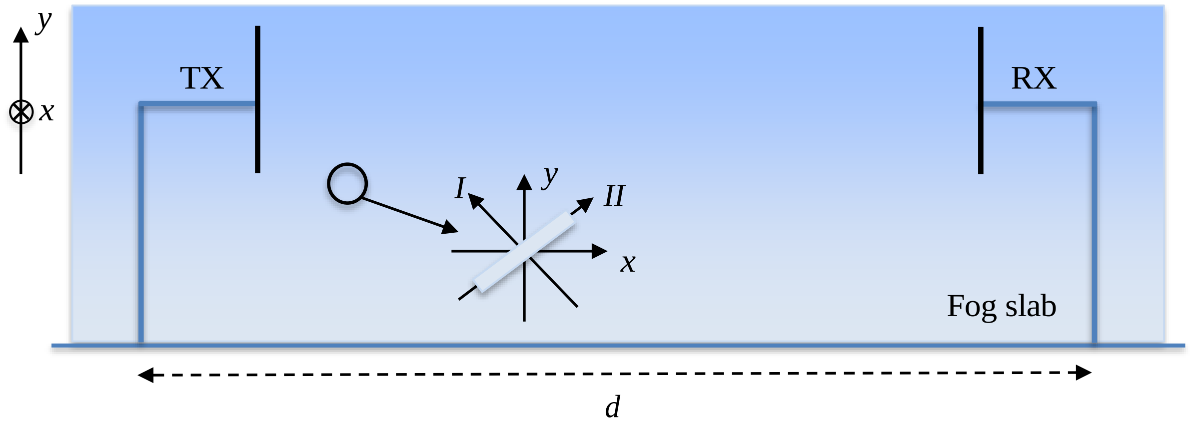
Problem 2

A terrestrial link, with path length d = 5 km and operating at f = 80 GHz, is subject to fog. Both the transmitter (TX) and the receiver (RX) use antennas with vertical polarization (along y); as shown in the figure below, the fog slab consists only of equioriented ice needles, which induce a differential attenuation (but same phase shift) for the components of the wave along I and II, namely $\Delta\gamma = \gamma_{II} - \gamma_I$ = 1.2 dB/km; the angle between axis II and axis x is 45°. For this link:

1) Determine the polarization of the wave in front of the receiver RX (specify: LH or RH for a circular/elliptical, tilt angle for a linear).
2) Which is the best polarization to be used to maximize the received power?

Assume that:

- the Earth surface is flat
- disregard the reflection from the ground and the attenuation due to gases



Solution

1) The electric field is aligned with y so both components on I and II will have the same amplitude, as they will be projected using sin(45°) and cos(45°):

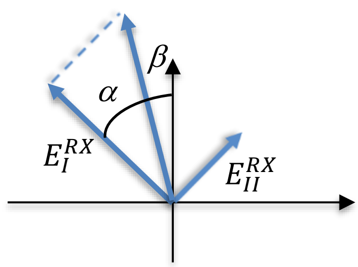
$$E_I^{TX} = E_{II}^{TX}$$

E_{II}^{TX} will be attenuated more than E_I^{TX}, specifically by $\Delta A_{dB} = \Delta\gamma\, d$ = 6 dB. This means that the ratio of the two electric fields in front of the receiver (as they have the same phase shift) will be:

$$\frac{E_{II}^{RX}}{E_I^{RX}} = 10^{-\frac{\Delta AdB}{20}} = 0.5$$

As there is no differential phase shift, the polarization will still be linear, but with a tilt angle. Making reference to the figure below:

$$\alpha = \tan^{-1}\left(\frac{E_{II}^{RX}}{E_I^{RX}}\right) = 26.6° \;\rightarrow\; \beta = 45° - \alpha = 18.4°$$



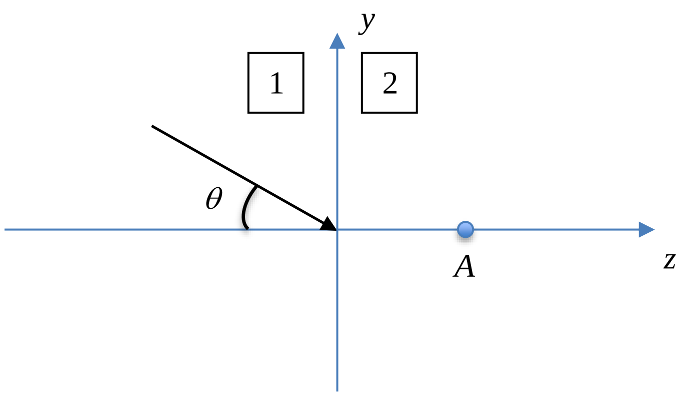
2) The best polarization to be used in this context is the linear polarization along axis I (antennas will need to be tilted as accordingly): in this case, the electric field will not be depolarized and will be subject to the lowest attenuation.

Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 4 into free space (assume μ_r = 1 for both media). The expression of the incident electric field is:

$$\vec{E}_i(z,y) = -j\vec{\mu}_x e^{-j\beta\cos\theta z} e^{j\beta\sin\theta y} \text{ V/m}$$

1) Determine the polarization of the incident field $\vec{E}_i$.
2) Determine the value of θ to maximize the power received in A(z = 1 m, y = 0 m, x = 10 m), where an isotropic antenna is located.
3) Calculate the power received by the antenna in A for the θ value determined at point 2)
4) Determine the value of θ to minimize the power received in A.



Solution

1) The wave has just one component, specifically the TE one, so the polarization is linear along -$\vec{\mu}_x$.

2) As the wave is a TE wave, there is no chance to have total transmission (this is possible only with the TM wave, when θ_i is the Brewster's angle. Therefore, the power density transmitted in the second medium will be maximized by minimizing the reflection coefficient, which occurs when θ_i = 0° (orthogonal incidence).

3) The reflection coefficients will be:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/\sqrt{\varepsilon_{r2}} - \eta_0/\sqrt{\varepsilon_{r1}}}{\eta_0/\sqrt{\varepsilon_{r2}} + \eta_0/\sqrt{\varepsilon_{r1}}} = 0.34$$

The power reaching A will be:

$$S_A = \frac{1}{2}\frac{|\vec{E}_i|^2}{\eta_0/\sqrt{\varepsilon_{r1}}}(1 - |\Gamma|^2) = 2.4 \text{ mW/m}^2$$

The power received in A is:

$$P_R = S_A\ A_E = S_t \frac{\lambda^2}{4\pi} G = 0.21\ \mu\text{W}$$

4) The power received in A will be minimized (zero) if θ_i is larger than the critical angle; in fact, total reflection (evanescent wave in the second medium) is possible as medium 1 is denser than medium 2 (electromagnetically). The critical angle is:

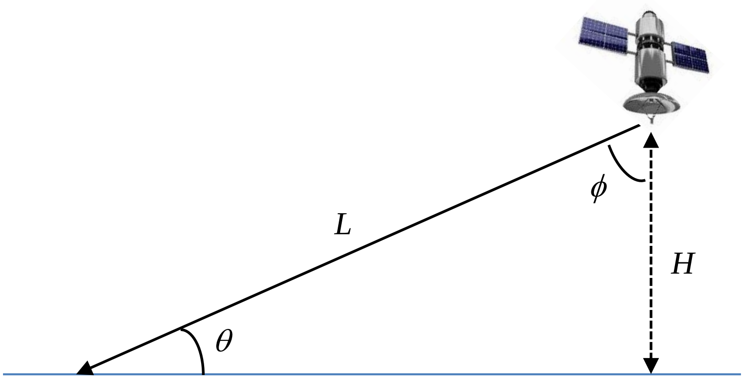
$$\theta_C = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

As a result, if $\theta_i \geq \theta_C$, no power will be received in A.

Problem 4

Consider a link from a LEO satellite to a ground station, operating at f = 30 GHz. Determine if the link operates properly down to the target elevation angle θ = 20°, knowing that the minimum signal-to-noise ratio (SNR) at the ground station must be 5 dB and that the link needs to be available for 99.9% of the time. The CCDF of the zenithal tropospheric attenuation is given by:

$$P\left(A_T^{dB}\right) = 100e^{-1.15A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in } \%)$$



Additional assumptions and data:

- use the simplified geometry depicted above (flat Earth)
- the specific attenuation of the troposphere is homogeneous vertically and horizontally
- ground station tracking the satellite optimally
- power transmitted by the satellite P_T = 100 W
- LEO satellite pointing always to the centre of the Earth
- radiation patter of the LEO satellite antenna (circular symmetry): $f_T = \cos(\phi)$
- disregard the cosmic background temperature contribution
- mean radiating temperature T_{mr} = 290 K
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 30 dB
- altitude of the LEO satellite: H = 800 km
- bandwidth of the receiver: B = 1 MHz
- internal noise temperature of the receiver: T_R = 350 K

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_T}{k[T_R + T_{mr}(1 - A_T)]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). As the link needs to be available for 99.9% of the time, the outage probability is P_{out} = 100%-99.9% = 0.1%. Using P_{out} in the CCDF expression, we obtain:

A_T^{dB} = 6 dB

Moreover, the attenuation along the slant path is:

A_S^{dB} = 17.56 dB → $A_T = 0.0175$

Also, $f_T = \cos(90° - \theta) = 0.342$, while $f_R = 1$.

Finally, $L = H/\sin(\theta) = 2339$ km.

Solving the equation above → SNR = 9 dB > 5 dB → the system operates correctly down to the target elevation angle.