Radio and Optical Wave Propagation – Prof. L. Luini,
July 15th, 2022 – Part 1

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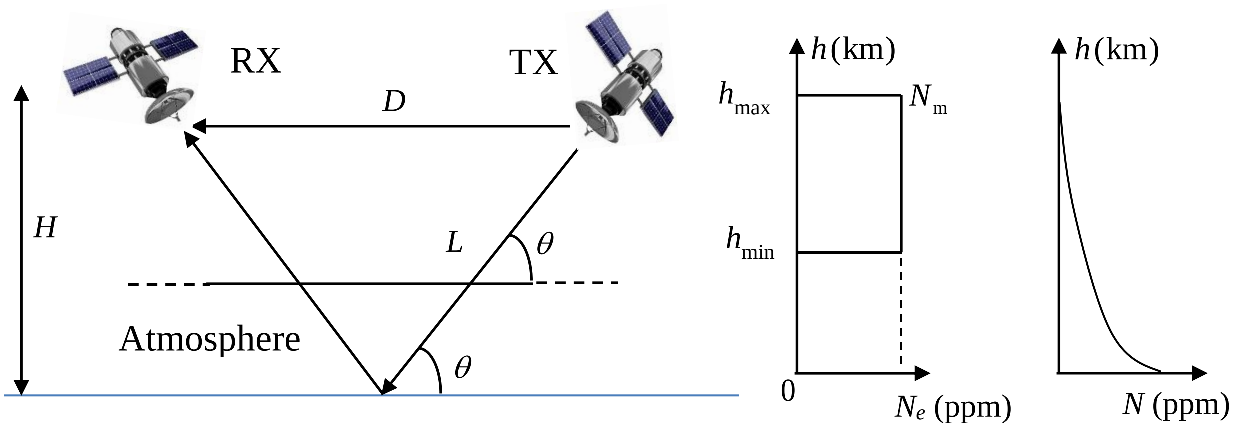
Problem 1

The figure below shows a bistatic radar system (carrier frequency f_1 = 1 GHz), consisting of two LEO satellites (same orbit, satellite height above the ground H = 500 km, elevation angle θ = 45°). The radar extracts information on the atmosphere by measuring the differential propagation time along the two paths (reflected one and direct one). Making reference to the right, where the electron content (N_e) profile ($h_{\min}$ = 100 km, $h_{\max}$ = 400 km, $N_m = 5\times10^{12}$ e/m^3, N homogeneous horizontally) and the refractivity profile ($N = 800e^{-h/40000}$, where h is in m and N is homogeneous horizontally) are shown, and to the left side, where the simplified geometry is reported:

1) Calculate the differential propagation time between the two paths, Δt, and highlight the contribution of the atmosphere to Δt.
2) Will the differential propagation time increase or decrease (compared to point 1) if the frequency changes to f_2 = 25 MHz?



Solution

1) The direct path propagation time is simply given by:

$$t_D = \frac{D}{c} = \frac{2H}{\mathrm{tg}(\theta)c} \approx 3.4 \text{ msec}$$

The wave crossing the atmosphere will be delayed twice by the troposphere and the ionosphere. The impact of the former can be calculated as (one-way zenithal tropospheric delay):

$$t_T^z = -\frac{h_0 N_0 10^{-6}}{c}\left[e^{-\frac{h}{h_0}}\right]_0^H$$

Considering the satellite height $H >> h_0$:

$$t_T^z = -\frac{h_0 N_0 10^{-6}}{c} = 0.107\ \mu\text{sec}$$

The impact of the latter can be calculated as (one-way zenithal ionospheric delay):

$$\text{TEC} = (h_{\max} - h_{\min})N_m = 1.5 \times 10^8\ \text{e/m}^2$$

$$t_I^z = \frac{40.3}{cf_1^2}\text{TEC} = 0.202\ \mu\text{sec}$$

The total propagation time along the reflected path is:

$$t_R = 2\frac{H}{\sin(\theta)c} + 2\frac{t_T^z}{\sin(\theta)} + 2\frac{t_I^z}{\sin(\theta)} \approx 4.7\ \text{msec}$$

The differential propagation time is:
$\Delta t = t_R - t_D \approx 1.3815$ msec

The contribution to Δt of the atmosphere is $\Delta t_{ATM} = 2\frac{t_T^z}{\sin(\theta)} + 2\frac{t_I^z}{\sin(\theta)} = 0.872$ µsec.

2) Using the second frequency (range of tens of MHz), it is worth checking if the wave can actually cross the ionosphere. For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as by:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\text{m}}}{f_2}\right)^2} \quad \Rightarrow \quad \theta_{min} = 53.6°$$

As $\theta < \theta_{min}$, the wave will be totally reflected and it will not be received by RX.

Problem 2

A plane EM wave at 30 GHz propagates across a layer of melting hydrometeors (path length d = 1 km, θ = 45° tilt angle), which is characterized by the following propagation constants (see sketch below):

$\gamma_I = 0.6413 + j628676$ 1/km

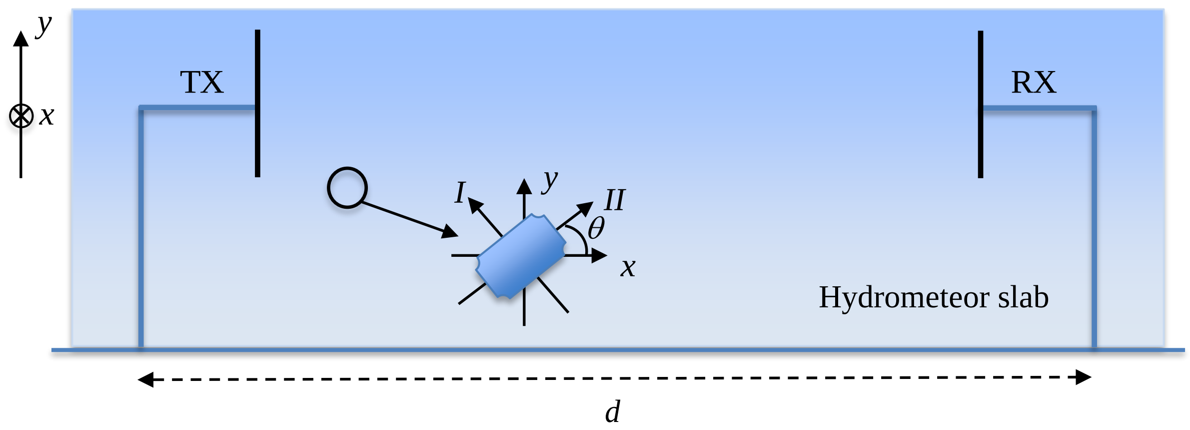
$\gamma_{II} = 0.6413 + j628677.57$ 1/km

Both the transmitter (TX) and the receiver (RX) employ linear antennas: as for the former, it is a vertical one; as for the latter, see point 2). For this link:

1) Determine the wave polarization in front of the receiver (no need to specify the tilt angle for a linear polarization, nor the rotation direction for a circular/elliptical polarization).
2) Based on point 1, determine the best direction of the RX antenna to maximize the received power.
3) Based on point 2, calculate the power received by RX.

Assume that:

- the Earth surface is flat; disregard the reflection from the ground and the attenuation due to gases; consider plane wave propagation
- use the following data: electric filed emitted at TX, $|\vec{E}_{TX}| = 10$ V/m, effective area of the RX antenna, A_{RX} = 1 m^2, assume free space for the calculation of the medium intrinsic impedance



Solution

1) As is clear from the propagation constants, the hydrometeor slab induces the same attenuation on both I and II components of the wave; on the other hand, the differential phase shift is:

$\Delta\beta = \beta_{II} - \beta_I = 1.57 = \pi/2$ rad/km

The TX antenna emits a vertical polarization, so the amplitude of components I and II are equal and given by $|\vec{E}_{TX}|\cos(45°) = 7.07$ V/m. Therefore, in front of RX, the two components I and II will have: 1) same amplitude (attenuated by the slab); 2) a differential phase shift of $\pi/2$ rad/km → the polarization will be circular.

2) Given the circular polarization determined at point 1, any direction of the linear RX antenna will collect the same amount of power.

3) The received power will be given by:

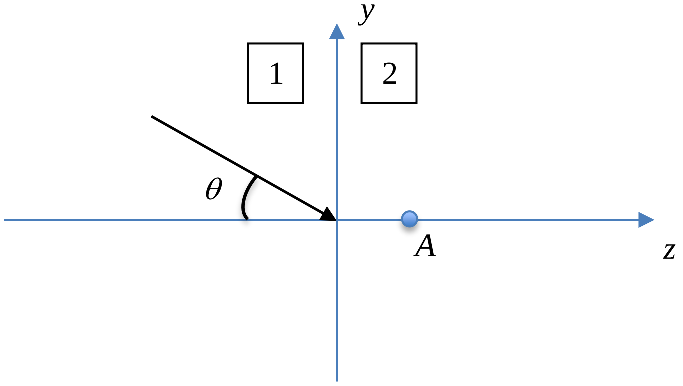
$$P_{RX} = \frac{1}{2} \frac{\left|\vec{E}_{TX} \cos(45°)\right|^2}{\eta_0} e^{-2\times 0.6413\times d} A_{RX} = 0.0184 \text{ W}$$

Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 4 (assume μ_r = 1 for both media) into free space. The expression of the incident electric field is:

$$\vec{E}_i(z,y) = -\vec{\mu}_x\, e^{-j\frac{\sqrt{2}}{2}\beta_1 z} e^{j\frac{\sqrt{2}}{2}\beta_1 y} \text{ V/m}$$

1) What is the polarization of the incident field (specify the details of the polarization)?
2) Determine the value of the electric field in A(z = 1 cm, y = 0 m).



Solution

1) The wave polarization is linear, specifically a TE component (along -x).

2) The incidence angle can be derived, for example, from the y component of β:
$\beta_y = \beta_1 \sin(\theta) = \beta_1\sqrt{2}/2 \rightarrow \sin(\theta) = \sqrt{2}/2 \rightarrow \theta = 45°$

To determine the transmitted wave, it is first necessary to calculate the refraction angle, which is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = \sin^{-1}(\sqrt{2}) \approx \sin^{-1}(1.4142)$$

This is the sign of an evanescent wave: this wave is totally reflected, but the electric field will penetrate in the second medium. The expression of the transmitted field will be:

$\vec{E}_t(z,y) = -\vec{\mu}_x(1+\Gamma_{TE})\, e^{-j\beta_{2z}z} e^{j\beta_{2y}y}$ V/m

As in A, y = 0 m $\rightarrow \vec{E}_t(z, y = 0 \text{ m}) = -\vec{\mu}_x\Gamma_{TE}\, e^{-j\beta_{2z}z}$ V/m

Let us calculate β_{2z}:

$$\beta_{2z} = \beta_2\cos(\theta_2) = \beta_2\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{1-[\sin(\theta_2)]^2} = \beta_0\sqrt{1-\sqrt{2}^2} = \beta_0\sqrt{-1}$$
$$= \pm j\beta_0 = -\,j\beta_0$$

The negative sign is chosen to obtain a physical solution:

$e^{-j\beta_{2z}z} = e^{-j(-\,j\beta_0)z} = e^{-\beta_0 z}$

The reflection coefficient can be calculated as:

$$\eta_1 = \frac{\eta_0}{\cos(\theta)\sqrt{\varepsilon_{r1}}} = 266.6\ \Omega$$

$$\eta_2 = \frac{\eta_0}{\cos(\theta_2)\sqrt{\varepsilon_{r2}}} = \frac{\eta_0}{-j} = j377\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_2}{\eta_2 + \eta_1} = 0.334 + j0.943$$

Therefore:
$\vec{E}_t(A) = \vec{E}_t(z = 0.01\text{ m}, y = 0\text{ m}) = -\vec{\mu}_x(1.334 + j0.943)e^{-0.01\,\beta_0} = -\vec{\mu}_x(0.202\ +\ j0.143)$ V/m

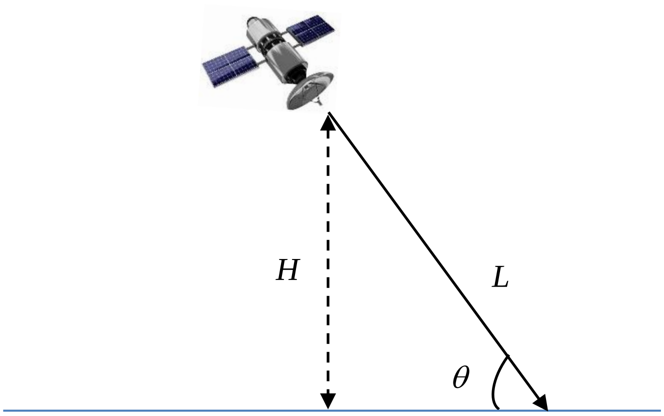
with $\beta_0 = 188.62$ rad/m.

Problem 4

Consider a satellite link, implementing adaptive coding and modulation, which adapts the data rate depending on the link signal to noise ratio (SNR). The link elevation angle is θ = 45° and the link operates at f = 20 GHz. The Complementary Cumulative Distribution Function (CCDF) of the zenithal tropospheric attenuation is given by:

$$P(A^Z_{dB}) = 100e^{-0.545\,A^Z_{dB}} \quad (A^Z_{dB} \text{ in dB and } P \text{ in \%})$$

Determine the yearly time for which 10 Mbit/s can be guaranteed to the user.



15 dB < SNR ≤ 20 dB	D = 20 Mbit/s
10 dB < SNR ≤ 15 dB	D = 10 Mbit/s
SNR ≤ 10 dB	D = 1 Mbit/s

Additional assumptions and data:

- both antennas pointed optimally
- disregard the cosmic background radiation
- power transmitted by each satellite P_T = 100 W
- mean radiating temperature T_{mr} = 290 K
- gain of both antennas G = 35 dB
- satellite altitude H = 600 km
- bandwidth of the receiver: B = 100 MHz
- internal noise temperature of the receiver: T_R = 300 K

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A}{k[T_R + T_{mr}(1 - A)]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = f_R = 1$, $L = H/\sin(\theta)$, $G_R = G_T = G$ (same antenna features). The target data rate is obtained for 10 dB < SNR ≤ 15 dB: to calculate the yearly time for which 10 Mbit/s can be guaranteed to the user, the threshold SNR$_{\min}$ = 10 dB is used. Therefore, the above equation can be solved for A (slant path attenuation in linear scale):

$$A = \frac{SNR_{min} kB(T_R + T_{mr})}{P_T G^2 (\lambda/4\pi L)^2 + SNR_{min} kBT_{mr}} = 0.0041$$

The slant path attenuation in dB is:

$$A_{dB} = -10\log_{10}(A) = 23.9 \text{ dB}$$

The zenithal path attenuation in dB is:

$A_{dB}^{Z} = A_{dB}\sin(\theta) = 16.9$ dB

Using such a value in the CCDF of the tropospheric attenuation:

P = 0.01%, which corresponds to approximately 0.89 hours in a year. Therefore, 10 Mbit/s can be guaranteed for 99.99% of the yearly time, i.e. always but 0.89 hours (53 minutes) in a year.