Radio and Optical Wave Propagation – Prof. L. Luini,
February 17th, 2022

1	2	3	4

do not write above

SURNAME AND NAME ______________________________

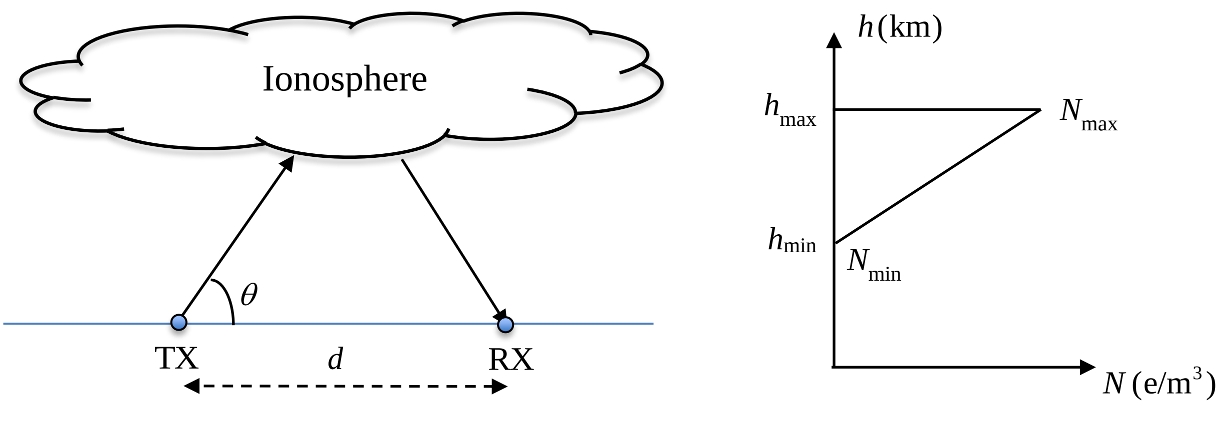
ID NUMBER ______________________________

SIGNATURE ______________________________

Problem 1

Making reference to the figure below, the ionosphere is modelled with the sketched electron density profile, where N_{max} = 4×10^{12} e/m^3, N_{min} = 4×10^{10} e/m^3, h_{min} = 100 km and h_{max} = 300 km. The elevation angle θ is 40° and the distance between TX and RX is d = 286 km.
For this scenario:
1) Determine the TX operational frequency f for the wave to reach RX.
2) Keeping the same elevation angle, what happens if the operational frequency is f_1 = 1 MHz?
3) Keeping the same elevation angle and the frequency determined at point 1), what happens if N_{max} changes to 10^{11} e/m^3.

Assume that the virtual reflection height is 1.2 of the height at which the wave is actually reflected.

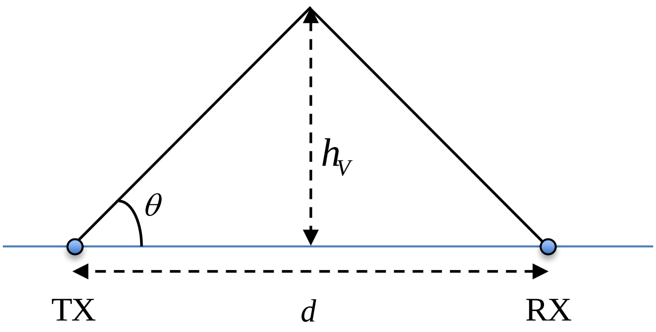


Solution
1) Exploiting the concept of virtual reflection height (see figure below), h_V is given by:

$$h_V = \frac{d}{2}\text{tg}(\theta) = 120 \text{ km}$$

from which, the actual reflection height is:

$$h = \frac{h_V}{1.2} = 100 \text{ km} = h_{\min}$$



For the reflection to occur at h_{min}, the operational frequency is:

$$f = \sqrt{\frac{81N_{\text{min}}}{1-[\cos(\theta)]^2}} \approx 2.8 \text{ MHz}$$

2) For any frequency lower than f (as in the case of f_1), given that elevation angle, the wave will be totally reflected at h_{min}, so RX will still be reached.

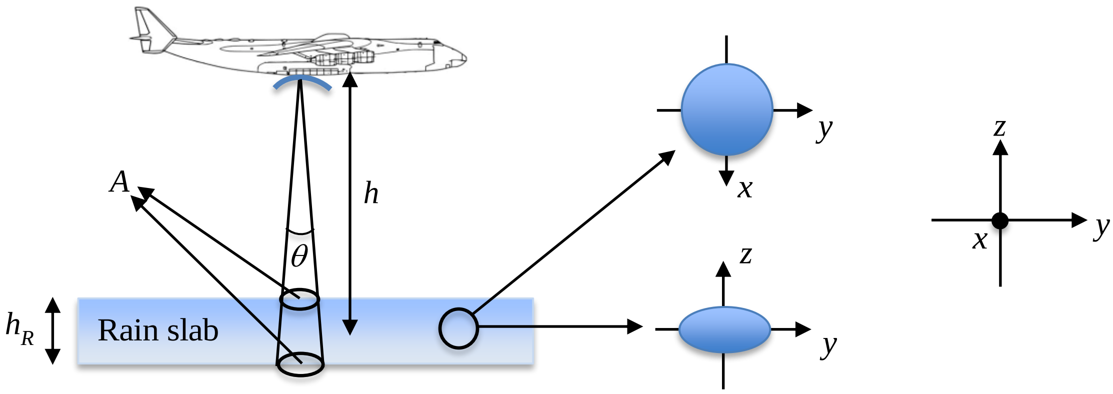
3) As the reflection occurs at h_{min}, the change in N_{max} will not affect the ionospheric link.

Problem 2

Making reference to the figure below, a pulsed radar onboard an airplane, operating with carrier frequency f = 4 GHz and pointed zenithally, is used to measure precipitation. The beam illuminates a volume V filled with rain at distance h = 10 km. The area of the volume is A = 314 m^2 and its height is h_R = 100 m. The rain drops, whose density is N = 150 drops/m^3, are oblate spheroids, all equi-oriented with major axis parallel to the ground. All drops have the same dimension and same backscatter section, i.e. σ = 2 mm^2.

1) Determine the radar transmit power P_T to guarantee that the power received back by the radar, P_R, is higher than 10 μW.
2) What is the best polarization to be used?

Consider the following data: radar antenna gain G = 30 dB; assume no atmospheric attenuation and neglect the cosmic background noise.



Solution

1) First, let us calculate the power density reaching the rain volume:

$$S = \frac{P_T}{4\pi h^2} G f \text{ W/m}^2$$

where G = 10^3, f = 1 (radar pointing to the volume).

The power reirradiated by a single rain drop is (with gain = 1 according to the definition of backscatter section), is:

$$P_d = S\sigma$$

Considering all the drops in the volume and under the assumption of Wide Sense Stationary Uncorrelated Scatterers (based on which we can sum the power reirradiated by the single drops), we obtain:

$$P_t = NAh_R S\sigma$$

Finally, the power received by the radar is:

$$P_R = \frac{P_t}{4\pi h^2} A_E = \frac{P_t}{4\pi h^2} G \frac{\lambda^2}{4\pi}$$

Combining all the equations and rearranging the terms:

$$P_R = \frac{NAh_R \sigma P_T G^2 \lambda^2}{h^4 (4\pi)^3}$$

The equation above can be inverted to obtain P_T:

$$P_T > \frac{P_R h^4 (4\pi)^3}{NAh_R \sigma G^2 \lambda^2} \approx 374.5 \text{ W}$$

2) There is no preferential polarization to be used as the drops have circular section as seen from the impinging wave.

Problem 3

Consider the downlink from a spacecraft to a ground station (zenithal pointing). The link operating frequency is f = 20 GHz. The atmospheric attenuation is only due to gases and clouds; in this condition, determine the maximum LNA noise temperature to guarantee an SNR higher than 12 dB. To this aim, consider the following data:

- the directivity of the both antennas is D = 42 dB and their efficiency is 0.6
- both antennas are optimally pointed
- the power transmitted by the satellite is P_T = 79 W
- the distance between the ground station and the satellite is H = 40000 km
- the antenna equivalent noise temperature is T_A = 90 K and the mean radiating temperature of the medium is T_{mr} = 120 K
- antennas are parabolic reflectors with Cassegrain configuration
- disregard the background cosmic noise
- the system bandwidth is B = 30 MHz

Solution

The wavelength is $\lambda = c/f$ = 0.015 m. The gain of the two antennas is:

$$G = D\eta \approx 9509.3 = 39.8 \text{ dB}$$

The tropospheric attenuation can be inferred from T_A and T_{mr} as:

$$A_{RF} = 1 - \frac{T_A}{T_{mr}} = 0.25 \rightarrow A_{RF} = 6 \text{ dB}$$

Considering all the terms, the received power is:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R A_{RF} \approx 1.6 \text{ pW}$$

being both f_T and f_R equal to 1.

The noise power depends on the total system equivalent noise temperature (no impact from the transmission line/waveguide, given the Cassegrain configuration):

$$T_{sys} = T_A + T_{LNA}$$

The SNR is therefore:

$$SNR = \frac{P_R}{k(T_A + T_{LNA})B} > 12 \text{ dB} = 16$$

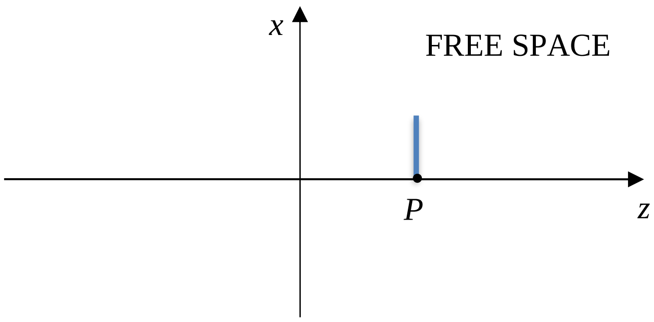
where k is the Boltzmann's constant (1.38×10^{-23} J/K). Inverting the equation $\rightarrow$ T_{LNA} < 150 K.

Problem 4

A uniform plane wave propagates from a medium characterized by ε_{r1} = 4 and μ_{r1} = 4 into free space. The incident electric field is

$$\vec{E}_i = \frac{1}{\sqrt{2}}(\vec{\mu}_x + \vec{\mu}_y)e^{-j335.1z} \text{ V/m}$$

1. What is the polarization of the incident wave?
2. Calculate the frequency of the wave.
3. Determine the wavelength of the wave in the first medium and in the second medium.
4. Calculate the power absorbed by a linear antenna in P(0,0,0.6375): for simplicity assume that the antenna is isotropic with efficiency equal to 1.



Solution

1) The wave polarization is linear, as there is no phase shift between the two components of the electric field along *x* and *y*.

2) The information on the frequency of the wave is embedded in phase constant β = 335.1 rad/m. In medium 1, its expression is:

$$\boldsymbol{\beta = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}\mu_{r1}}}$$

Therefore f = 4 GHz.

3) The wavelength in the first medium is:

$$\lambda = \frac{c}{f\sqrt{\varepsilon_{r1}\mu_{r1}}} = 0.01875 \text{ m}$$

and in free space:

$$\lambda = \frac{c}{f} = 0.075 \text{ m}$$

4) To calculate the power absorbed by the antenna in P, first, let us first calculate the reflection coefficient:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0$$

as:

$$\eta_2 = \eta_0\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} = \eta_1 = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = \eta_0$$

There is total transmission, i.e. the electric field in the second medium is the same as the one in the first medium.

Considering the antenna of the problem, it receives only the component of the field parallel to the antenna itself ($\vec{\mu}_x$). As a result, the power absorbed will be:

$$P = SA_E = \frac{1}{2}\frac{\left|\vec{E}_{tx}\right|^2}{\eta_2}\frac{\lambda_2^2}{4\pi}G \approx 0.3\ \mu\text{W}$$

where G = 1 for an isotropic antenna with efficiency equal to 1.