Radio and Optical Wave Propagation – Prof. L. Luini,
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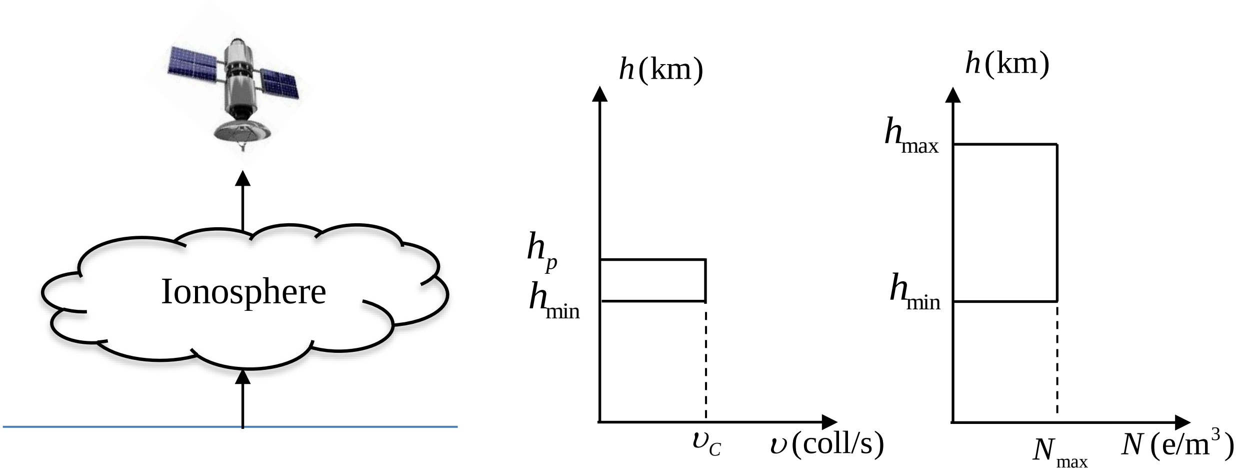
Exercise 1

Making reference to the figure below, a ground station transmits a monochromatic plane wave to a satellite along a zenithal link at f = 20 MHz. The satellite altitude is d = 900 km. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where $h_{\max}$ = 400 km, $h_{\min}$ = 100 km and $N_{max} = 4{\cdot}10^{12}$ e/m^3. The collision frequency is constant from $h_{\min}$ up to h_p = 110 km and its value is $\upsilon_C = 10^5$ collisions/s (left side). Can the satellite be reached? If yes, calculate the power received by the satellite assuming the following features for the system: transmit power of ground station P_T = 1 kW, ground and satellite antenna gain G = 35 dB, antennas optimally pointed, no tropospheric attenuation.



Solution

Let us calculate the critical frequency:

$$f_C \approx 9\sqrt{N_{\max}} = 18 \text{ MHz}$$

In order to properly calculate the power received from the satellite, we need to include in the link budget the attenuation due to the ionosphere. To this aim, let us first calculate the equivalent conductivity of the ionosphere, which is:

$$\sigma = \frac{Ne^2 \nu_C}{m(\nu_C{}^2 + \omega^2)} = 7.2 \cdot 10^{-7} \text{ S/m}$$

where $m = 9 \cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6 \cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{Ne^2}{m\varepsilon_0} = 1.285 \cdot 10^{16} \text{ rad}^2/\text{s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu_C{}^2 + \omega^2} = 0.1862$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 3.15 \cdot 10^{-4} + j0.181 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between $h_{\min}$ and h_p. Therefore

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 2.74 \text{ dB/km}$$

$$A_{dB} = \alpha_{dB}(h_p - h_{\min}) = \alpha_{dB}(110 - 100) = 27.4 \text{ dB}$$

In linear scale: 0.0018.

Therefore the received power is given by

$$P_R = P_T G_T f_T \left(\lambda/4\pi d\right)^2 G_R f_R A_l = 32.7 \text{ } \mu\text{W}$$

where:

$$\lambda = \frac{2\pi}{\beta_0} = 15 \text{ m}$$

Exercise 2

Consider a transmitting antenna at height h from the ground. Considering $f = 5$ GHz, for this setup:

1) Determine the coverage area, and its radius R, for an antenna height of 50 m (assume that the refractivity gradient on the ground is $dN/dh = 0$).
2) Assume now that there is a receiver at distance R mounted on a mast with same height h. Determine whether the first Fresnel's ellipsoid is free.
3) If the answer to point 2) is no, then determine the minimum antenna height to free the first Fresnel's ellipsoid.
   If the answer to point 2) is yes, keeping the same value of R and h of point 1), determine the limit value of the refractivity gradient, which still guarantees that the first Fresnel's ellipsoid is free.

Solution

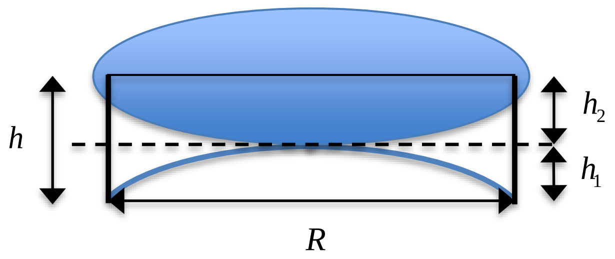
1) The maximum distance covered by the antenna is given by (refractivity gradient zero → equivalent Earth radius factor $k = 1$):

$$R = \sqrt{2hR_{eq}} = \sqrt{2hkR_E} = 25.24 \text{ km}$$

$$A = \pi R^2 \approx 2001 \text{ km}^2$$

2) Making reference to the sketch below, with h is fixed to 50 m, while h_1 is:

$$h_1 = \frac{1}{2}\frac{(R/2)^2}{kR_E} = 12.5 \text{ m}$$



Therefore $h_2 = h - h_1 = 37.5$ m.
The semi-minor axis of the first Fresnel's ellipsoid is:

$a = \sqrt{\lambda R}/2 = 19.5$ m being $\lambda = 0.06$ m at 5 GHz.

As $h_2 > a$, the first Fresnel's ellipsoid is free.

3) The propagation conditions that can lead to problems in the link are those for which the refractivity gradient becomes positive: in this case, the equivalent Earth radius will decrease, the equivalent Earth surface will be more curved, i.e. h_1 will increase and h_2 will decrease. The limit value is that for which $h_2 = a$:

$$h_2 = h - h_1 = h - \frac{1}{2}\frac{(R/2)^2}{kR_E} = a$$

From such equation, k can be derived:

$$k = \frac{1}{2}\frac{(R/2)^2}{(h-a)R_E} = 0.4098$$

As a result

$$\frac{dn}{dh} = \frac{1}{R_E}\left(\frac{1}{k} - 1\right)10^6 \approx 226$$

Exercise 3

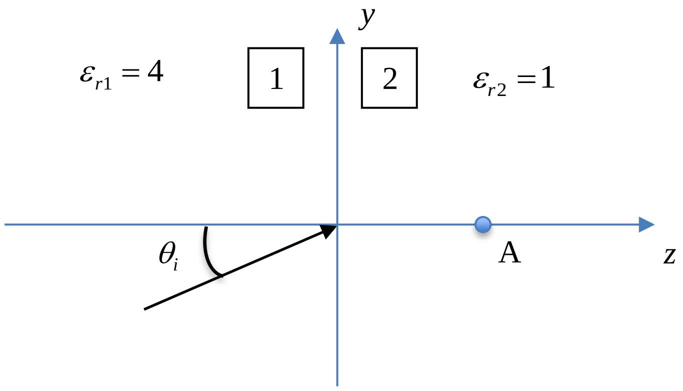
A plane sinusoidal EM wave (f = 5 GHz) propagates from a medium with electric permittivity ε_{r1} = 4 into vacuum with incidence angle θ_i (assume μ_r = 1 for both media). The expression for the electric field is:

$$\vec{E}_i(z,y) = \left(10\cos\theta\,\vec{\mu}_y - 10\sin\theta\,\vec{\mu}_z\right)e^{-j\beta_z z}e^{-j\beta_y y}\ \text{V/m}$$

1) Determine the condition for which there is no reflected wave (if possible)

2) Determine the condition for which there is no refracted wave (if possible)

3) Calculate the power density, along direction z, reaching point A(0,0,2), assuming θ_i = 28°



Solution

1) The considered wave is a TM wave, therefore the condition to have no reflected wave is that the incidence angle is equal to Brewster's angle:

$$\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 26.56°$$

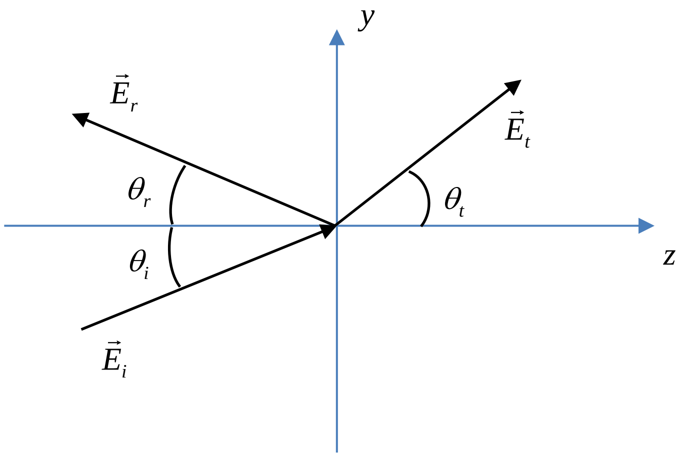
2) Total reflection, i.e. no refracted wave, can occur, both for TE and TM waves, when the second medium is electromagnetically less dense than the first one, which is the case of the present problem. In this case, in fact, we can calculate the critical angle θ_C:

$$\theta_C = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

For $\theta_i < \theta_C$, the wave is partially refracted, for $\theta_i \geq \theta_C$, the wave is totally reflected: the latter is therefore the condition to obtain no refracted wave.

3) As the considered incidence angle is lower than the critical angle, part of the incident wave will be refracted into the second medium. Making reference to the sketch below, let us check the refraction angle:

$$\theta_t = \sin^{-1}\left(\sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 69.87°$$



Let us calculate the intrinsic impedances for the TM wave:

$$\eta_1^{TM} = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}}\cos\theta_i = 166.4\ \Omega$$

$$\eta_2^{TM} = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}}\cos\theta_t = 129.7\ \Omega$$

$$\Gamma^{TM} = \frac{\eta_2^{TM} - \eta_1^{TM}}{\eta_2^{TM} + \eta_1^{TM}} = -0.124$$

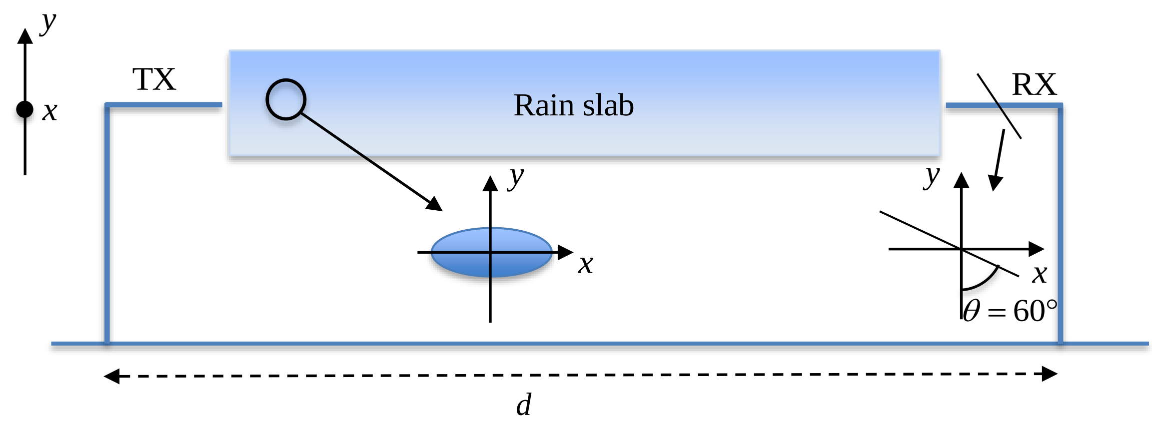
Therefore, the power density, along direction *z*, reaching point A(0,0,2) is given by:

$$S_z^t(A) = S_z^i\left(1 - \left|\Gamma^{TM}\right|^2\right) = \frac{1}{2}\frac{\left|\vec{E}_i\right|^2}{\eta_1}\cos\theta_i\left(1 - \left|\Gamma^{TM}\right|^2\right) = 0.23\ \text{W/m}^2$$

Exercise 4

A terrestrial link, with path length d = 1 km and operating at f = 30 GHz, is subject to rain. As shown in the figure below, rain drops are all horizontally aligned, while RX makes use of a dipole antenna with 60° tilt (see the sketch below). Determine whether TX should use vertical or horizontal linear polarization to maximize the power received by RX.

Assumptions: consider the field as a plane wave, that the Earth is flat and that there are no reflections from the ground. Also assume that the specific attenuation at 30 GHz and 0° elevation angle is $A_{spec} = aR^b$,where a_V = 0.2291, b_V = 0.9129 for vertical polarization, a_H = 0.2403, b_H = 0.9485 for horizontal polarization. Also assume that the rain rate R = 120 mm/h is constant along the path and that the propagation constant β is the same for both V and H.



Solution:

Given the drop alignment, neither the vertical (V) nor the horizontal (H) linearly polarized waves will undergo depolarization.

First we need to calculate the specific attenuation for V and H:

$$A_{spec}^{H} = a_H R^{b_H} = 22.5 \text{ dB/km} \rightarrow \alpha_H = \frac{A_{spec}^{H}}{8.686} = 2.5944 \quad \text{Np/km}$$

$$A_{spec}^{V} = a_V R^{b_V} = 18.1 \text{ dB/km} \rightarrow \alpha_V = \frac{A_{spec}^{V}}{8.686} = 2.0859 \quad \text{Np/m}$$

α_H and α_V are the attenuation constants for the two directions.

If TX uses the vertical polarization, the field received by RX is:

$$E_V = \left|\vec{E}_0\right| e^{-\alpha_V d} \cos\theta = \left|\vec{E}_0\right| 0.0621 \text{ V/m}$$

$$E_H = \left|\vec{E}_0\right| e^{-\alpha_H d} \sin\theta = \left|\vec{E}_0\right| 0.0647 \text{ V/m}$$

As a result, TX should use the horizontal polarization (0.0647 > 0.0621).