Radio and Optical Wave Propagation – Prof. L. Luini,
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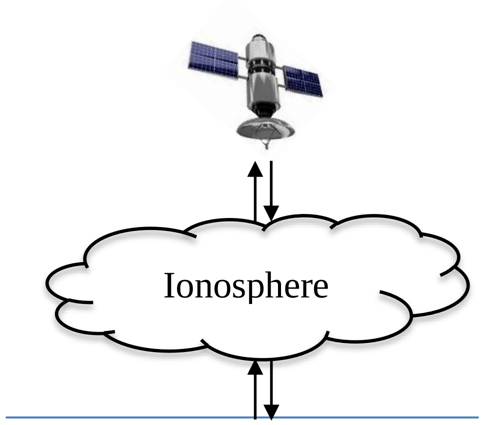
Problem 1

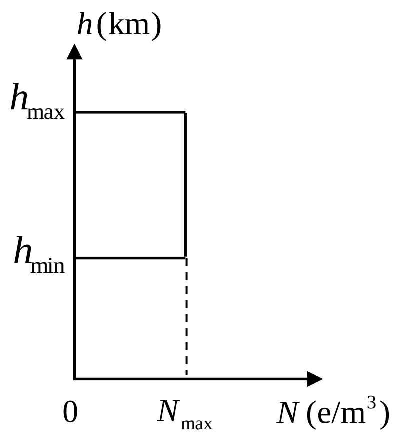
Making reference to the figure below, a satellite, whose altitude is d = 500 km, is used as a radar altimeter. To this aim, the satellite transmits an electromagnetic pulse (zenithal pointing) and the altitude of the ground is measured by knowing the time required for the pulse to reach back the satellite after the reflection on the Earth's surface. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where h_{max} = 470 km, h_{min} = 70 km and $N_{max} = 9\times10^{12}$ e/m^3. For this system:

1) Determine the ratio P_R/P_T (where P_R is the power received back by the radar and P_T is the power transmitted by the radar), when the altimeter operates at 150 MHz.

2) Determine the altimeter accuracy when the radar operates at 20 MHz.

Assumptions: the radar antenna is a reflector with diameter D = 10 m, the antenna efficiency is η = 0.9, the backscatter section of the ground is σ = 100 m^2.





Solution

1) For the radar to work properly, the pulse needs to cross the ionosphere, i.e., for zenithal pointing, the operational frequency needs to be higher than the critical frequency f_C, whose value is:

$$f_C \approx 9\sqrt{N_{\max}} = 27 \text{ MHz}$$

As the operational frequency f = 150 MHz > f_C, the pulse reaches the ground and it is reflected back to the radar. To calculate P_R, let us first calculate the power density reaching the ground:

$$S = \frac{P_T}{4\pi d^2} f_T G_T$$

where:

$$G_T = \frac{4\pi}{\lambda^2}\eta\left(\frac{D}{2}\right)^2 \pi \approx 222 \text{ and } f_T = 1$$

The power reaching back the radar is:

$$P_R = \frac{S\sigma}{4\pi d^2} f_R A_E$$

where:

$$A_E = \eta\left(\frac{D}{2}\right)^2 \pi \approx 70.7 \text{ m}^2 \text{ and } f_R = 1$$

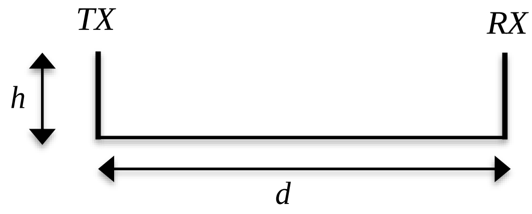
Thus:

$$\frac{P_R}{P_T} = \frac{G_T \sigma A_E}{\left(4\pi d^2\right)^2} = 1.6\times 10^{-19}$$

2) As the operational frequency is lower than f_C, the pulse is completely reflected by the ionosphere and the altimeter does not operate properly.

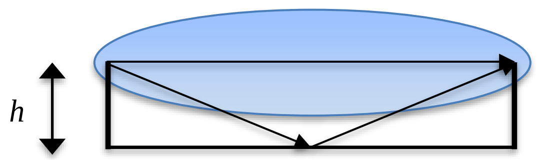
Problem 2

Consider a terrestrial link operating at f = 10 GHz with transmitter TX and receiver RX at distance d = 1 km. Both TX and RX are set at height h from the ground, whose reflection coefficient is Γ = -0.6. For this link, determine the antenna height to maximize the power received at RX. To this aim, assume: flat Earth, no atmospheric impairments.



Solution

The optimum antenna height should: 1) guarantee that first Fresnel's ellipsoid is free; 2) guarantee that the direct and reflected rays combine constructively at the receiver.



The semi-minor axis of the first Fresnel's ellipsoid is:
$a = \sqrt{\lambda d}/2 = 2.74$ m being $\lambda = 0.03$ m.
This is the minimum height to be guaranteed. The two rays combine at the receiver with different phases, depending on h. The total electric field is given by:

$$E = E_0\left(1 + \Gamma e^{-j\beta\delta}\right)$$

where E_0 is the field received from the direct wave and:

$$\beta = {}^{2\pi}/_{\lambda} = 209.44 \text{ 1/m}$$
$$\delta = \frac{2hh}{d}$$

Therefore:

$$E = E_0\left(1 + \Gamma e^{-j\beta\frac{2hh}{d}}\right)$$

In order to achieve the best combination of the rays, we must impose (considering that Γ is negative):
$$e^{-j\beta\frac{2hh}{d}} = -1 \rightarrow -\beta\frac{2hh}{d} = \pi + n2\pi \rightarrow h = \sqrt{-\frac{d}{2\beta}(\pi + 2n\pi)}$$
The first acceptable value of h is obtained for n = -1 → h = 2.74 m = a. This is a limit situation, so n = -2 → h = 4.74 m represents a better more conservative choice.

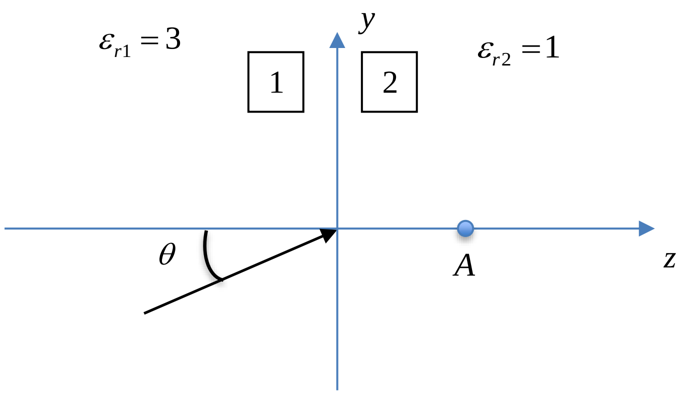
Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 3 into free space (assume μ_r = 1 for both media). There are two possible incident electric fields, whose expressions are:

$$\vec{E}_1(z,y)=\vec{\mu}_x e^{-j\beta\cos\theta z}e^{-j\beta\sin\theta y}\ \text{V/m}$$

$$\vec{E}_2(z,y)=\left(\sin\theta\,\vec{\mu}_z-\cos\theta\,\vec{\mu}_y\right)e^{-j\beta\cos\theta z}e^{-j\beta\sin\theta y}\ \text{V/m}$$

1) Determine which electric field and which θ value should be used to maximize the power received by the antenna located in A(x = 1 m, y = 0 m, z = 1 m)
2) Under the conditions at point 1), calculate the power received in A (the antenna gain is 10 dB).



Solution

1) $\vec{E}_1$ is the electric field of a TE wave, while $\vec{E}_2$ represents a TM wave. Both waves carry the same power (the absolute value of the electric field is 1 V/m in both cases), so the choice depends just on the reflection coefficients. The latter can be zero only for the TM case, specifically when θ corresponds to the Brewster angle:

$$\theta_B=\sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}}\right)=30°$$

Therefore if $\theta=\theta_B$, and we pick $\vec{E}_2$, the wave is totally transmitted, and the power received at A is maximized.

2) Checking Snell's law:

$$\sin\theta_i\sqrt{\varepsilon_{r1}}=\sin\theta_t\sqrt{\varepsilon_{r2}}\quad\Rightarrow\quad\theta_t=\sin^{-1}\left(\sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right)=60°$$

As the reflection coefficient Γ = 0 → the transmission coefficient T = 1. Therefore the power density reaching A will be:

$$S_t^z = S_i^z \rightarrow S_t \cos\theta_t = S_i \cos\theta \rightarrow S_t = \frac{1}{2}\frac{\left|\vec{E}_1\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}}\frac{\cos\theta}{\cos\theta_t} = 4 \text{ mW/m}^2$$

The power received in *A* is:

$$P_R = S_t A_E = S_t \frac{\lambda^2}{4\pi} G = 3.5 \ \mu\text{W}$$

Problem 4

Consider a link from a LEO satellite to a ground station with elevation angle $\theta = 90°$, operating at $f = 30$ GHz. The satellite is used to measure the rain rate from space. The signal-to-noise ratio (SNR) measured at the ground station before the rain event is $SNR_1 = 23$ dB; during the rain event, the SNR drops to $SNR_2 = 9.15$ dB. Assuming that the rain rate is uniform horizontally and vertically, that the rain height is $h = 4$ km and that the specific attenuation due to rain is given by $\alpha = 0.23R^{0.93}$ dB/km.
1) Calculate the power transmitted by the LEO satellite assuming that, in non-rainy conditions, no additional attenuation is induced by the atmosphere.

2) Derive the value of the rain rate affecting the link during the precipitation event.

Additional assumptions and data:

- antennas optimally pointed
- disregard the cosmic background temperature contribution
- mean radiating temperature (rainy case) $T_{mr} = 290$ K
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R = 20$ dB
- altitude of the LEO satellite: $H = 800$ km
- bandwidth of the receiver: $B = 1$ MHz
- internal noise temperature of the receiver: $T_R = 359$ K

Solution

1) In case of no rain, the signal-to-noise ratio (SNR) is given by:

$$SNR_1 = \frac{P_R}{P_N} = \frac{P_T G\left(\lambda/4\pi H\right)^2 G}{kT_R B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). Note that there is no attenuation at all and therefore also the antenna noise temperature is 0 K (according to the assumptions).

By inverting the equation above → $P_T = 100$ W.

2) In case of rain, the SNR becomes

$$SNR_2 = \frac{P_T G\left(\lambda/4\pi H\right)^2 GA}{k\left[T_R + T_{mr}(1-A)\right]B}$$

where A is the rain attenuation (in linear scale), which decreases the received power and increases the receiver noise. Solving for A:

$$A = \frac{SNR_2 kB\left(T_R + T_{mr}\right)}{SNR_2 kT_{mr}B + P_T G^2\left(\lambda/4\pi H\right)^2} = 0.0722$$

In dB:

$A_{dB} = 11.417$ dB

As: $A_{dB} = 0.23R^{0.93}h$ → $R = 15$ mm/h