Radio and Optical Wave Propagation – Prof. L. Luini,
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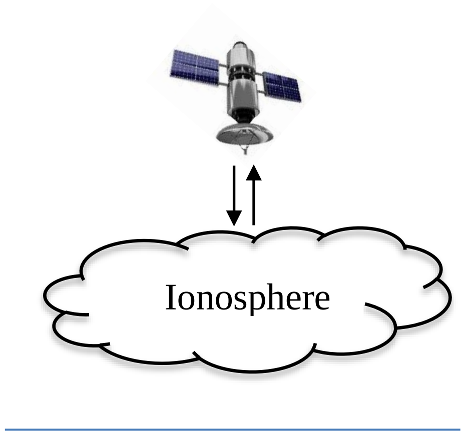
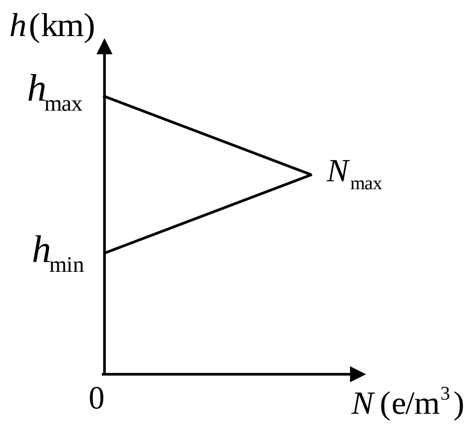
Problem 1

Making reference to the figure below, a Low Earth Orbit (LEO) satellite, pointing to the Earth center, features a radar intended to measure the position of the ionosphere peak electron content. The satellite altitude is $D = 800$ km and the peak electron content is $N_{max} = 6\times10^{12}$ e/m^3. The electron content profile is symmetric and shown in the figure below (right side), where $h_{max} = 400$ km and $h_{min} = 100$ km.

1) Calculate the radar operational frequency f_{LEO} to measure the position of N_{max}.
2) Calculate the error in measuring the position of N_{max} using f_{LEO}.
3) What happens if the new operational frequency becomes $f_1 = 0.9\, f_{LEO}$? (discuss qualitatively)
4) What happens if the new operational frequency becomes $f_2 = 1.1\, f_{LEO}$? (discuss qualitatively)





Solution

The operational frequency of the radar f_{LEO} can be obtained by using the following expression:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{max}}}{f_{LEO}}\right)^2}$$

where θ is the elevation angle, here set to 90°.

Solving for the frequency f_{LEO}, we obtain:

$$f_{LEO} = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta)\right]^2}} = 22.04 \text{ MHz}$$

Using this frequency, the wave will be reflected exactly at $N_{\max}$.

2) As a matter of fact, the ionosphere will slow down any EM wave propagating through it, but the radar will consider the speed of light propagation velocity c to infer the position of the peak electron content. The time required for the pulse to reach the ionosphere peak, T, depends both on the relative distance between the satellite and the peak, but also on the electron content of the ionosphere. Therefore T is:

$$T = \frac{d}{c} + \frac{1}{2c}\frac{81}{f_{LEO}^2}TEC = T_{FS} + T_{IONO}$$

where $d = D - h_p = D - (h_{\min} + (h_{\max} - h_{\min})/2) = 550$ km and TEC is the total electron content, calculated as the integral of N between the peak of the ionosphere and $h_{\max}$. The latter can be simply calculated as:

$$TEC = \int_{h_p}^{h_{\max}} N(h)dh = \frac{N_{\max}\left(h_{\max} - h_p\right)}{2} = 4.5\times10^{17} \text{ e/m}^2$$

Therefore, the ionospheric delay is:

$$T_{IONO} = 1.25\times10^{-4} \text{ s}$$

Considering that the pulse reaches back the radar, the error in estimating the peak position is:

$$\Delta s = 2cT_{IONO} = 37.5 \text{ km}$$

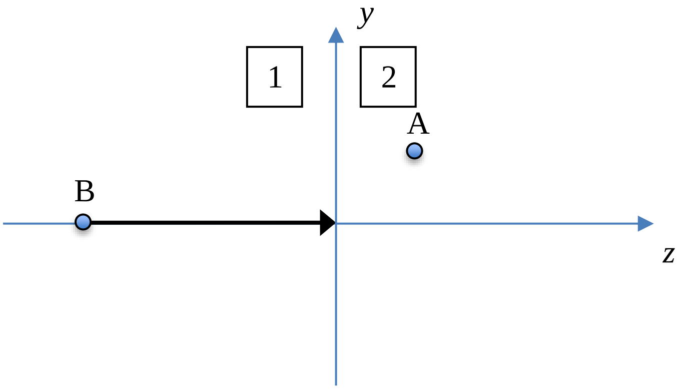
3) For $f < f_{\text{LEO}}$, the pulse will be reflected at a point of the profile between h_p and $h_{\max}$.

4) For $f > f_{\text{LEO}}$, the pulse will cross the peak and will reach the ground.

Problem 2

A plane sinusoidal wave (f = 12 GHz) propagates orthogonally from a medium characterized by ε_{r1} = 2, σ_1 = 0.0134 S/m, μ_{r1} = 1 into a medium characterized by ε_{r2} = 4-j3, μ_{r2} = 1. The incident electric field in B(x = 0 m, y = 0 m, z = -1 m) is $\vec{E}_i(B) = 10\vec{\mu}_y$ V/m. In this context:

1) Calculate the full expression of the electric field in medium 1, i.e. $\vec{E}_i(z) = E_0 e^{-\gamma_1 z}\vec{\mu}_y$.
2) Calculate the value of the magnetic field and of the power density in A(x = 0.1 m, y = 0.01 m, z = 0.1 m).



Solution

1) The plane wave is a TEM wave with vertical polarization. To answer the first question, the propagation constant in the first medium is required. To this aim, it is worth checking the loss tangent:

$$\tan\delta = \frac{\sigma_1}{\omega\varepsilon_{r1}} = 0.01$$

As medium 1 is a good dielectric, the attenuation and phase constants can be simply calculated as:

$$\alpha_1 \approx \frac{\sigma_1}{2}\sqrt{\frac{\mu_1}{\varepsilon_1}} = 1.7848 \text{ Np/m}$$

$$\beta_1 \approx \omega\sqrt{\varepsilon_1\mu_1} = 355.67 \text{ rad/m}$$

Therefore E_0 can be calculated as: $\vec{E}_i(B) = E_0 e^{-\gamma_1 z_B}\vec{\mu}_y = 10\vec{\mu}_y \quad \Rightarrow \quad E_0 = \dfrac{10}{e^{-\gamma_1 z_B}} = \text{-}1.3124 + \text{j}1.0461$ V/m, which yields $|E_0| = 1.6783$ V/m. As expected, due to the lossy properties of medium 1, the amplitude of the electric field decreases moving from B to the interface.

2) To calculate the field in the second medium, we need the reflection coefficient. The intrinsic impedances of the two media (no approximations are possible for the second medium as the loss tangent is Im(ε_{r2})/Re(ε_{r2}) = 0.75) are:

$$\eta_1 \approx \sqrt{\frac{\mu_1}{\varepsilon_1}} = 266.4\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_0\mu_{r2}}{j\omega\varepsilon_0\varepsilon_{r2}}} = 159.8 + j53.3\ \Omega$$

The reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.231 + j0.154$$

The electric field at the boundary between the two media is given by:

$$\vec{E}_t(0) = \vec{E}_i\ (0)\ (1+\Gamma) = \vec{E}_0 \vec{\mu}_y (1+\Gamma) = (-1.171 + j0.603)\vec{\mu}_y \ \text{ V/m}$$

The propagation constant in the second medium is:

$$\gamma_2 = \sqrt{j\omega\mu_2\, j\omega\varepsilon_2} = 177.8 + j533.5 \ \ \text{m}^{-1}$$

The full expression of the electric field in A is:

$$\vec{E}_t(z_A) = \vec{E}_t(0)e^{-\gamma_2 z_A} = \left(2.273 - j1.014\right)10^{-8}\,\vec{\mu}_y \ \text{ V/m}$$

Thus, the full expression of the magnetic field in the second medium is:

$$\vec{H}_t(z_A) = -\frac{\vec{E}_t(z_A)}{\eta_2}\vec{\mu}_x = (-1.09 + j9.98)10^{-11}\,\vec{\mu}_x \ \text{ A/m}$$

The power density in point A is:

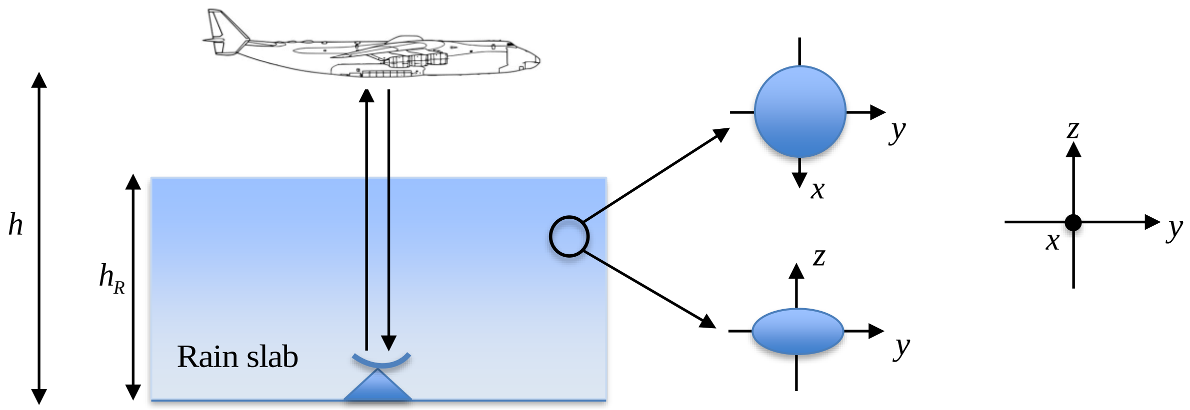
$$S(z_A) = \frac{1}{2}\frac{\left|\vec{E}_t(z_A)\right|^2}{\left|\eta_2\right|}\cos\left(\sphericalangle \eta_2\right) = 1.74\cdot 10^{-18} \ \text{ W/m}^2$$

Problem 3

Making reference to the figure below, a ground-based pulsed radar, operating with carrier frequency of 24 GHz and pointed zenithally, is used to identify aircrafts flying at altitude h = 10 km. The radar operates in all weather conditions, including rain, as depicted below, where a rain slab of constant (both horizontally and vertically) rain rate R is considered. The rain drops are oblate spheroids, all equi-oriented with major axis parallel to the ground and the rain height is h_R = 3 km. In this context:

1) Determine the best linear polarization to be used to minimize the impact of rain on the radar.
2) Calculate the maximum rain rate value tolerated by the radar, which requires a minimum received power P_R of 1 pW to operate correctly.

Consider the following data: radar transmit power P_T = 1 kW; radar antenna gain G = 50 dB; aircraft backscatter section σ = 10 m^2, coefficients for the specific attenuation due to rain: a = 0.1415 and b = 0.9833; no attenuation due to clouds and gases.



Solution

1) Any linear polarization transmitted by the radar will be subject to the same effects induced by the rain drops: in fact, for the geometry indicated in the figure, the wave will see a circular section for the drops (on the xy plane). As a result, no polarization and same specific attenuation, regardless of the chosen linear polarization.

2) First, let us calculate the power density reaching the aircraft:

$$S_A = \frac{P_T}{4\pi h^2} G f A_R$$

where G = 10000, f = 1 (radar pointing to the aircraft) A_R is the rain attenuation in linear scale.
The power reirradiated by the aircraft (with gain = 1 according to the definition of backscatter section), is:

$$P_A = S_A \sigma$$

The power density reaching the radar is:

$$S_R = \frac{P_A}{4\pi h^2} A_R$$

Finally, the power received by the radar is:

$$P_R = S_R A_E$$

where A_E is the equivalent area, expressed as:

$$A_E = G\frac{\lambda^2}{4\pi} = 1.2434\,\mathrm{m}^2$$

Combining all the expressions:

$$P_R = \frac{P_T G\sigma}{\left(4\pi h^2\right)^2} A_R^2 A_E$$

Solving for A_R:

$$A_R = \sqrt{\frac{P_R\left(4\pi h^2\right)^2}{P_T G\sigma A_E}} = 0.0356$$

In dB:

$$A_R^{dB} = -10\log 10(A_R) = 14.48 \text{ dB}$$

The rain attenuation is given by:

$$A_R^{dB} = aR^b h_R$$

Therefore, the maximum rain rate is:

$$R = \left(\frac{A_R^{dB}}{ah_R}\right)^{\frac{1}{b}} = 36.22 \text{ mm/h}$$

Problem 4

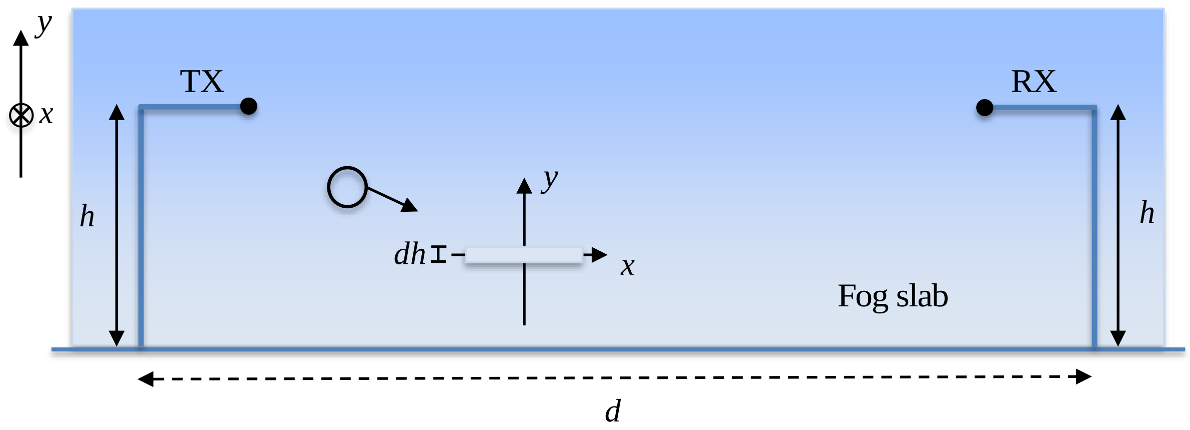
A terrestrial link, with path length d = 9 km and operating at f = 40 GHz, is subject to fog. Both the transmitter (TX) and the receiver (RX) use antennas with horizontal polarization (along x), which are mounted at height h = 7 m from the ground; as shown in the figure below, the fog slab consists only of equi-oriented ice needles, whose thickness dh is negligible.

For this link:

1) Determine the polarization of the wave in front of the receiver RX.
2) Determine if the antenna height h is sufficient to guarantee full visibility for the link; if not, propose the minimum value of h to achieve full visibility.
3) After fixing h according to the requirements in point 2), decrease the distance d to achieve the best combination at RX between the direct ray and the reflected ray.

Assume that:

- for points 2) and 3), the wavelength is the one in free space
- the Earth surface is flat
- the relative permittivity of ice is ε_r = 3.6 and its conductivity is σ = 0
- the ground reflection coefficient is Γ = -0.8+j0.4



Solution

1) The wave emitted by TX is horizontally polarized; as a result, the field will interact with the ice needles, which will simply cause a decrease in the propagation velocity $v = c/\sqrt{\varepsilon_r} \approx 1.58 \cdot 10^8$ m/s: in fact, the ice needles will not induce any attenuation (σ = 0 and ε_r is real), and the wave will not be affected by depolarization because the field lies along one of the symmetry axes of the particle. Therefore, the wave at the RX will still have horizontal polarization.

2) The condition of full visibility is achieved if the first Fresnel's ellipsoid is free. The semi-minor axis of the ellipsoid is given by:

$$a = \frac{\sqrt{\lambda d}}{2} \approx 4.108 \text{ m}$$

where

$$\lambda = \frac{c}{f} = 0.0075 \text{ m}$$

3) The direct and reflected ray will combine at the receiver according to the following expression:

$E = E_0\left(1 + \Gamma e^{-j\beta\delta}\right)$

where E_0 is the field received from the direct wave.

Also:

$\beta = {}^{2\pi}/_{\lambda} = 837.758$ 1/m

$\delta = \frac{2hh}{d} = 10.9$ mm

Therefore

$E = E_0(0.2014 - j0.4028)$ and $|E| = |E_0| 0.4504$ (destructive combination)

In order to maximize the value of E, we need to maximize the factor $\Gamma e^{-j\beta\delta}$, which can be rewritten as follows:

$$\Gamma e^{-j\beta\delta} = |\Gamma| e^{j\sphericalangle\Gamma} e^{-j\beta\delta} = |\Gamma| e^{j(\sphericalangle\Gamma - \beta\delta)}$$

As a result, to maximize this factor, we must impose:

$$e^{j(\sphericalangle\Gamma - \beta\delta)} = 1 \quad \Rightarrow \quad \sphericalangle\Gamma - \beta\delta = n2\pi$$

with n being an integer value. Therefore:

$$d = \frac{2h^2\beta}{\sphericalangle\Gamma - n2\pi} \approx 9.1618 \text{ km}$$

This result is obtained by setting n = -1; in fact, for $n = 0 \rightarrow d > 30$ km, while for $n = 1 \rightarrow d$ is negative. The decrease in d is therefore achieved for n = -1.