Radio and Optical Wave Propagation – Prof. L. Luini,
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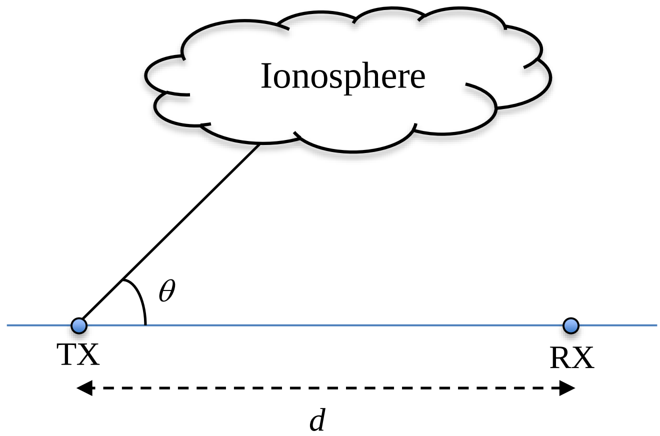
Problem 1

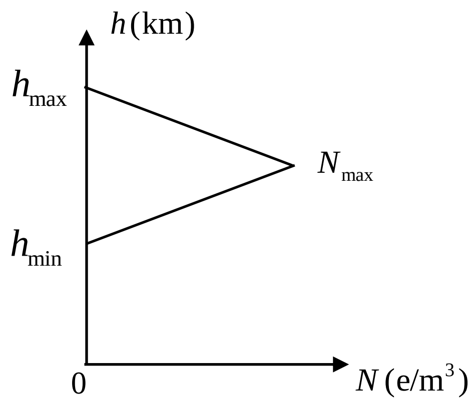
Making reference to the figure below, the transmitter TX, working at f = 18 MHz, reaches the user RX, at a distance d, by exploiting the ionophere. TX transmits with elevation angle θ = 60°. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where h_{max} = 450 km, h_{min} = 100 km and N_{max} = 3×10^{12} e/m^3.

1) Calculate the distance d.

2) Calculate the distance d when θ = 70°.

Assume that the virtual reflection height h_V is 1.2 of the height at which the wave is actually reflected.





Solution:

1) Considering the figure below, given the distance between the TX and RX and the elevation angle:

$$d = \frac{2h_V}{\tan\theta}$$

First we need to find where the reflection occurs, which can be obtained by inverting the following formula:

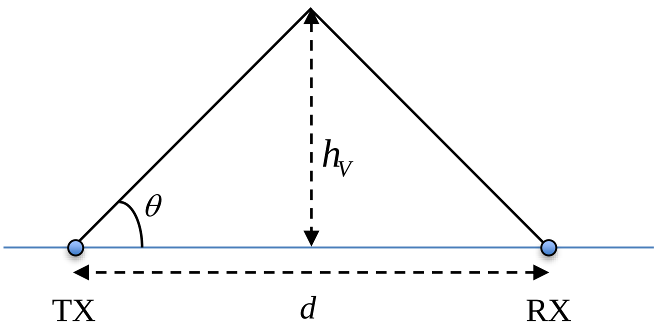
$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N}}{f}\right)^2}$$

This yields:

$$N = \frac{\left[1-(\cos\theta)^2\right]f^2}{81} \approx 3\times10^{12}\ \text{e/m}^3$$

As $N = N_{max}$, the reflection occurs at the height of the profile peak, i.e.:
$h_R = h_{\min} + (h_{\max} - h_{\min})/2 = 275$ km

The virtual reflection height is therefore $h_V = 1.2\ h_R = 330$ km.



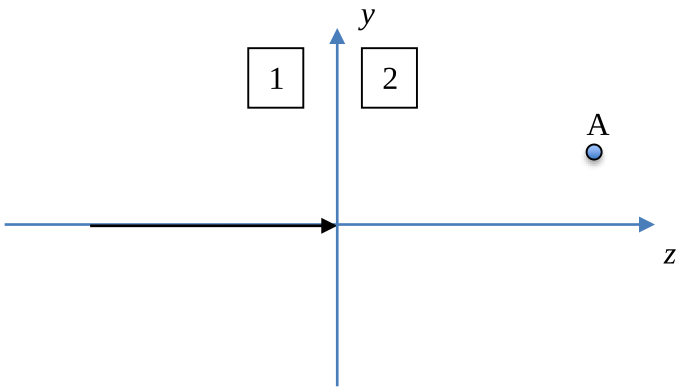
Therefore:

$$d \approx 381\,\text{km}$$

2) When the elevation angle increases beyond 60°, RX cannot be reached because the wave crosses the ionosphere.

Problem 2

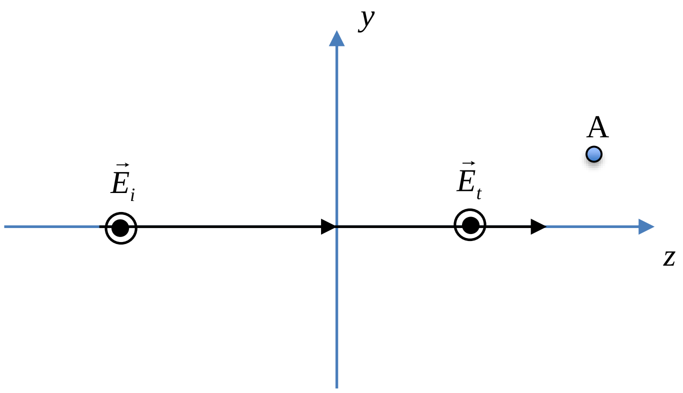
A plane sinusoidal wave at f = 3 GHz propagates in the vacuum and impinges orthogonally on a medium characterized by ε_{r2} = 4, μ_{r2} = 1 and σ_2 = 0.6676 S/m. The electric field in (z = 0 m) is $\vec{E}_0 = -5e^{j2\pi}\vec{\mu}_x$ V/m. Write the full expression of the electric field in the second medium and calculate the power received the parabolic antenna in A(z = 0.2 m, y = 0.2 m, x = 0.2 m) that has the following features: diameter d = 1.6 m, efficiency η = 0.5, same polarization as the wave polarization.



Solution:

The problem concerns a TEM plane wave polarized along -x, i.e.:

$\vec{E}_i = -E_0\vec{\mu}_x e^{-j\beta_1 z} = -5\vec{\mu}_x e^{-j\beta_1 z}$ V/m



The intrinsic impedances of the two media (no approximations are possible for the lossy medium as the loss tangent is equal to 1) are:

$$\eta_1 = \eta_0 = 377\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_0\mu_{r2}}{\sigma + j\omega\varepsilon_0\varepsilon_{r2}}} = 146.3 + j60.6\ \Omega$$

The reflection coefficient is therefore:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.4271 + j0.1647$$

The electric field at the boundary between the two media is given by:

$$\vec{E}_t(0) = \vec{E}_i\ (1+\Gamma) = -5\vec{\mu}_x(0.5783 + j0.1647) = (\text{-}2.8916\ \text{-}\ j0.8233)\vec{\mu}_x\ \text{V/m}$$

The full expression of the electric field in the second medium is:

$$\vec{E}_t(z) = \vec{E}_t(0)e^{-\gamma_2 z} = \vec{E}_t(0)e^{-(\alpha_2 + j\beta_2)z} \text{ V/m}$$

Therefore, for z = 0.2 m (the other coordinates are useless as it is a plane wave):

$$\vec{E}_t(z = 0.2 \text{ m}) = \vec{E}_t(0)e^{-\gamma_2 0.2} = 1.95\times10^{-5} + j2.56\times10^{-5} \text{ V/m}$$

The propagation constant in the second medium is:

$$\gamma_2 = \sqrt{j\omega\mu_2(\sigma + j\omega\varepsilon_2)} = 57.2 + j138.2 \quad \text{m}^{-1}$$

The equivalent area of the antenna is:

$$A_E = \eta\left(\frac{d}{2}\right)^2 \pi = 1 \text{ m}^2$$

The power received by the antenna in A is:

$$P = S_t A_E = \frac{1}{2}\left|\vec{E}_t(z = 0.2 \text{ m})\right|^2 \text{Re}\left\{\frac{1}{\eta_2}\right\} A_E = \frac{1}{2}\frac{\left|\vec{E}_t(z = 0.2 \text{ m})\right|^2}{\left|\eta_2\right|}\cos\left(\angle\eta_2\right) A_E \approx 3 \text{ pW}$$

Problem 3

Given a transmitter for TV broadcasting operating at frequency f = 40 GHz installed on a tower with height h = 30 m, calculate:

1) The radius of the area covered by the transmitter in standard propagation conditions.

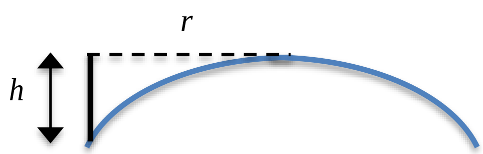
2) The transmit power P_T to guarantee that the power density reaching the user at the end of the coverage area is S_{min} = 1 nW/m^2. To this aim assume: transmit antenna is isotropic; the area is fully covered by fog, which is characterized by the specific attenuation γ = 0.2 dB/km.

Solution:

1) Under standard propagation conditions, the equivalent Earth radius is R_E = 4/3 R_{earth} = 8495 km.

The radius of the area A covered by the antenna is given by the inversion of:

$$h = \frac{1}{2}\frac{r^2}{R_E} \rightarrow r = \sqrt{2hR_E} = 22.6 \text{ km}$$



2) The attenuation along the path from the antenna to the user at the edge of the coverage area is:

$A_{dB} = \gamma r = 4.515$ dB $\rightarrow$ A = 0.3536

Therefore, the power density reaching the edge of the coverage area will be:

$$S = \frac{P_T}{4\pi r^2} A$$

In fact, the gain and directivity function are both 1 for an isotropic antenna.

By imposing $S = S_{min}$ and inverting the expression above, we obtain the transmit power:

$P_T \approx 18.1$ W

Problem 4

Consider a zenithal downlink (elevation angle θ = 90°) from a satellite to a ground station, operating at f = 19 GHz, in which the signal goes through a uniform ice cloud (thickness h = 4 km) consisting of equioriented ice needles. The propagation constants for the ice cloud are γ_H = 0.02+j1.194 km^{-1} and γ_V = 0.02+j1.19557 km^{-1} (V and H are associated to the vertical and horizontal wave polarization, respectively) and they are uniform throughout the whole cloud. Knowing that the satellite transmits a left-end circular polarization (LHCP):
1) Determine the polarization in front of the receiver.

2) Calculate the signal-to-noise ratio of the ground receiver.

Additional data:

- antennas are optimally pointed
- the antenna on the ground receives LHCP waves
- cloud temperature T_{ice} = -2 °C
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 20 dB
- power transmitted by the satellite: P_T = 50 W
- altitude of the satellite: H = 500 km
- bandwidth of the ground receiver: B = 2.5 MHz
- internal noise temperature of the receiver: T_R = 300 K
- receiver waveguide loss L_{WG}^{dB} = 1 dB
- receiver waveguide physical temperature t_{WG} = 20 °C

Solution:

1) A LHCP wave consists of two orthogonal linear components with the same amplitude and a differential phase shift of 90°. Looking at the propagation constants, the ice cloud causes the same amount of attenuation on both components, as expected due to the spherical shape of the droplets. In fact, the real part of such constants is:

$$\alpha = 0.02\,\text{Np/km} = 0.1736\,\text{dB/km}$$

which means that at the receiver the two components will still have the same amplitude.

The cloud also causes a total differential phase shift of:

$$\Delta\beta = \beta_H - \beta_V = \frac{\pi}{2} \text{ rad/km}$$

The total phase shift added by the ice cloud is:

$$\Delta\phi = \Delta\beta h = 2\pi \text{ rad}$$

As a result the total differential phase shift between the two components will be:

$$\Delta\phi_{TOT} = \Delta\phi + \frac{\pi}{2} = \frac{\pi}{2}$$

Therefore at the receiver the two linear components will still produce a LHCP wave.

2) The signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi H\right)^2 G_R f_R A_C L_{WG}}{k\left(T_R + T_A + T_{WG}\right)B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T_A is the antenna noise temperature, T_{WG} is the noise temperature added by the waveguide, $f_R = f_T = 1$ (antenna optimally pointed), A_C is the attenuation induced by the ice cloud and L_{WG} is the waveguide loss; finally $\lambda = c/f = 0.0158$ m.

Calculating the terms in the equation above

- $A_C^{dB} = \alpha h = 0.7 \quad \rightarrow \quad A_C = 0.8521$
- $L_{WG} = 0.7943$
- $T_A = T_{ice}\,(1\text{-}A_C) = 40.1$ K
- $T_{WG} = t_{WG}\,(1\text{-}L_{WG}) = 60.2$ K

we obtain:

SNR = 154.7 = 21.9 dB