Radio and Optical Wave Propagation – Prof. L. Luini,
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SURNAME AND NAME ______________________________

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Problem 1

Consider a terrestrial link of d = 10 km, with both antennas having the same height h, operating at f = 20 GHz:
1) Determine the best propagation conditions (refractivity gradient) allowing to minimize the value of h.
2) Determine the minimum value of h, assuming the same propagation conditions at point 1), to guarantee full visibility between the transmitter and the receiver.

Solution

1) The best propagation conditions to minimize h are given by dN/dh = -155 1/km: in fact, for that specific value, the equivalent Earth radius approaches infinity, which means that the EM rays tend to follow the Earth's curvature. In other words, the Earth appears flat.

2) Given dN/dh = -157 1/km, the condition to guarantee full visibility is that $h > a$, where a is the semi-minor axis of the first Fresnel's ellipsoid, given by:

$a = \sqrt{\lambda d}/2 = 6.12$ m, being $\lambda = 0.015$ m at 20 GHz.

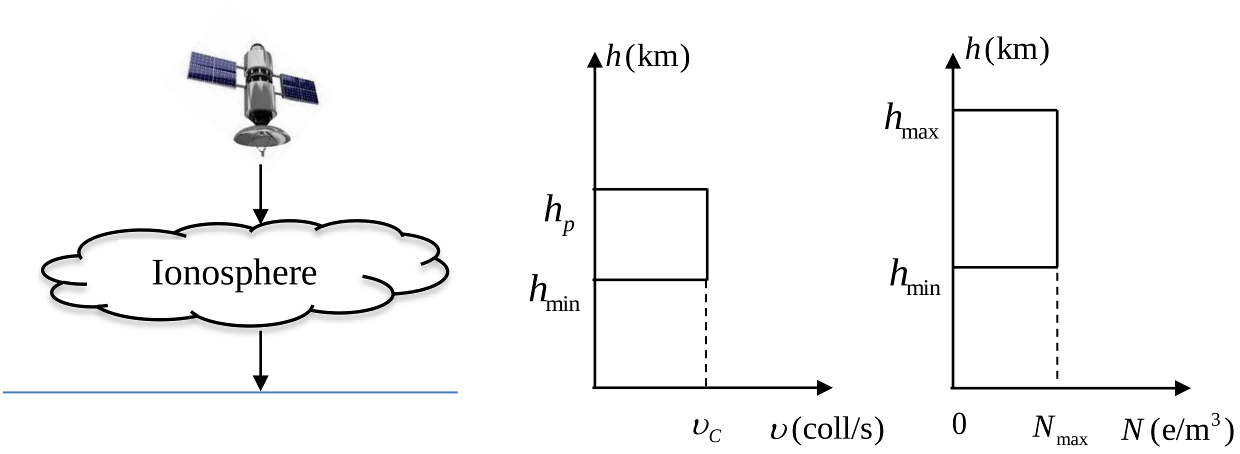
Problem 2

Making reference to the figure below, a satellite transmits Earth observation data to a ground station using two channels centered around the carrier frequencies f_1 = 20 MHz and f_2 = 25 MHz (zenithal link). The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where h_{max} = 450 km and h_{min} = 50 km. The collision frequency is constant from h_{min} up to h_p = 60 km and its value is $\upsilon_C = 10^4$ collisions/s.

1) Determine the satellite altitude H (> h_{max}) and the peak electron content N_{max}, knowing that the data travel time is T_1 = 2.2 ms at f_1 and T_2 = 2 ms at f_2. For this point, disregard the presence of υ_C.

2) Calculate the power received by the ground station for the link operating at f_1. Assume that both antennas (ground and satellite) have a gain of 30 dB and that they are optimally pointed. Also assume that the power transmitted by the satellite is P_T = 10 W.

Assumption: no effects induced by the troposphere (neither delay, nor attenuation).



Solution

1) The total travel time T is due to the free space and to the ionosphere, the latter being frequency dependent. T is defined as:

$$T = T_{FS} + T_{IONO} = \frac{H}{c} + \frac{1}{2c}\frac{81}{f^2}TEC$$

where TEC is the total electron content. As a result, taking the difference between T_1 and T_2:

$$\Delta T = T_1 - T_2 = \frac{81}{2c}TEC\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)$$

$$TEC = \frac{2c}{81}\Delta T \frac{1}{\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)} = 1.646 \cdot 10^{18} \text{ e/m}^2$$

N_{max} can be obtained from:

$$N_{\max} = \frac{TEC}{\left(h_{\max} - h_{\min}\right)} = 4.11 \cdot 10^{12} \text{ e/m}^3$$

Finally, the height of the satellite is given, for example, by:

$$H = c\left(T_1 - \frac{1}{2c}\frac{81}{f_1^2}TEC\right) \approx 493 \text{ km}$$

2) First it is necessary to consider the attenuation induced by the ionosphere.

The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{N_{\max} e^2 \nu_C}{m(\nu_C{}^2 + \omega^2)} = 7.4 \cdot 10^{-8} \text{ S/m}$$

where $m = 9 \cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6 \cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{N_{\max} e^2}{m\varepsilon_0} = 1.32 \cdot 10^{16} \text{ rad}^2\text{/s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu_C{}^2 + \omega^2} = 0.1641$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 3.441 \cdot 10^{-5} + j0.1698 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between $h_{\min}$ and h_p. Therefore

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.298 \text{ dB/km}$$

$$A_{IONO} = \alpha_{dB}(h_p - h_{\min}) \approx 3 \text{ dB} = 0.5$$

The received power by the ground station is calculated as:

$$P_R = P_T G_T f_T \left(\lambda / 4\pi H\right)^2 G_R f_R A_{IONO} = 29.5 \ \mu\text{W}$$

Problem 3

A plane sinusoidal EM wave propagates from a medium with electric permittivity $\varepsilon_{r1} = 9$ into free space (assume $\mu_r = 1$ for both media) with incident angle $\theta_i = 30°$. The expression for the incident electric field is:

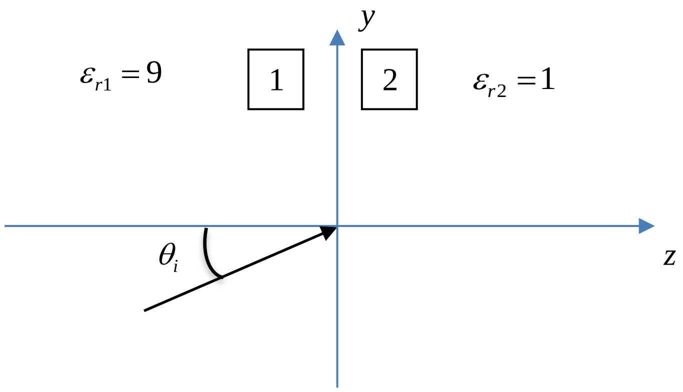
$$\vec{E}(z,y) = \left[2\vec{\mu}_x + j\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right)\right]e^{-j\cos\theta 188.5z}e^{-j\sin\theta 188.5y} \text{ V/m}$$

1) Determine the frequency of the EM wave.

2) Determine the polarization of the incident EM wave.

3) Determine the power density, propagating along *z*, associated to the TE component of the transmitted wave.

4) Answer the question at point 3), but for incident angle $\theta_i = 10°$.



Solution

1) The frequency of the incident EM wave can be derived from the phase constant $\beta = 188.5$ rad/m:

$$\beta = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}} \quad \Rightarrow \quad f = \frac{c\beta}{2\pi\sqrt{\varepsilon_{r1}}} = 3 \text{ GHz}$$

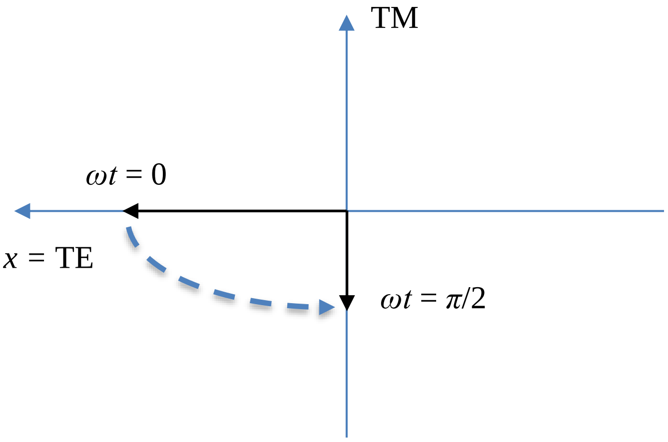
2) The polarization of the incident wave is left-hand elliptical, as the two TE and TM components have different amplitude and a phase shift of $-\pi/2$. In fact, setting *y* and *z* to 0, and expressing the dependence on time, we can easily understand the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re}\left\{\left[2\vec{\mu}_x + j\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right)\right]e^{j\omega t}\right\} = 2\cos(\omega t)\,\vec{\mu}_{TE} + \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TM} \text{ V/m}$$

Thus, for $t = 0$ → $\vec{E}(0,0)\Big|_{\omega t=0} = 2\vec{\mu}_{TE}$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0,0)\Big|_{\omega t=\pi/2} = -\vec{\mu}_{TM}$ V/m

Looking from behind the wave along its propagation direction, we can see the following:



3) Checking Snell's law:

$$\sin\theta_i\sqrt{\varepsilon_{r1}} = \sin\theta_t\sqrt{\varepsilon_{r2}} \quad \Rightarrow \quad \sin\theta_t = \sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = 1.5$$

This is a hint of an evanescent wave in the second medium; therefore, the power density propagating along z in the second medium is zero.

4) Considering the new incident angle:

$$\theta_t = \sin^{-1}\left[\sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right] \approx 31.4°$$

No evanescent wave in this case. The power density can be calculated through the reflection coefficient:

$$\eta_1^{TE} = \frac{\eta_1}{\cos\theta_i} = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}\cos\theta_i} = 127.6\ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_t} = 441\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0.55$$

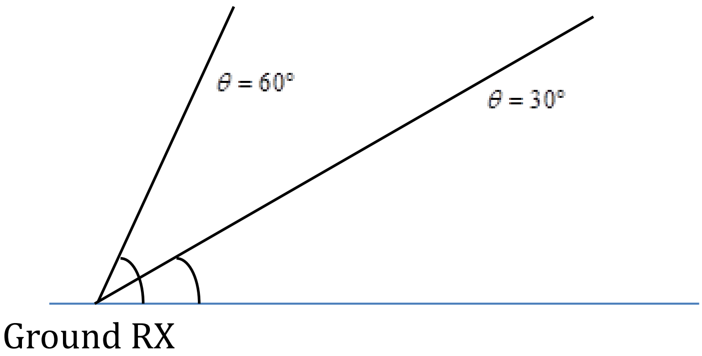
The power density of the transmitted wave along z is:

$$S_{t,TE}^z = S_{i,TE}^z\left(1 - \left|\Gamma^{TE}\right|^2\right) = S_{i,TE}\cos\theta_i\left(1 - \left|\Gamma^{TE}\right|^2\right) = \frac{1}{2}\frac{\left|\vec{E}^{TE}\right|^2}{\eta_1}\cos\theta_i\left(1 - \left|\Gamma^{TE}\right|^2\right) = 10.9\ \ \text{mW/m}^2$$

Problem 4

A ground receiver, whose internal noise temperature is T_R = 300 K, is pointed to a satellite with elevation angle θ = 30°: with this elevation, the antenna noise temperature is T_A = 120 K and the atmospheric attenuation is A = 2.5 dB. Determine the improvement in the *SNR* if the link elevation changes to θ = 60°.

Assumptions: disregard the cosmic background radiation; consider the troposphere to be homogeneous (both horizontally and vertically); consider the same distance D between the satellite and the ground receiver at both elevation angles.



Solution

First, it is necessary to find the mean radiating temperature, T_{mr}, which can be calculated by inverting the following equation:

$$T_A = T_{mr}(1 - A_l) \quad \text{where} \quad A_l = 10^{-\frac{A}{10}} = 0.5623$$

Inverting the equation, we obtain:

$$T_{mr} = \frac{T_A}{1 - A_l} = 274.2 \text{ K}$$

The attenuation, in dB, can be scaled using the sine of the elevation angle, i.e.:

$$A(\theta) = \frac{A_Z}{\sin\theta}$$

Therefore, the zenithal attenuation A_Z is:

$$A_Z = A(30°)\sin(30°) = 1.25 \text{ dB and } A(60°) = A_z / \sin(60°) = 1.44 \text{ dB}$$

In linear scale, A'_l = 0.72. Therefore, the new antenna noise temperature is:

$$T'_A = T_{mr}(1 - A'_l) = 77.53 \text{ K}$$

The target *SNR* at 60° elevation angle is:

$$SNR' = \frac{P_T G_T f_T \left(\lambda / 4\pi D\right)^2 G_R f_R A'_l}{k\left(T_R + T'_A\right) B}$$

Considering that:

$$T_R + T_A = 420 \text{ K} \;\text{ and }\; T_R + T'_A = 377.53 \text{ K} \;\Rightarrow\; T_R + T'_A = 0.899\left(T_R + T_A\right)$$
$$A'_l = 1.28 A_l$$

we can express the target SNR as a function of the *SNR* at 30° elevation:

$$SNR' = \frac{P_T G_T f_T \left(\lambda / 4\pi D\right)^2 G_R f_R A_l}{k\left(T_R + T_A\right) B} \frac{1.28}{0.899} = 1.424\, SNR$$