Radio and Optical Wave Propagation – Prof. L. Luini,
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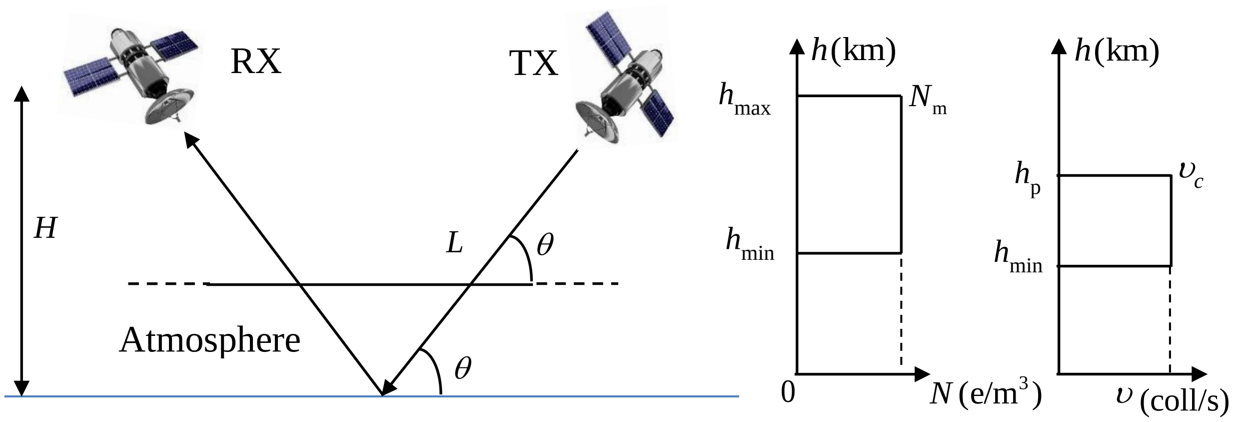
Problem 1

The figure below shows a bistatic radar system consisting of two LEO satellites (same orbit, satellite height above the ground H = 500 km). The radar extracts information on the ground measuring the power received at RX by reflection. Making reference to the right, where the electron content profile ($h_{\min}$ = 50 km, $h_{\max}$ = 400 km, $N_m = 4\times10^{12}$ e/m^3, N homogeneous horizontally) and the collisions frequency profile (h_p = 80 km, $\upsilon_c = 10^4$ coll/s, υ homogeneous horizontally) are shown, and to the left side, where the simplified geometry is reported:

1) Determine the minimum value of θ for the system to avoid reflection from the atmosphere, when the operational frequency is f = 21 MHz.
2) Considering to work with θ calculated at point 1, determine which type of target is on the ground, according to the associated backscatter section, σ: if $\sigma < 100$ m^2 → rural area; if $\sigma \geq 100$ m^2 → urban area.

Additional data: the gain of TX antenna is G = 45 dB, the transmit power is P_T = 100 W, the power density reaching RX is $S_{RX} = 10^{-18}$ W/m^2.



Solution

1) For the wave to avoid total reflection due to the atmosphere (ionosphere at 21 MHz), the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1-\left(\frac{9\sqrt{N_{\mathrm{m}}}}{f_1}\right)^2} \quad \Rightarrow \quad \theta_{min} = 59°$$

2) To calculate correctly the link budget, first it is necessary to consider the attenuation induced by the ionosphere. The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{N_{\mathrm{max}} e^2 \nu_C}{m(\nu_C^2+\omega^2)} = 6.5 \cdot 10^{-8} \text{ S/m}$$

where $m = 9 \cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6 \cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{N_{\mathrm{max}} e^2}{m\varepsilon_0} = 1.285 \cdot 10^{16} \text{ rad}^2/\text{s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu_C^2 + \omega^2} = 0.26$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 2.4 \cdot 10^{-5} + j0.2252 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between h_{min} and h_p, and that the zenith attenuation can be scaled to the slant path using the cosecant law. Therefore:

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.21 \text{ dB/km}$$

$$A_{IONO} = \alpha_{dB}\left(h_p - h_{\mathrm{min}}\right)/\sin(\theta) \approx 7.3 \text{ dB} \rightarrow A_l = 0.186$$

Working at f = 21 MHz, the tropospheric effects can be neglected, but not the ionospheric ones. The power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l = 1.37 \cdot 10^{-7} \text{ W/m}^2$$

where $L = H/\sin(\theta) = 583.3$ km. The power density reaching RX is:

$$S_{\mathrm{RX}} = \frac{S\sigma}{4\pi L^2} A_l$$

Combining both equations:

$$S_{\mathrm{RX}} = \frac{P_T}{4\pi L^2} G A_l \frac{\sigma}{4\pi L^2} A_l = P_T G \sigma \left(\frac{A_l}{4\pi L^2}\right)^2$$

Inverting the equation to solve for σ:

$$\sigma = \frac{S_{\mathrm{RX}}}{G P_T}\left(\frac{4\pi L^2}{A_l}\right)^2 = 167.7 \text{ m}^2 \rightarrow \text{urban area}$$

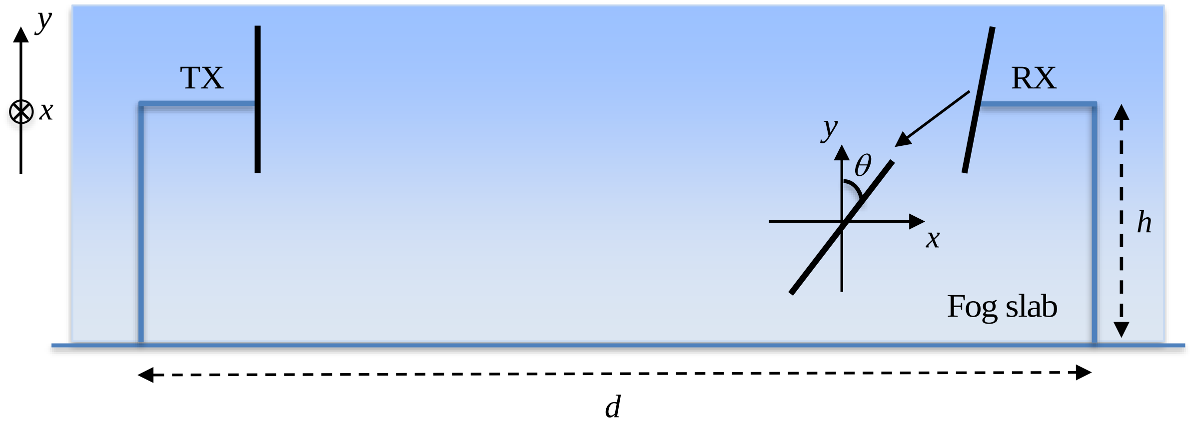
Problem 2

A terrestrial link, with path length d = 5 km and operating at f = 80 GHz, is subject to fog, consisting of spherical droplets. The transmitter (TX) uses a vertical antenna, while the receiver (RX) antenna is tilted by an angle of θ = 30° (see sketch below) due to strong winds. For this link:

1) Determine the height h for the link to operate properly. Use such value of h for points 2 and 3.
2) Determine the polarization of the wave in front of the receiver RX (specify: LH or RH for a circular/elliptical, tilt angle for a linear).
3) Calculate the power received by the RX antenna?

Assume that:

- the Earth surface is flat; disregard the reflection from the ground and the attenuation due to gases; consider plane wave propagation
- use the following data: electric filed emitted at TX, $|\vec{E}_{TX}| = 10$ V/m, effective area of the RX antenna, A_{RX} = 1 m^2; attenuation due to fog, A_f = 3 dB.



Solution

1) For the link to operate properly, the first Fresnel's ellipsoid must be free, i.e.:

$$h = \frac{\sqrt{\lambda d}}{2} = 2.165 \text{ m}$$

2) As fog consists of small spherical droplets, which are isotropic, and the TX antenna is vertical, the polarization reaching RX will still be vertical.

3) To calculate the power received at RX, we must consider the antenna tilt angle and the attenuation due to fog. Considering plane wave propagation, the received power is:

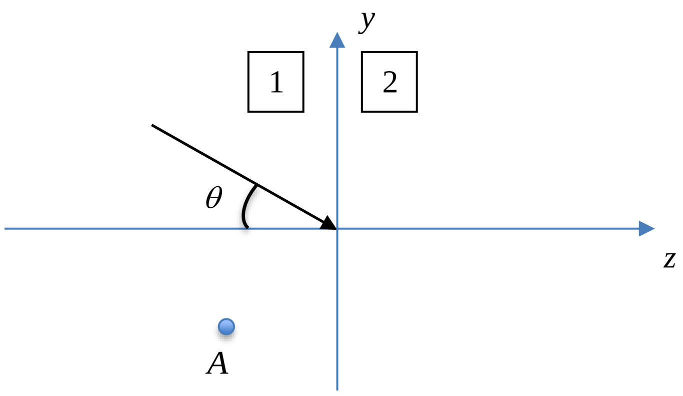
$$P_{RX} = \frac{1}{2}\frac{\left|\vec{E}_{TX}\cos(\theta)\right|^2}{\eta_0} A_{RX} A_{f,lin} \approx 50 \text{ mW}$$

Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 3 into free space (assume μ_r = 1 for both media). The expression of the incident electric field is:

$$\vec{E}_i(z,y) = E_0\left(\frac{\sqrt{3}}{2}\vec{\mu}_y + \frac{1}{2}\vec{\mu}_z + \frac{j}{2}\vec{\mu}_x\right)e^{-j\beta\frac{\sqrt{3}}{2}z}e^{j\frac{\beta}{2}y} \text{ V/m}$$

1) Determine the incidence angle θ.
2) Determine the polarization of the incident field $\vec{E}_i$.
3) Determine the polarization of the reflected field $\vec{E}_r$.
4) Calculate the value of E_0 by knowing that the power density of the electric field in A(x = -1 m, y = -1 m, z = -1 m) is S_T = 0.5 W/m^2 (consider only the contribution of the reflected field).



Solution

1) The incidence angle can be derived, for example, from the y component of β:
$\beta_y = \beta\sin(\theta) = \beta/2 \rightarrow \sin(\theta) = 1/2 \rightarrow \theta = 30°$

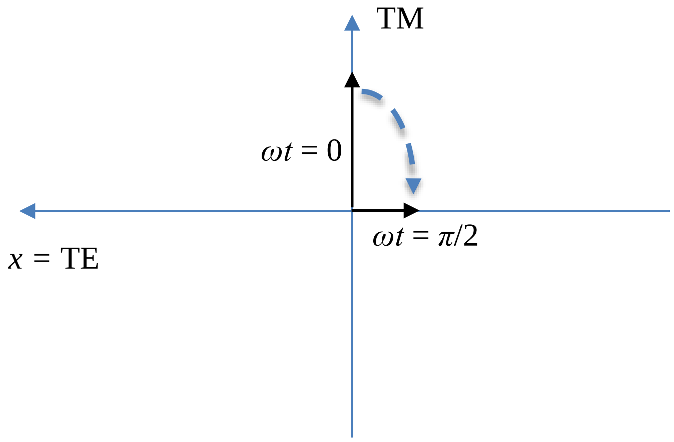
2) The wave has two components: a TE one, along x, and a TM one, given by the combinations of the two fields along y and z. The absolute value of the components is E_0 V/m (TM) and $E_0/2$ V/m (TE), and the differential phase shift is $\pi/2$ → the wave polarization is elliptical. The rotation direction can be calculated considering the two components in time:

$\vec{E}(0,0,t) = \frac{E_0}{2}\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE} + E_0\cos(\omega t)\vec{\mu}_{TM}$ V/m

Thus, for t = 0 → $\vec{E}(0,0,0) = E_0\vec{\mu}_{TM}$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0,0,0) = -\frac{E_0}{2}\vec{\mu}_{TE}$ V/m

The wave is RHEP.



3) Considering that the wave also has a TM component, it is worth checking the Brewster angle:

$$\theta_B = \mathrm{tg}^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

As $\theta = \theta_B$ → the TM component is totally transmitted into the second medium. As a result, the polarization of the reflected wave will be linear (along x).

4) To determine E_0, it is first necessary to calculate the reflection coefficient for the TE component. To this aim, the transmission angle is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 60°$$

The TE reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) - \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})}{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) + \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})} = 0.5$$

The reflected field is therefore given by:

$$\vec{E}_r(z,y) = j\frac{E_0}{2}\Gamma\vec{\mu}_x e^{j\beta\frac{\sqrt{3}}{2}z} e^{j\frac{\beta}{2}y}\ \mathrm{V/m}$$

The power density reaching A will be:

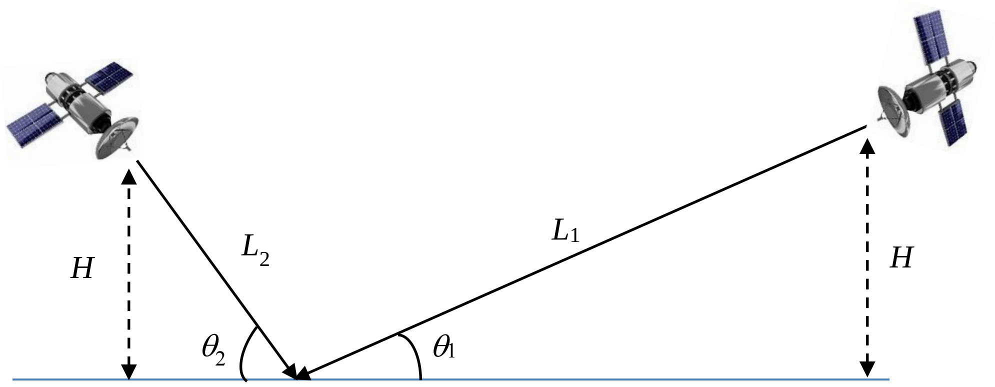
$$S_T = \frac{1}{2}\frac{\left|\vec{E}_r(A)\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}} = \frac{1}{2}\frac{\left|\frac{E_0}{2}\Gamma\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}} = \frac{\Gamma^2 E_0^2\sqrt{\varepsilon_{r1}}}{8\eta_0}$$

Solving for E_0:

E_0 = 59 V/m

Problem 4

Consider a ground station, implementing orbit diversity (i.e. which always selects the satellite with the best SNR): it can be potentially served by two satellites. Satellite 1 (elevation θ_1 = 20°) operates at f_1 = 20 GHz , while satellite 2 (elevation θ_2 = 40°) operates at f_2 = 30 GHz. The ground station is affected by a constant rain rate (rain height h_R = 3 km, both horizontally and vertically) R = 10 mm/h (at 20 GHz, k = 0.0939, α = 1.0199). Determine if the ground station connects to satellite 1 or to satellite 2.



Additional assumptions and data:

- use the simplified geometry depicted above (flat Earth)
- typical frequency scaling for rain attenuation from 20 GHz to 30 GHz
- ground station tracking the satellites optimally
- both satellites can maintain a perfect pointing to the ground station
- power transmitted by each satellite P_T = 100 W
- mean radiating temperature T_{mr} = 290 K
- LEO satellite antenna and ground antenna: parabolic reflectors with diameter D = 0.5 m and efficiency η = 0.6
- altitude of the LEO satellites: H = 800 km
- bandwidth of the receiver: B = 100 MHz
- internal noise temperature of the receiver: T_R = 350 K

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_R}{k[T_R + T_{mr}(1 - A_R) + T_C A_R]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = f_R = 1$, $L = H/\sin(\theta)$, $G_R = G_T$ (same antenna features). Depending on the link, several quantities will change. As for rain attenuation A_R:

Zenith: $A_{R1}^Z = kR^\alpha h_R = 2.94$ dB → Slant: $A_{R1}^S = \frac{A_{R1}^Z}{\sin(\theta_1)} = 8.62$ dB → Linear scale: $A_{R1}^L = 0.1373$

Zenith: $A_{R2}^Z = A_{R1}^Z \left(\frac{f_2}{f_1}\right)^{1.72} = 5.92$ dB → Slant: $A_{R2}^S = \frac{A_{R2}^Z}{\sin(\theta_2)} = 9.22$ dB → Linear scale $A_{R2}^L = 0.1198$

As for the antenna gains, the effective areas of all antennas, at both frequencies, is:

$$A_E = \eta\pi\frac{D^2}{4} = 0.1178\ \mathrm{m}^2$$

The gain of the both the satellite and ground antennas at 20 GHz is:

$$G_1 = \frac{4\pi}{\lambda_1^2}A_E = 6588.7 = 38.2\ \mathrm{dB}$$

The gain of the both the satellite and ground antennas at 30 GHz is:

$$G_2 = \frac{4\pi}{\lambda_2^2}A_E = 14825 = 41.7\ \mathrm{dB}$$

λ_1 and λ_2 are associated to f_1 and f_2, respectively.

Therefore:

$$SNR_1 = \frac{P_T G_1(\lambda_1/4\pi L_1)^2 G_1 A_{R1}^L}{k[T_R + T_{mr}(1 - A_{R1}^L) + T_C A_{R1}^L]B} = 22.72\ \mathrm{dB}$$

$$SNR_2 = \frac{P_T G_2(\lambda_2/4\pi L_2)^2 G_2 A_{R2}^L}{k[T_R + T_{mr}(1 - A_{R2}^L) + T_C A_{R2}^L]B} = 31.1\mathrm{dB}$$

It is more convenient to establish the connection with satellite 2.