Radio and Optical Wave Propagation – Prof. L. Luini,
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Problem 1

Given a transmitter for TV broadcasting operating at frequency f = 15 GHz, installed on a tower, and using an antenna that can be considered isotropic, calculate:

1) The height of the tower, h, to guarantee a circular coverage area A = 1000 km^2; to this aim consider standard propagation conditions.
2) Using the same conditions at point 1), determine the transmit power to guarantee that the signal can be correctly received for 99% of the yearly time by users at the edge of the coverage area (ground level) when the receiver sensitivity is P_R = 0.1 pW and the receiver antenna is isotropic. To this aim consider that the atmospheric attenuation A can be modelled using the following yearly Complementary Cumulative Distribution Function (probability expressed in percentage values, A expressed in dB):

$$P(A) = 100e^{-0.9A}$$

Solution

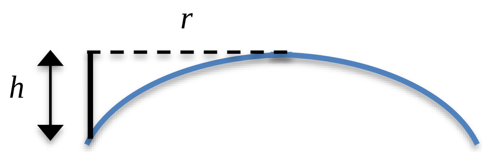
1) Under standard propagation conditions, the equivalent Earth radius is $R_E = 4/3R_{earth}$ = 8495 km.

The radius of the area A covered by the antenna is:

$$r = \sqrt{\frac{A}{\pi}} = 17.84 \text{ km}$$

The antenna height is:

$$h = \frac{1}{2}\frac{r^2}{R_E} = 18.73 \text{ m}$$



2) Using the same conditions as in point 1), first it is necessary to calculate the atmospheric attenuation exceeded for 100-99 = 1% of the yearly time. This is achieved by setting P = 1:

$100e^{-0.9A} = 1$ → A = 5.1 dB = 0.31

The transmit power is determined using the link budget equation:

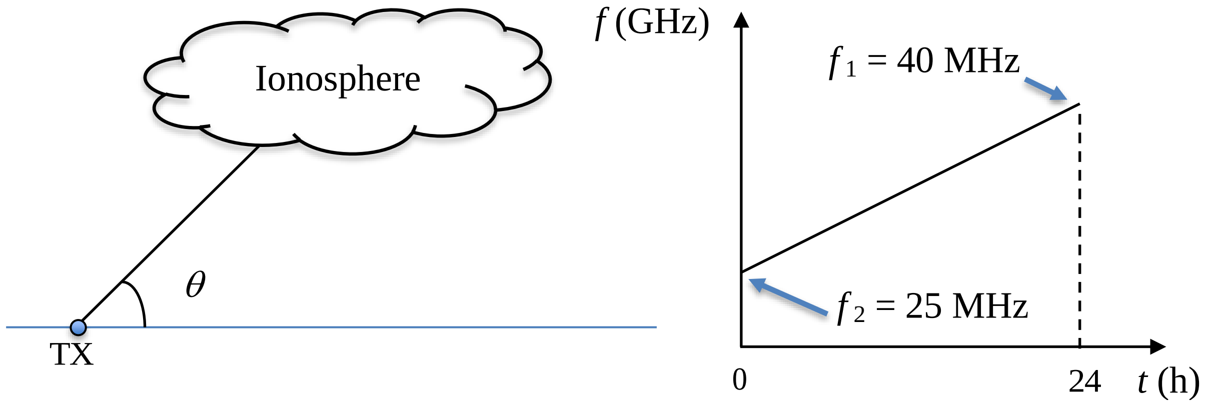
$$P_R = P_T\left(\frac{\lambda}{4\pi r}\right)^2 A$$

Note that, in the equation, neither G (gain) nor f (directivity function) are included, as they are all 1 (isotropic antenna have G = 1 and the directivity function is 1 for any direction). By inverting the equation and by knowing the wavelength $\lambda = c/f$ = 0.02 m:

$$P_T = \frac{P_R}{\left(\frac{\lambda}{4\pi r}\right)^2 A} = 40.7\ \text{W}$$

Problem 2

Making reference to the figure below, a ground station points to a spacecraft with variable frequency f, according to the graph reported below (right side). The elevation angle is fixed to $\theta = 20°$ and the maximum electron content along the profile is $N_{max} = 2\times10^{12}$ e/m^3. Calculate the percentage of time during the day for which the link to the spacecraft cannot be established.



Solution

To determine if the ionosphere can be crossed, we need to use the following equation to find the frequency f, given θ and N_{max}:

$$\cos\theta = \sqrt{1 - \left(\frac{9\sqrt{N_{\max}}}{f^*}\right)^2} \quad \Rightarrow \quad f^* = \frac{9\sqrt{N_{\max}}}{\sin\theta} = 37.2 \text{ MHz}$$

If $f > f^*$, the wave crosses the ionosphere, otherwise it is completely reflected.

The trend of f in the figure is given by (t expressed in hours):

$$f = \frac{f_1 - f_2}{24} t + f_2$$

By imposing that $f > f^*$:

$$\frac{f_1 - f_2}{24} t^* + f_2 > f^* \quad \Rightarrow \quad t^* > \frac{24\left(f^* - f_2\right)}{f_1 - f_2} \approx 19.52 \text{ h}$$

The time percentage of the day for which the link cannot be established is therefore $P = 81.33$ %.

Problem 3

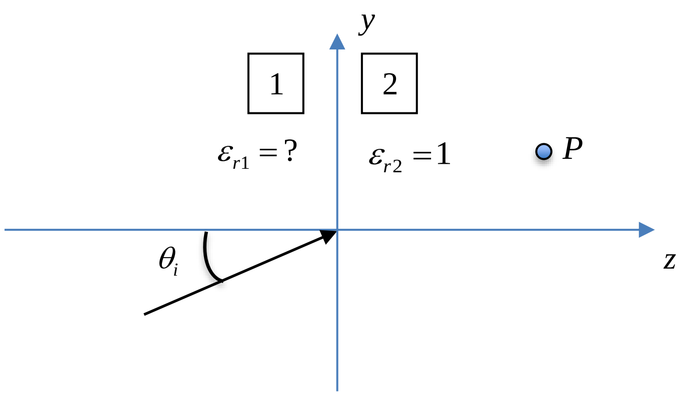
A plane sinusoidal EM wave (frequency f = 5 GHz) propagates from a dielectric medium into vacuum with incidence angle θ_i = 20° (assume μ_r = 1 for both media). The expression for the electric field is:

$$\vec{E}(z,y) = E_0 \vec{\mu}_x e^{-j196.81z} e^{-j71.63y} \text{ V/m}$$

1) Determine the dielectric permittivity ε_{r1} of the first medium.

2) Determine the polarization of the incident EM wave.

3) Calculate the absolute value of E_0 by knowing that the power received by the linear antenna located in P(x = 1 m, y = 1 m, z = 2 m), with effective area A_E = 2 m^2 and parallel to x, is P_R = 5.5 mW.



Solution

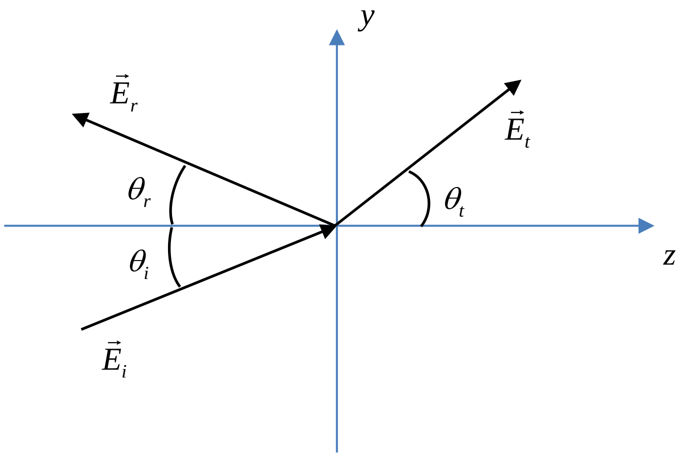
1) The electric permittivity of the first medium can be derived from the propagation constant β, which, in turn, can be determined from the electric field equation. For example:

$$e^{-j71.63y} = e^{-j\beta \sin\theta\, y} \quad \Rightarrow \quad \beta = \frac{71.63}{\sin\theta} = 209.44 \text{ rad/m}$$

$$\beta = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}} \quad \Rightarrow \quad \varepsilon_{r1} = \left(\frac{c\beta}{2\pi f}\right)^2 = 4$$

2) The polarization of the wave is simply linear; specifically, the wave is a TE wave.

3) The incidence of the wave on the discontinuity will give birth to a reflected wave and a transmitted wave.



The transmission angle is given by:

$\sqrt{\varepsilon_{r1}\mu_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\sin\theta_t \rightarrow \theta_t \approx 43.2°$

To find the refracted wave, let us first calculate the intrinsic impedances for the TE wave:

$$\eta_1^{TE} = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}\cos\theta_i} = 200.6\ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}\cos\theta_t} = 516.8\ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0.44$$

The power received by the antenna will be:

$$P_A = S_t A_E = S_i \frac{\cos\theta_i}{\cos\theta_t}\left(1 - \left|\Gamma^{TE}\right|^2\right) A_E = \frac{1}{2}\frac{\left|E_0\right|^2}{\eta_1}\frac{\cos\theta_i}{\cos\theta_t}\left(1 - \left|\Gamma^{TE}\right|^2\right) A_E \quad \text{mW}$$

By inverting the equation above → $\left|E_0\right| = 1$ V/m

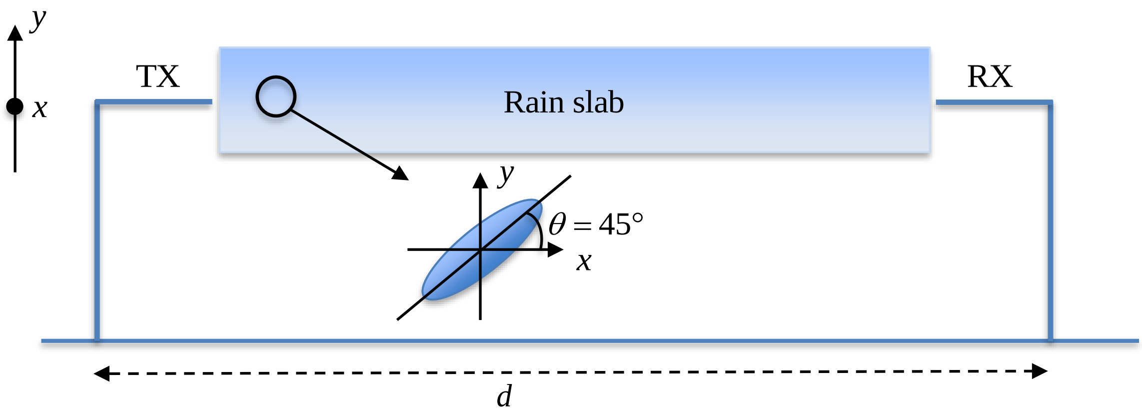
Problem 4

A terrestrial link, with path length d = 10 km and operating at f = 20 GHz, is subject to rain. Both the transmitter (TX) and the receiver (RX) use parabolic antennas having the same polarization; as shown in the figure below, rain drops are tilted 45°. For this link:

1) Determine the best wave polarization to maximize the SNR at RX
2) Assuming that in the condition of point 1) the path attenuation due to rain is A_R = 15 dB, determine the power to be transmitted to guarantee a minimum SNR of 30 dB.
3) Keeping the same hardware and increasing the frequency to 40 GHz, will the SNR increase of decrease?

Assume:

- the gain G of both antennas (parabolic type) is 30 dB
- assume that both antennas are optimally pointed and have no side lobes
- the receiver noise figure is NF = 3 dB
- the physical temperature of rain is T_r = 275 K
- the bandwidth of the system is B = 0.5 GHz
- the same antenna can be used both at 20 and 40 GHz, with the same efficiency η = 0.6



Solution

1) Given the geometry of the drops, the best polarization is the linear one, with 45° degree counter-clockwise tilt, such that the electric field lies along the semi-minor axis of the drop: in this way rain attenuation is minimized and there is no depolarization

2) The transmit power P_T depends on the link budget equation (the wavelength is $\lambda = c/f$ = 0.015 m):

$$SNR = \frac{P_T G \left(\frac{\lambda}{4\pi d}\right)^2 G A_R}{k T_{sys} B}$$

where f_S and f_R are assumed to be 1, as antennas are optimally pointed.

The noise power depends on the total system equivalent noise temperature:

$$T_{sys} = T_A + T_{RX} = T_r\left(1 - A_R\right) + 290\left(10^{\frac{NF}{10}} - 1\right) = 554.9 \text{ K}$$

By inverting the SNR equation, P_T can be determined:

$$P_T = \frac{SNR(kT_{sys}B)}{G^2\left(\frac{\lambda}{4\pi d}\right)^2 A_R} = 8.5 \text{ W}$$

3) The increase in frequencies affects various elements of the link budget. In fact:

- $\lambda = c/f = 0.0075$ m
- The antenna gain: the diameter of the parabolic antenna is (subscript 1 and 2 refer 20 GHz and 40 GHz, respectively)

  $$A_{eff} = \eta A = \frac{\lambda_1^2}{4\pi} G_1 \quad \Rightarrow \quad \eta\left(\frac{D}{2}\right)^2 \pi = \frac{\lambda_1^2}{4\pi} G_1 \quad \Rightarrow \quad D = \frac{\lambda_1}{\pi}\sqrt{\frac{G_1}{\eta}} = 0.1949 \text{ m}$$

  $$G_2 = \eta\left(\frac{\pi D}{\lambda_2}\right)^2 = 4000 = 36 \text{ dB}$$

- Rain attenuation can be scaled up in frequency using the following empirical relationship:

  $$\frac{A_2}{A_1} = \left(\frac{f_2}{f_1}\right)^{1.72} \rightarrow A_2 = 49.4 \text{ dB}$$

- The noise temperature will increase too:

  $$T_{sys} = T_A + T_{RX} = T_r\left(1 - A_R\right) + 290\left(10^{\frac{NF}{10}} - 1\right) = 563.6 \text{ K}$$

Using the same link budget, the new SNR is 1.42.