Radio and Optical Wave Propagation – Prof. L. Luini,
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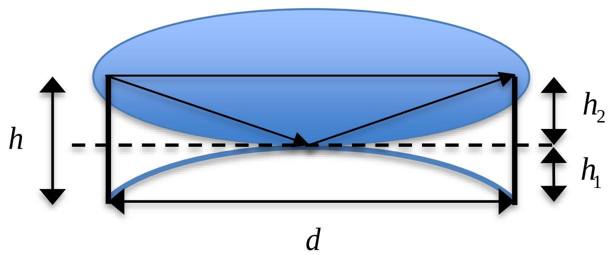
Problem 1

Consider a point-to-point link on the surface of the Moon (no atmosphere, radius of the Moon R = 1737 km). The distance between the transmitter and the receiver is d = 10 km and the operational frequency is f = 30 GHz. Considering the same height h for both antennas:
1) Determine the minimum value of h to guarantee full visibility between the transmitter and the receiver.
2) Determine whether the direct and reflected rays combine constructively or destructively at the receiver. To this aim, assume: plane wave reflection on the Moon surface (i.e. surface is locally flat); horizontal polarization; EM characteristics of the Moon surface: $\varepsilon_r = 4$, $\mu_r = 4$, $\sigma = 0$.

Solution

1) Making reference to the sketch below, the antenna height depends both on the surface curvature (h_1) and on the Fresnel's ellipsoid, which needs to be free to have full visibility (h_2)



The former can be calculated simply as (no atmosphere → no ray bending → no need of equivalent Moon radius):

$$h_1 = \frac{1}{2}\frac{(d/2)^2}{R} = 7.2 \text{ m}$$

The latter is calculated as (a is the semi-minor axis of the first Fresnel's ellipsoid):

$$h_2 = a = \sqrt{\lambda d}/2 = \ 5 \text{ m}$$

The total antenna height is therefore:

$h = h_1 + h_2 = 12.2$ m

2) Defining E_0 as the electric field at the receiver associated to the direct ray, the combination of the direct and reflected rays can be assessed as:

$$E = E_0(1+\Gamma e^{-j\beta\delta}) \text{ V/m}$$

where Γ is the ground reflection coefficient and δ is the differential path, i.e. the difference between the path travelled along the reflected ray and the one travelled along the direct ray. By definition of the Fresnel's ellipsoid → $\delta = \lambda/2$.

Therefore:

$$E = E_0(1+\Gamma e^{-j\beta\delta}) = E_0(1+\Gamma e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{2}}) = E_0(1-\Gamma) \text{ V/m}$$

The reflection coefficient Γ can be calculated using the assumptions. Specifically, labelling free space (no atmosphere on the Moon) as "medium 1" and the Moon surface as "medium 2", we can calculate the intrinsic impedances of both media for the TE wave (for the horizontal polarization, the electric field is completely parallel to the discontinuity). To this aim we first need to calculate the incidence angle θ_i (from the geometry) and refraction angle θ_r (using Snell's law):

$$\theta_i = \tan^{-1}\left(\frac{d}{2h_2}\right) = 89.94°$$

$$\theta_r = \sin^{-1}\left(\frac{\sin\theta_i}{\sqrt{\mu_r\varepsilon_r}}\right) = 14.48°$$

Therefore:

$$\eta_1^{TE} = \frac{\eta_0}{\cos\theta_i} = 3.77\times10^5 \ \Omega$$

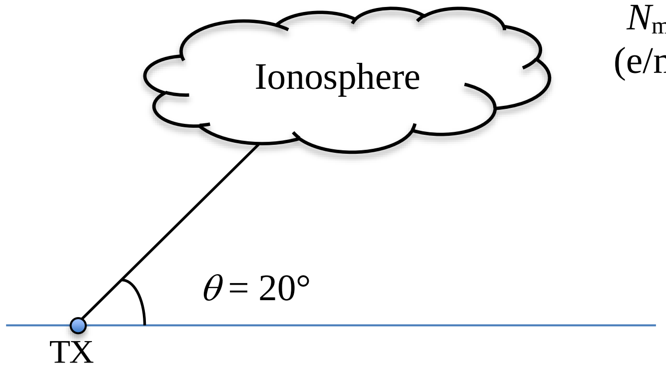
$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_t}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 389 \ \Omega$$

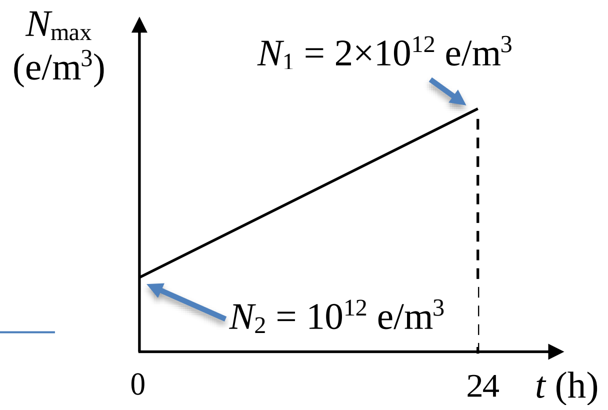
$$\Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.99$$

As a result → $E \approx 2E_0$, i.e. the rays combine constructively.

Problem 2

Making reference to the figure below, a ground station points to a spacecraft (elevation angle θ = 20°) with operational frequency f = 30 MHz. For a specific day, the maximum ionospheric electron content evolves in time as reported below (right side). Calculate the percentage of time during the day for which the link to the spacecraft can be established.





Solution

To determine if the ionosphere can be crossed, we need to invert the following equation, given θ and f:

$$\cos\theta = \sqrt{1 - \left(\frac{9\sqrt{N'_{\max}}}{f}\right)^2} \quad \Rightarrow \quad N'_{\max} = \frac{f^2}{81}\left[1-(\cos\theta)^2\right] = \frac{f^2}{81}(\sin\theta)^2 = 1.3\times10^{12}\ \text{e/m}^3$$

If $N_{\max} < N'_{\max}$, the wave crosses the ionosphere, otherwise it is completely reflected.

The trend of $N_{\max}$ in the figure is given by (t expressed in hours):

$$N_{\max} = \frac{N_1 - N_2}{24} t + N_2$$

By imposing that $N_{max} < N'_{max}$:

$$\frac{N_1 - N_2}{24} t + N_2 < N'_{\max} \quad \Rightarrow \quad t < \frac{24\left(N'_{\max} - N_2\right)}{N_1 - N_2} = 7.2\ \text{h}$$

The time percentage of the day is therefore P = 30 %.

Problem 3

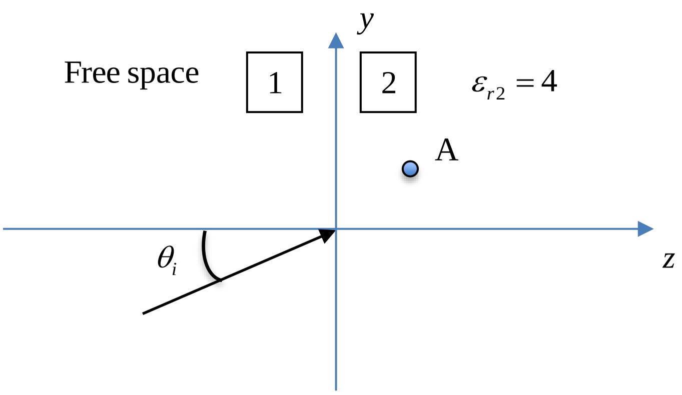
A plane sinusoidal EM wave propagates from free space into a medium with electric permittivity ε_{r2} = 4 (assume μ_r = 1 for both media) with incident angle θ_i. The expression for the incident magnetic field is:

$$\vec{H}_i(z,y) = -\vec{\mu}_x \, e^{-j\cos\theta_i 94.25z} e^{-j\sin\theta_i 94.25y} \text{ mA/m}$$

1) Write the expression of the electric field.

2) Considering a dipole antenna placed in A(1,1,1), determine the incident angle θ_i and the antenna orientation to maximize the power received by the antenna.

3) Calculate the power received by the antenna assuming its gain is G = 3 dB.



Solution

1) The electric field associated to the wave is:

$$\vec{E}_i(z,y) = 0.001\eta_0\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right)e^{-j\cos\theta 94.25z}e^{-j\sin\theta 94.25y} = 0.377\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right)e^{-j\cos\theta 94.25z}e^{-j\sin\theta 94.25y}$$

V/m

The wave frequency is:

$$\beta = \frac{2\pi f}{c} \quad \Rightarrow \quad f = \frac{c\beta}{2\pi} = 4.5 \text{ GHz}$$

2) The wave is a TM wave: if the incident angle is the Brewster's angle, the wave is completely transmitted in the second medium. Such angle is calculated as:

$$\theta_i = \theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 63.4°$$

The refraction angle is given by;

$$\theta_t = \sin^{-1}\left[\sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right] = 26.6°$$

Thus, to maximize the power received in A, the antenna must have the same direction as the electric field in the second medium, which, in turn, is orthogonal to the propagation direction of the wave in the second medium, according to θ_t.

3) The power density of the wave in the second medium is:

$$S_{z,2} = S_{z,1} \quad \Rightarrow \quad S_2 \cos\theta_t = S_1 \cos\theta_i \quad \Rightarrow \quad S_2 = \frac{1}{2}\frac{|E_1|^2}{\eta_1}\frac{\cos\theta_i}{\cos\theta_t} = 9.425\times10^{-5}\ \text{W/m}^2$$

$$P = S_2 A_{eff} = S_2 \frac{\lambda^2}{4\pi} G$$

Considering $G = 3.16$ and $\lambda = \dfrac{c}{f\sqrt{\varepsilon_{r2}}} = 0.0333\ \text{m}$ → $P = 2.63\times10^{-8}\ \text{W}$

Problem 4

A terrestrial link, with path length d = 3 km and operating at f = 30 GHz, is subject to rain. The electric field emitted by TX is polarized along $\vec{\mu}_y$ and, as shown in the figure below, rain drops are horizontally aligned. Both antennas at TX and RX are suited for vertical polarized waves. The atmospheric attenuation for the horizontal polarization, A_H, can be modelled using the following Complementary Cumulative Distribution Function (probability expressed in percentage values, A_H expressed in dB):

$$P(A_H) = 100e^{-0.9A_H}$$

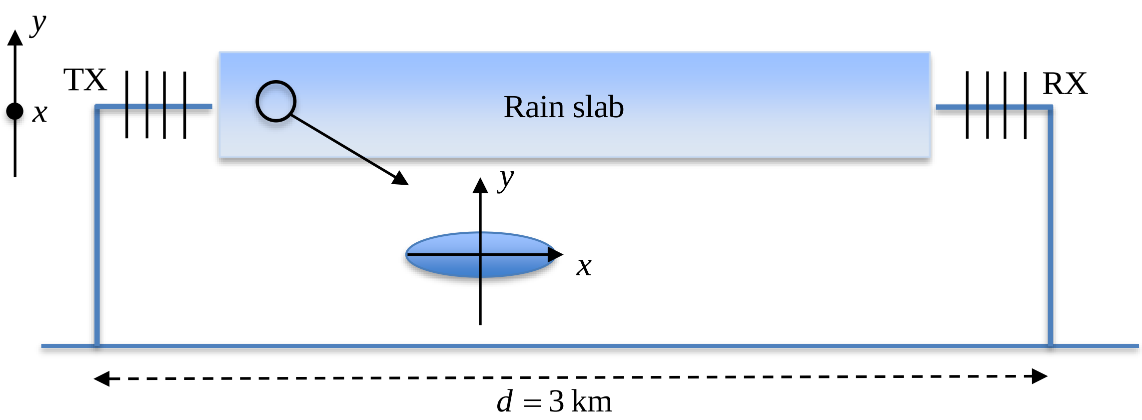
while for the vertical polarization (same units):

$$P(A_V) = 100e^{-1.15A_V}$$

Determine the power to be transmitted by TX to guarantee a minimum *SNR* of 10 dB for 99.999% of the yearly time. Use the following data:

- the gain of both antennas is 10 dB
- assume that both antennas are optimally pointed and have no side lobes
- the receiver equivalent noise temperature is T_R = 120 K
- the physical temperature of rain is T_r = 278 K
- the bandwidth is B = 1 GHz

Assumptions: consider the field as a plane wave, that the Earth is flat, that there are no reflections from the ground.



Solution

The wave polarization is vertical and the antennas are suited to receive that polarization. The electric field is aligned with one of the drop symmetry axis, therefore drops will cause no depolarization. In order to guarantee the target SNR for 99.999% of the time, the atmospheric attenuation needs to be lower than a given value. This value can be determined by setting $P(A_V)$ = 100%-99.999% = 0.001%. As a result, A_V = 10 dB = 0.1. This value can be included in the link budget to determine the transmit power (the wavelength is $\lambda = c/f$ = 0.01 m):

$$SNR = \frac{P_T G \left(\frac{\lambda}{4\pi d}\right)^2 G A_V}{k T_{sys} B}$$

where f_S and f_R are assumed to be 1, as antennas are optimally pointed.

The noise power depends on the total system equivalent noise temperature:

$$T_{sys} = T_A + T_{RX} = T_r\left(1 - A_V\right) + T_{RX} = 370.2 \text{ K}$$

By inverting the SNR equation, P_T can be determined.

$P_T = 72.6$ W