Radio and Optical Wave Propagation – Prof. L. Luini,
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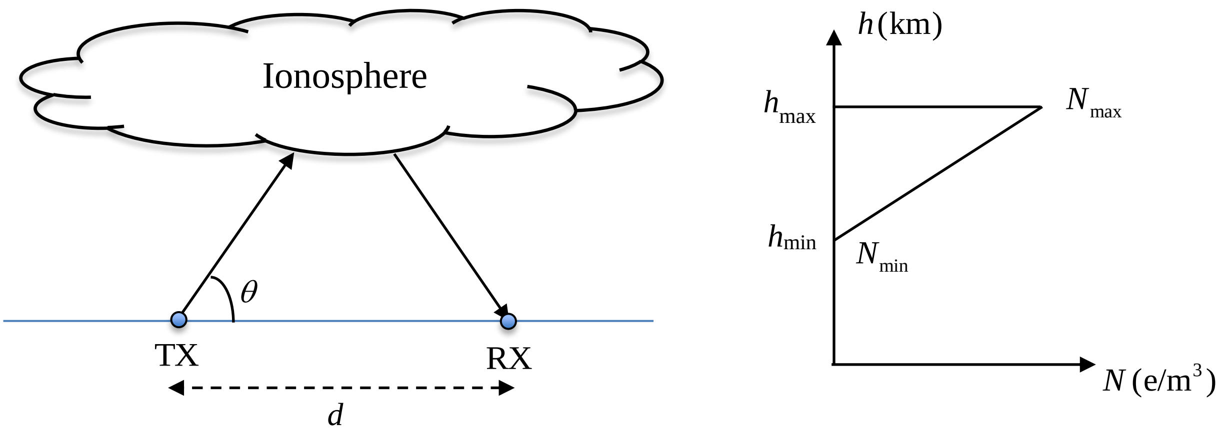
Problem 1

Making reference to the figure below, we want the transmitter TX to reach the user RX at distance d by exploiting the ionosphere (elevation angle θ = 40°). The ionosphere is modelled with the electron density profile sketched in the figure (right side), where $N_{\max} = 6\times10^{12}$ e/m^3, $N_{\min} = 4\times10^{10}$ e/m^3, $h_{\min}$ = 50 km and $h_{\max}$ = 400 km.

1) Determine the distance d to minimize the effects due to the ionosphere on the TX → RX link.
2) Determine the operational frequency f to achieve the conditions at point 1) and to maximize, at the same time, the link data rate.

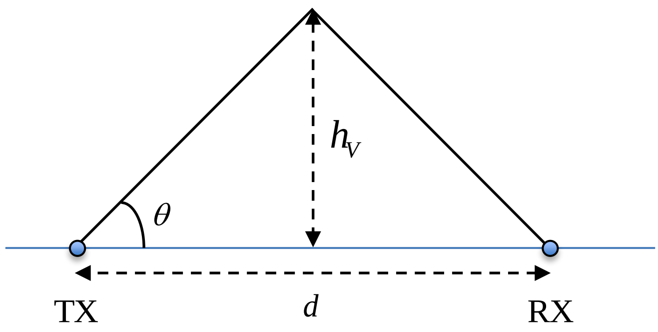
Assume: the virtual reflection height h_V is 1.2 times h_R, the height at which the wave is actually reflected; the Earth is flat.



Solution

1) The effects of the ionosphere (delay, attenuation and depolarization) are minimized if the reflection occurs at N_{min}. Considering the figure below, the distance can be found by inverting the following expression:

$$h_V = 1.2\, h_{\min} = {}^{d}/_{2}\tan\theta \rightarrow d = \frac{2.4\, h_{\min}}{\tan\theta} = 143 \text{ km}$$



2) The link operational frequency f can be determined by inverting the following equation:

$$\cos\theta = \sqrt{1 - \left(\frac{f_P}{f_m}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N_{\min}}}{f_m}\right)^2}$$

Solving for the frequency f_m, we obtain:

$$f_m = \sqrt{\frac{81 N_{\min}}{1 - [\cos(\theta)]^2}} = 2.8 \text{ MHz}$$

Any frequency lower than f_m would induce total reflection, so there is a degree of freedom in choosing the link frequency. On the other hand, the higher the frequency, the higher the data rate. Therefore → $f = f_m$.

Problem 2

Consider a terrestrial link (f = 1 GHz) with path length D = 15 km and with both antennas having the same height h. The yearly propagation conditions are characterized by a random change in the refractivity gradient between -80 km^{-1} to 20 km^{-1}.

1) Determine h for the link to work at best.
2) Using the value of h calculated at point 1), determine if the direct and reflected rays combine constructively or destructively at the receiver, under standard propagation conditions (assume that the ground behaves like a perfect electric conductor).

Solution

1) Making reference to the figure below, the link works at best in full visibility conditions. This is achieved by considering that the first Fresnel's ellipsoid is free. The two conditions corresponds to:

- dN/dh = -80 km^{-1} → marked refraction (rays curved towards the Earth surface)
- dN/dh = 20 km^{-1} → sub refraction (rays curved towards the sky)

Therefore, for the link to operate properly, we need to consider the most critical situation, which is associated to dN/dh = 20 km^{-1}. In this case, the equivalent Earth radius is given by (R_0 = 6371 km is the average Earth radius):

$$R_{eq} = R_0 \left(\frac{1}{1 + R_0 \frac{dN}{dh} 10^{-6}} \right) = 5651 \text{ km}$$

Therefore,

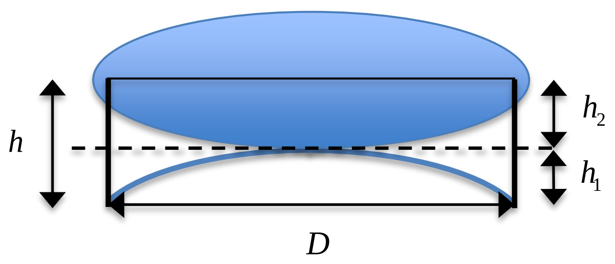
$$h_1 = \frac{1}{2} \frac{(D/2)^2}{R_{eq}} = 4.98 \text{ m}$$

For the first Fresnel's ellipsoid to be free from obstacles ($\lambda = c/f$ = 0.3 m):

$$h_2 = \sqrt{\lambda D}/2 \approx 33.54 \text{ m}$$

Therefore the proper minimum height for the two antennas is:

$h = h_1 + h_2$ = 38.52 m



2) In standard propagation conditions:

$$R_{eq} = R_0 \frac{4}{3} = 8495 \text{ km}$$

Therefore h_1 becomes:

$$h_1 = \frac{1}{2} \frac{(D/2)^2}{R_{eq}} = 3.31 \text{ m}$$

As a result:
$h_2 = h_1 - h = 35.21$ m

The electric field at the receiver is:
$$E = E_0 e^{-j\beta D}\left(1 + \Gamma e^{-j\beta\frac{2h_2h_2}{D}}\right)$$

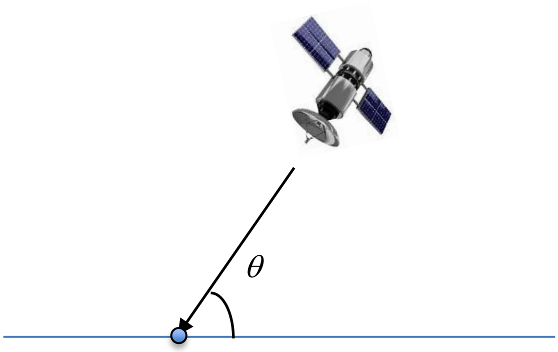
If the ground is assimilated to a perfect electric conductor → $\Gamma = -1$.
The term indicating constructive or destructive conditions is:
$$\left(1 + \Gamma e^{-j\beta\frac{2h_2h_2}{D}}\right) = 1.949\ - j0.315$$

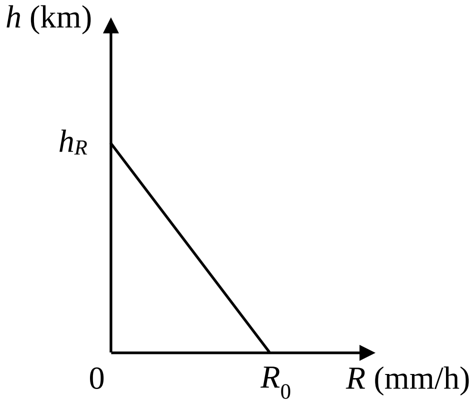
In fact:
$$|E| = |E_0|\left|1 + \Gamma e^{-j\beta\frac{2h_2h_2}{D}}\right| = |E_0|1.97$$

The two rays combine constructively.

Problem 3

Consider a ground station pointing with elevation angle θ = 50° to a GEO satellite (distance d = 38000 km). The satellite transmits at f = 15 GHz and it is under rainy conditions; the rain rate, whose vertical profile is given below (h_R = 5 km, R_0 = 30 mm/h), is constant horizontally.





Determine the power to be transmitted from the satellite to guarantee a minimum signal-to-noise ration SNR = 15 dB at the ground station. Assume both antennas has gain G = 38 dB and that they are optimally pointed. Also assume for the calculation of the specific rain attenuation γ → k = 0.0485 and α = 1. Disregard the other contributions to tropospheric attenuation. Also assume: equivalent noise temperature of the LNA, T_R = 300 K; waveguide loss A_{WG} = 2 dB; physical temperature of the waveguide T_{WG} = 280 K; mean radiating temperature T_{mr} = 290 K, receiver bandwidth B = 20 MHz.

Solution

First, let us calculated the total zenithal rain attenuation:

$$A = \int_0^{h_R} \gamma(h)\, dh = \int_0^{h_R} kR(h)^\alpha\, dh = \int_0^{h_R} k\left(R_0 - \frac{R_0}{h_R}h\right)^\alpha dh\Bigg|_{\alpha=1} = \int_0^{h_R} k\left(R_0 - \frac{R_0}{h_R}h\right)\; dh =$$

$$= kR_0\int_0^{h_R} dh - k\frac{R_0}{h_R}\int_0^{h_R} h\, dh = kR_0 h_R - \frac{kR_0 h_R}{2} = \frac{kR_0 h_R}{2} = 3.64\text{ dB}$$

The attenuation, scaled to the link elevation, and in linear scale is:

$A_{lin} = 10^{-\frac{A}{10}} = 0.3351$

The transmit power can be derived from inverting the SNR equation:

$$SNR = \frac{P_R}{P_N} = \frac{P_T \mathrm{G} f_T (\lambda/4\pi d)^2 \mathrm{G} f_R A_{lin}}{k_B T_{sys} B}$$

where k_B is the Boltzmann constant and the system equivalent noise temperature is:

$$T_{sys} = T_A + T_T + T_R = \left(2.73A_{lin} + T_{mr}(1 - A_{lin})\right) + T_{WG}\left(1 - A_{lin}^{WG}\right) + T_R = 597\text{ K}$$

Inverting the equation above, and setting SNR = 15 dB → $P_T \approx 223$ W.

Problem 4

A uniform sinusoidal plane wave propagates in a medium characterized by relative electric permittivity ε_r = 1, magnetic permeability μ_r = 9 and conductivity σ = 0.1 S/m. The expression of the electric field is (E_0 = 1 V/m):

$$\vec{E}(z,t) = E_0\ e^{-\alpha z}\cos(2\pi 10^9 t - \beta z)\vec{\mu}_y \text{V/m}$$

For such a wave, calculate the power received by an isotropic antenna located at P(0.1λ,0.1λ,0.1λ), which has efficiency η_A = 0.9.

Solution

Let us first check the loss tangent for the wave:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon} \approx 1.8$$

No approximations can be applied; therefore, the propagation constant is calculated as:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 45.7\ + j77.7\ 1/\text{m}$$

The wavelength is given by:

$$\lambda = \frac{2\pi}{\beta} = 0.0808\ \text{m}$$

Therefore P is in (0.00808 m, 0.00808 m, 0.00808 m).

The power received at P by the antenna is:

$$P_R = SA_e = \frac{1}{2}\frac{\left|\vec{E}(P)\right|^2}{|\eta|}\cos(\measuredangle\eta)\ A_e = \frac{1}{2}\frac{|E_0|^2}{|\eta|}e^{-2\alpha z_P}\cos(\measuredangle\eta)\ \frac{\lambda^2}{4\pi}D\eta_A$$

where:

D = 1 (isotropic antenna with directivity 1)

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = 679\ + j\ 399\ \Omega$$

Therefore:

P_R = 0.12 μW