Satellite Communication and Positioning Systems – Prof. L. Luini
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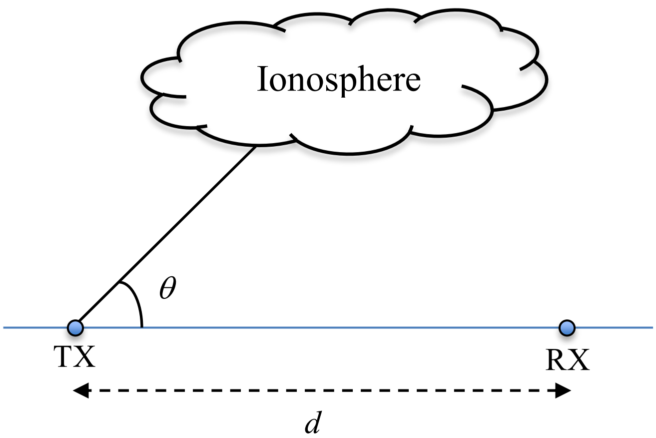
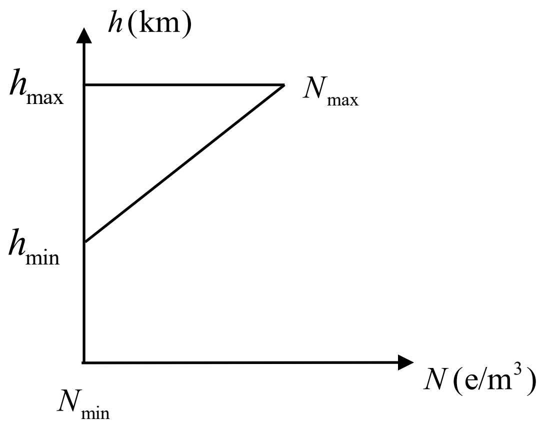
Problem 1

The transmitter TX aims at communicating with RX by exploiting the ionosphere. The ionosphere can be modelled with the symmetric electron density profile sketched in the figure (right side), where $h_{\max}$ = 400 km, $h_{\min}$ = 100 km, $N_{max} = 4\times10^{12}$ e/m^3 and $N_{min} = 10^{11}$ e/m^3. The elevation angle θ can vary between $\theta_1 = 80°$ and $\theta_2 = 20°$.

1) Determine the minimum (d_1) and maximum (d_2) distances between TX and RX covered by the ionospheric link.
2) Calculate the operational frequency range associated to d_1 and d_2.
3) What is the best polarization to be used for the users positioned at d_1 and d_2?

Assume: the virtual reflection height h_V is 1.1 of the height at which the wave is actually reflected.





Solution

1) Considering the figure below, the distance between the TX and RX is given by:

$$d = 2\frac{h_V}{\tan\theta}$$

Considering the actual reflection ($h_R = h_V/1.1$):

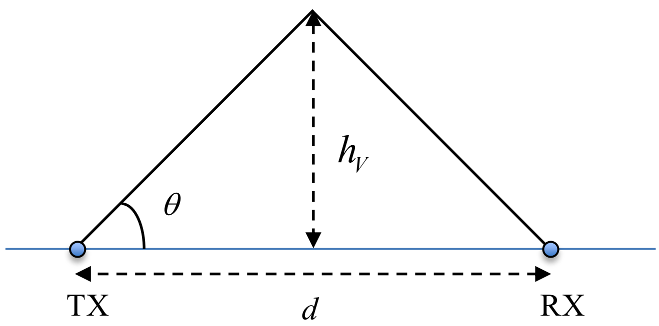
$$d = 2.2\frac{h_R}{\tan\theta}$$

The minimum distance d_1 is associated to the highest elevation angle θ_1 and to the lowest reflection point (h_{min}):

$$d_1 = 2.2 \frac{h_{min}}{\tan\theta_1} = 38.8 \text{ km}$$

The maximum distance d_2 is associated to the lowest elevation angle θ_2 and to the highest reflection point (h_{max}):

$$d_2 = 2.2 \frac{h_{max}}{\tan\theta_2} = 2417.8 \text{ km}$$



2) The relationship between the operational frequency, the electron content in the ionosphere and the geometry of the link is:

$$\cos\theta = \sqrt{1 - \left(\frac{f_P}{f}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N}}{f}\right)^2}$$

The user at distance d_1 is associated with the following frequency:

$$f_1 = \sqrt{\frac{81 N_{min}}{1 - (\cos\theta_1)^2}} = 2.89 \text{ MHz}$$

The user at distance d_2 is associated with the following frequency:

$$f_2 = \sqrt{\frac{81 N_{max}}{1 - (\cos\theta_2)^2}} = 52.63 \text{ MHz}$$

3) For the user in d_1, as the wave does not propagate in the ionosphere, any polarization can be used. For the user in d_2, as the wave propagates in the ionosphere, it is potentially subject to Faraday rotation; however, Faraday rotation affects only linearly polarized waves: the best polarization is RHCP or LHCP.

Problem 2

Referring to the figure below, a plane EM wave at 30 GHz, propagating along the $-\vec{\mu}_z$ direction, crosses a layer of anisotropic particles (layer thickness h = 1 km), which is characterized by the following propagation constants:

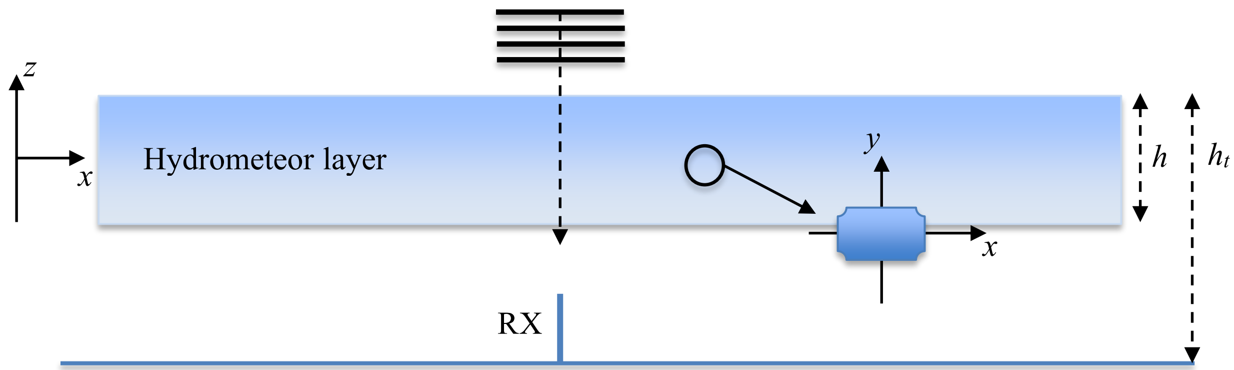
$\gamma_y = 0.6413 + j5658.7 = \alpha_y + j\beta_y$ 1/km

$\gamma_x = 2.9439 + j5658.7 = \alpha_x + j\beta_x$ 1/km

The electric field at the top of the layer is

$$\vec{E}(h_t) = \vec{\mu}_x + 0.1e^{j\frac{\pi}{2}}\vec{\mu}_y \text{ V/m}$$

Determine the best antenna at RX to maximize the received power.



Solution

1) First, it is necessary to determine the wave polarization at the top of the layer. In the space-time domain:

$$\vec{E}(h_t) = \cos(\omega t)\vec{\mu}_x + 0.1\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_y = E_x\vec{\mu}_x + E_y\vec{\mu}_y \text{ V/m}$$

For $\omega t = 0$ → $E_x = 1$ V/m e $E_y = 0$ V/m

For $\omega t = \frac{\pi}{2}$ → $E_x = 0$ V/m e $E_y = -0.1$ V/m

Considering the electric field rotation direction and the differential amplitude for E_x and E_y, the wave has a RHEP.

2) After crossing the anisotropic layer, the wave is depolarized. As the phase constants along x and y are the same, the depolarization is totally ascribable to the differential attenuation.

The absolute value of the y component after the layer is:

$$|E_y(h_t - h)| = |E_y(h_t)|e^{-\alpha_y h} = 0.1e^{-\alpha_y h} = 0.0527 \text{ V/m}$$

The absolute value of the x component after the layer is:

$$|E_x(h_t - h)| = |E_x(h_t)|e^{-\alpha_x h} = e^{-\alpha_x h} = 0.0527 \text{ V/m}$$

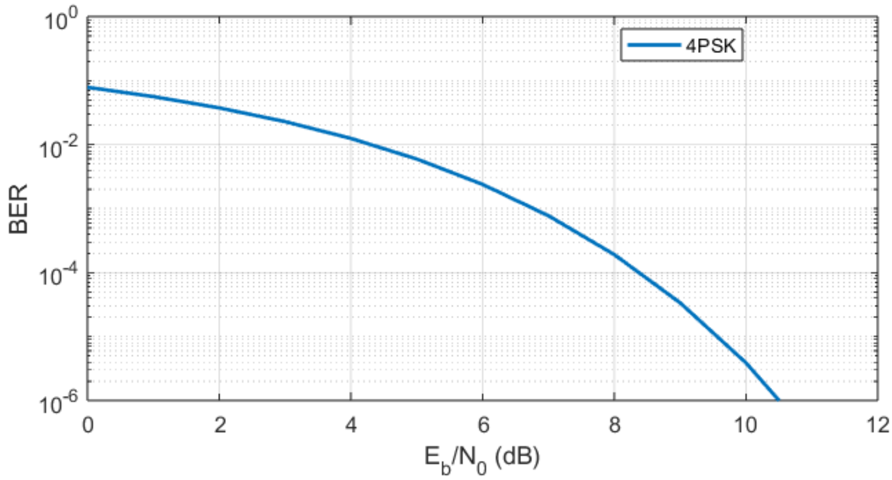
As the amplitude of the two components is the same, the wave reaching RX has RHCP: the best antenna must be designed to receive that kind of wave.

Problem 3

Consider the downlink from an Earth-observation satellite to a ground station at elevation angle of $\theta = 35°$, transmitting 40 Mb/s with 4PSK modulation. The link operational frequency is $f = 18.7$ GHz. Calculate the required ground station pointing accuracy P_{ACC} to guarantee a BER lower than 4×10^{-6}. Consider:

- the gain of both antennas (parabolic type) is $G = 36.5$ dB and their efficiency is $\eta = 0.51$
- the radiation pattern of the antennas is $\cos^2(\phi)$, ϕ being the off-boresight angle
- the antenna ground antenna points optimally at the satellite, while the satellite antenna points at the Earth's centre
- pointing loss of the satellite antenna $P_{SAT}^{LOSS} = 0.2$ dB
- the power transmitted by the satellite is $P_T = 9$ W
- the distance between the ground station and the satellite is $H = 1200$ km
- no satellite on-board losses and no waveguide losses in the receiver
- atmospheric attenuation along the path $A = 8$ dB
- mean radiating temperature $T_{mr} = 265$ K
- receiver noise figure $NF = 7$ dB

Assumptions: stratified atmosphere; flat Earth.



Solution

The wavelength is $\lambda = c/f = 0.016$ m. The gain of the two antennas is:

$G_T = G_R = G = 36.5 \text{ dB} = 4466.8$

The atmospheric attenuation is:

$A = 8 \text{ dB} \approx 0.1585$

The received power is:

$$P_R = P_T G f_T P_{SAT}^{LOSS} \left(\frac{\lambda}{4\pi H}\right)^2 G f_R P_G^{LOSS} A$$

where $f_T = [\cos(\phi)]^2 = 0.329$ ($\phi = 90° - \theta = 55°$) and $f_R = 1$ (optimal pointing), while P_{SAT}^{LOSS} and P_G^{LOSS} are the pointing losses of the satellite and ground antennas, respectively. The ground station pointing loss includes the pointing accuracy of the ground station (P_{ACC} expressed in degrees and P_G^{LOSS} in dB):

$$P_G^{LOSS} = 12\left(\frac{P_{ACC}}{70\lambda / D}\right)^2$$

where D is the diameter of the antennas, which can be calculated from the antenna efficiency and the gain as:

$$A_{eff} = \eta A_g = \frac{\lambda^2}{4\pi}G \quad \Rightarrow \quad \eta\left(\frac{D}{2}\right)^2\pi = \frac{\lambda^2}{4\pi}G \quad \Rightarrow \quad D = \frac{\lambda}{\pi}\sqrt{\frac{G}{\eta}} = 0.478 \text{ m}$$

As for the noise power, the LNA equivalent noise temperature is:

$$T_R = 290\left(10^{\frac{NF}{10}} - 1\right) = 1163.4 \text{ K}$$

while the antenna equivalent noise temperature is:

$$T_A = T_C A + T_{mr}(1 - A) = 223.4 \text{ K}$$

To guarantee a BER lower than 4×10^{-6} → $E_b/N_0 > 10$ dB = 10

The SNR is given by:

$$SNR = \frac{P_R}{k_B(T_R + T_A)B}$$

The SNR is also linked to E_b/N_0 as follows:

$$SNR = \frac{E_b R}{N_0 B}$$

The bandwidth is chosen as the minimum one to support the symbol rate R_S. Considering that 4PSK modulation, the symbol rate, $R_S = R/2 = 20$ MS/s (R being the data rate). Thus $B = R_S = 20$ MHz. Therefore, if $E_b/N_0 > 10$, SNR > 20 = 13 dB.

Imposing SNR > 20 = 13 dB and solving for P_G^{LOSS}:

$$P_G^{LOSS} = 0.7547 = 1.223 \text{ dB}$$

Finally, by inverting P_G^{LOSS}:

$$P_{ACC} < 70\frac{\lambda}{D}\sqrt{\frac{P_G^{LOSS}}{12}} = 0.75°$$

Problem 4

Compare a GNSS fully based on GEO satellites and another one fully based on MEO satellites: briefly highlight advantages and disadvantages of both systems.

Solution

The advantages of a GEO-based GNSS are:

- Satellites are always visible: less satellites are required
- As GEO satellites appear as fixed with respect to any user on the Earth, no Doppler effect: simpler satellite acquisition and tracking

The disadvantages of a GEO-based GNSS are:

- Given their orbit and the geometry, the PVT accuracy is limited by the high DOP
- Due to the large distance of the satellites, higher transmit power is required
- As GEO satellites appear as fixed with respect to any user on the Earth, no Doppler effect: lower accuracy in estimating the user velocity
- Poor coverage at high-latitudes/poles

The advantages of a MEO-based GNSS are:

- The PVT accuracy is increased by a potentially lower DOP
- Due to the shorter distance of the satellites, lower transmit power is required
- As MEO satellites appear as moving with respect to any user on the Earth, Doppler effect: higher accuracy in estimating the user velocity
- Good overall coverage, also at poles

The disadvantages of a GEO-based GNSS are:

- The same satellite is not always visible: more satellites are required
- As MEO satellites appear as moving with respect to any user on the Earth, Doppler effect: more complex satellite acquisition and tracking