Satellite Communication and Positioning Systems – Prof. L. Luini
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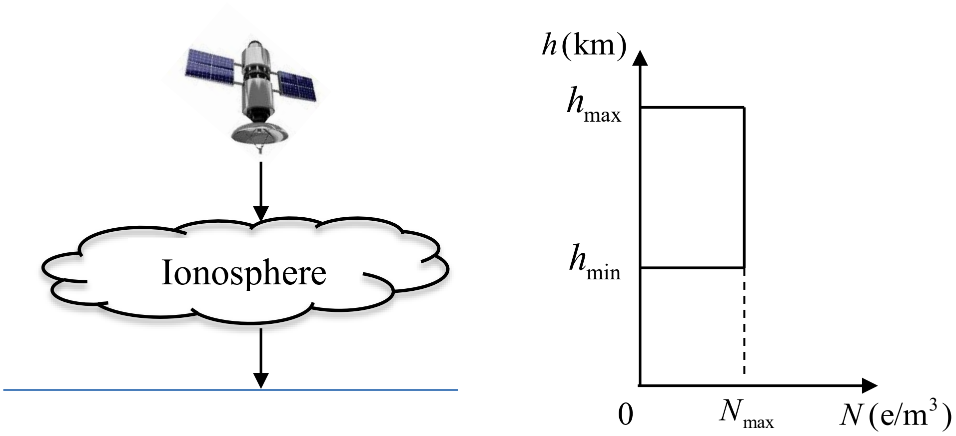
Problem 1

Referring to the figure below, a satellite at altitude h_{max} = 500 km transmits Earth observation data to a High Altitude Platform (HAP) at h_{min} = 30 km, using two channels centered about the carrier frequencies f_1 = 20 MHz and f_2 = 25 MHz (zenithal link). The ionosphere can be modelled with the electron density profile sketched in the figure (right side).

1) Determine the peak electron content N_{max}, knowing that the data travel time is T_1 = 2.0339 ms at f_1 and T_2 = 1.8657 ms at f_2.

2) What happens to the communication system if, due to a sudden ionospheric anomaly, the value of N_{max} determined at point 1) doubles?



Solution

1) The total travel time T is due to the free space and to the ionosphere, the latter being frequency dependent. T is defined as:

$$T = T_{FS} + T_{IONO} = \frac{H}{c} + \frac{1}{2c}\frac{81}{f^2}TEC$$

where TEC is the total electron content. As a result, taking the difference between T_1 and T_2:

$$\Delta T = T_1 - T_2 = \frac{81}{2c}TEC\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)$$

$$TEC = \frac{2c}{81}\Delta T \frac{1}{\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)} = 1.392 \cdot 10^{18}\ \mathrm{e/m^2}$$

N_{max} can be obtained from:

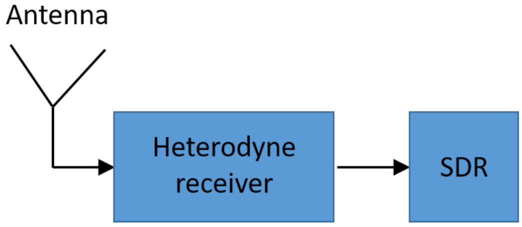
$$N_{max} = \frac{TEC}{(h_{max} - h_{min})} = 2.96 \cdot 10^{12}\ \ \mathrm{e/m^3}$$

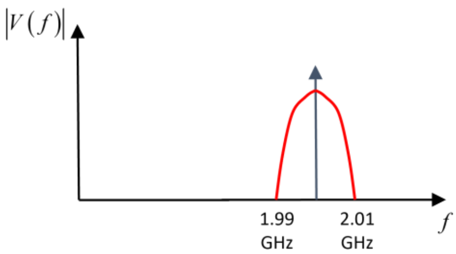
2) If the value of the maximum electron content becomes $N'_{max} = 5.89 \cdot 10^{12}\ \ \mathrm{e/m^3}$, the critical frequency of the link is $f_C \approx 9\sqrt{N'_{max}}$ = 21.84 MHz. As f_1 is lower than f_C, the associated signal will be reflected, while the signal associated with $f_2 > f_C$ will still cross the ionosphere.

Problem 2

Consider the receiver depicted below (left side), consisting in a single-stage heterodyne receiver followed by a Software Defined Radio (SDR) card, which has the task to post-process the received signal digitally. The maximum sampling rate of the SDR card is 100 Mb/s. The radio frequency signal to be received is shown below (right side).

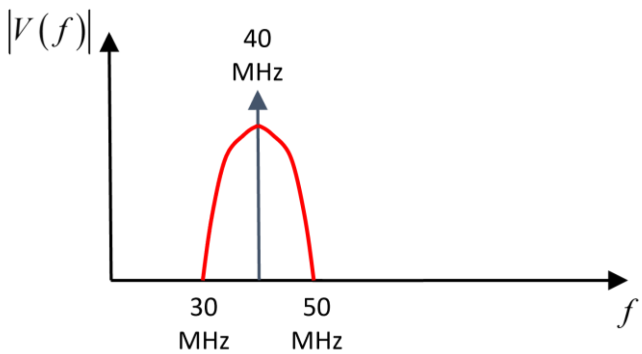
1. What is the first element of the heterodyne receiver?
2. Determine the maximum carrier intermediate frequency to allow a proper digitalization of the received signal.
3. Define the cut-off frequency (frequencies) of the intermediate frequency filter (assume that the filter is ideal).
4. Determine the possible values of the mixer local oscillator frequency to achieve the condition at point 2.





Solution

The first element is the LNA to reduce the noise as much as possible.
When an analog signal is digitalized, the sampling frequency f_s must be chosen according to the Nyquist-Shannon sampling theorem, which states that $f_s \geq 2f_{max}$, where f_{max} is the maximum frequency of the signal. The maximum f_s for the SDR is 100 MHz, therefore, when the signal is shifted to intermediate frequency, $f_{max} \leq f_s/2 = 50$ MHz; pushing the SDR to its limit, this means that after the down-conversion, we obtain:



Therefore the maximum carrier intermediate frequency is $f_{IF} = 40$ MHz and the cut-off frequencies of the ideal IF filter are: 30 MHz and 50 MHz.

After the mixer, the intermediate carrier frequency f_{IF} is given by:

$$f_{IF} = f_{RF} \pm f_{LO}$$

Therefore, the possible values of the mixer local oscillator frequency are:

$$f_{LO} = f_{RF} \pm f_{IF} = 2 \text{ GHz} \pm 40 \text{ MHz} = 2.04 \text{ GHz or } 1996 \text{ GHz}$$

Problem 3

Briefly explain the key features of a typical GNSS receiver antenna that help guarantee a high performance of the receiver.

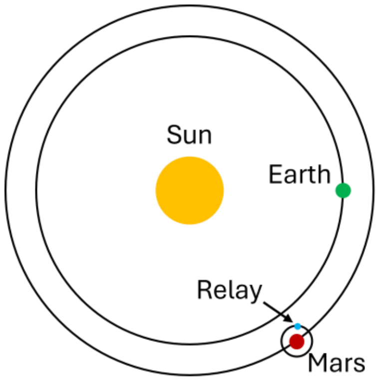
Solution

1. The antenna needs to receive RHCP waves.
2. The antenna needs to have a large radiation pattern to acquire as many satellites as possible, but with low gain at very low elevation angles (e.g. $< 5°$) to avoid satellites with low SNR.
3. Low sensitivity to LHCP waves.

Problem 4

Referring to the figure below, consider a direct link from a ground station on the Earth to a relay satellite orbiting Mars. The link frequency is f = 30 GHz. The tropospheric attenuation along the slant path, A, is modelled using the following Complementary Cumulative Distribution Function (probability expressed in percentage values, A expressed in dB):

$$P(A) = 100e^{-1.15A} \quad (P \text{ in \% and } A \text{ in dB})$$



Design the antenna on the relay satellite to guarantee a minimum *SNR* of 10 dB for 99.9% of the yearly time. Specifically:

- What kind of antenna is the most suitable one?
- Determine the antenna dimension

Use the following data:

- the gain of the transmitter antenna on the Earth is G_E = 70 dB
- assume that both antennas are optimally pointed
- the transmitted power is P_T = 2 kW
- the distance between the ground station and the satellite is H = 110 million km
- the receiver LNA equivalent noise temperature is T_{LNA} = 120 K
- assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
- the bandwidth is B = 8.25 MHz

Solution

A parabolic reflector antenna needs to be used to achieve a high gain. Also, circular polarization is needed for geometric reasons.

To guarantee the target SNR for 99.9% of the time, the atmospheric attenuation needs to be lower than a given value. This value can be determined by setting $P(A)$ = 100%-99.9% = 0.1%. As a result, A = 6 dB = 0.25. This value can be included in the link budget to determine the gain of the ground antenna (the wavelength is $\lambda = c/f$ = 0.01 m):

$$SNR = \frac{P_T G_E \left(\frac{\lambda}{4\pi H}\right)^2 G_{sat} A}{k T_{sys} B}$$

where f_S and f_R are assumed to be 1, as antennas are optimally pointed.

The noise power depends on the total system equivalent noise temperature:

$$T_{sys} = T_A + T_{LNA} = T_C + T_{LNA} = 122.73 \text{ K}$$

For the antenna equivalent noise temperature, only the background cosmic radiation is considered, as any emission from the Earth can be considered to be negligible due to the large distance.

By inverting the SNR equation, G_{sat} can be determined.

$G_{sat} = 5.3296 \cdot 10^5 = 57.27$ dB

Considering the relationship between the gain and the antenna effective area:

$$A_{eff} = \eta A_{geom} = \frac{\lambda^2}{4\pi} G_{sat} = 4.24 \text{ m}^2$$

η is the antenna efficiency and A_{geom} is its physical area. Assuming a reasonable value for η, i.e. 0.6:

$$A_{geom} = \left(\frac{D}{2}\right)^2 \pi = \frac{A_{eff}}{\eta} = 7.07 \text{ m}^2$$

D is the antenna diameter, which turns out to be:

$D = 3$ m