Satellite Communication and Positioning Systems – Prof. L. Luini
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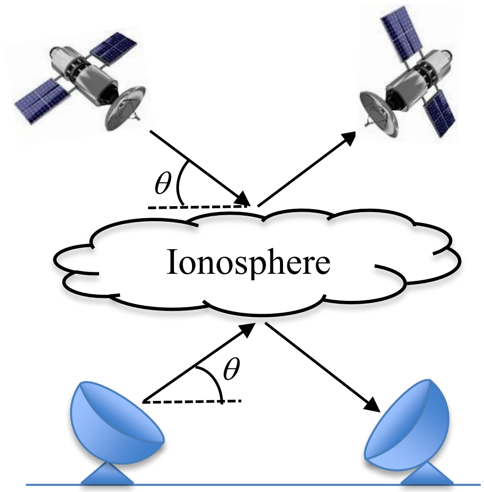
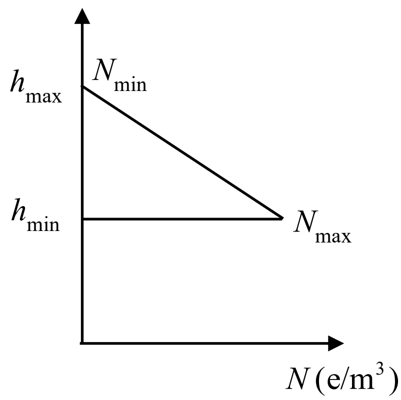
Problem 1

Referring to the figure below (left side), consider two bistatic radars, which aim at measuring the altitude of the lower layer (h_{min}) and of top layer (h_{max}) of the ionosphere – see the electron content profile depicted on the right side (N_{max} = 4×10^{12} e/m^3 and N_{min} = 9×10^9 e/m^3). For both radars:

1. Determine the operational frequency to properly measure h_{max}, knowing that θ = 60°.
2. Taking as reference θ = 60°, assuming that the radars can change the operational frequency, should θ be increased or decreased to improve the accuracy in measuring h_{min}? Justify the answer.

Assumption: neglect tropospheric effects.





Solution

1) <u>Ground-based radar</u>

Let us calculate the frequency associated to N_{max}:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f_{\max}}\right)^2} \quad \Rightarrow \quad f_{\max} = \frac{9\sqrt{N_{\max}}}{\sin\theta} \approx 20.785 \text{ MHz}$$

The ground based radar cannot be used to measure h_{max}: in fact, a wave associated to any frequency lower than or equal to f_{max} will be totally reflected at h_{min}, while a wave associated to any frequency higher than f_{max} will completely cross the ionosphere.

<u>Space-borne radar</u>

Let us calculate the frequency associated to N_{min}:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\min}}}{f_{\min}}\right)^2} \quad \Rightarrow \quad f_{\min} = \frac{9\sqrt{N_{\min}}}{\sin\theta} \approx 0.986 \text{ MHz}$$

The space-borne radar can be used to properly measured h_{max}: in fact, a wave associated to any frequency lower than or equal to f_{min} will be totally reflected at h_{max}.

2) <u>Ground-based radar</u>

Based on the discussion at point 1) above, the ground-based radar can measure h_{min} by using any frequency lower than or equal to f_{max} (total reflection at h_{min}). Considering that any radar estimates the position of a target from the pulse travel time, neglecting tropospheric effects, the accuracy in measuring h_{min} does not depend on θ: in fact, the wave will not enter the ionosphere (no additional ionospheric delay).

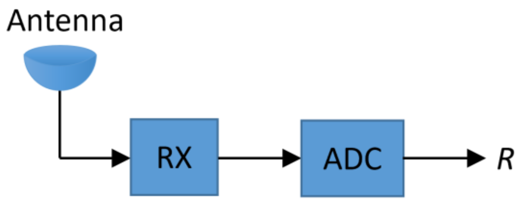
<u>Space-borne radar</u>

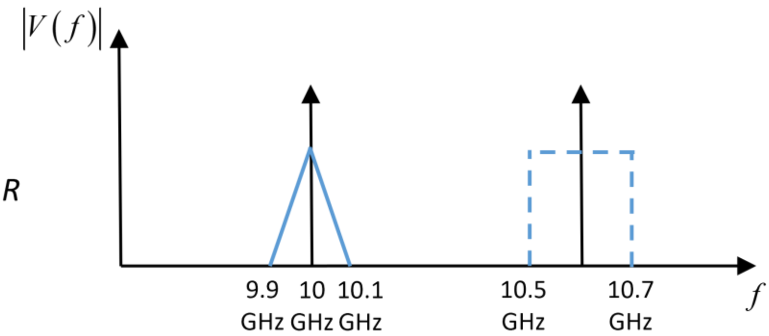
The situation changes for the space-borne radar: as the wave will partially travel across the ionosphere, longer paths through it (i.e. smaller values of θ) imply a higher ionospheric delay and also a higher depolarization (for linearly polarized waves), i.e. increase uncertainty in measuring h_{min}. Therefore, larger values of θ should be preferred to improve the radar accuracy.

Problem 2

Referring to the picture below (left side), a receiver consists in a single-stage heterodyne block followed by an analog-to-digital converter (ADC). The radio frequency signals are shown below (right side), the target signal being characterized by the triangularly shaped spectrum (solid line):

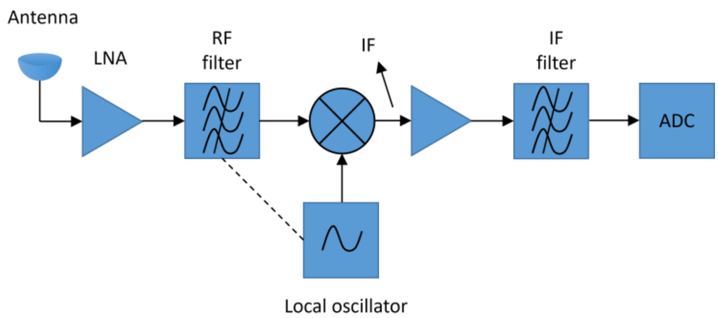
1. Draw the block scheme of the single-stage heterodyne receiver.
2. Considering the local oscillator frequency f_{LO} = 10.3 GHz, specify the features of the simplest radio frequency filter to use in the heterodyne receiver (type and cut off frequency/frequencies).
3. Draw the spectrum at intermediate frequency and specify the features of the most suitable intermediate frequency filter to use in the heterodyne receiver (type and cut off frequency/frequencies).
4. Specify the minimum sample rate R to properly digitalize the signal.





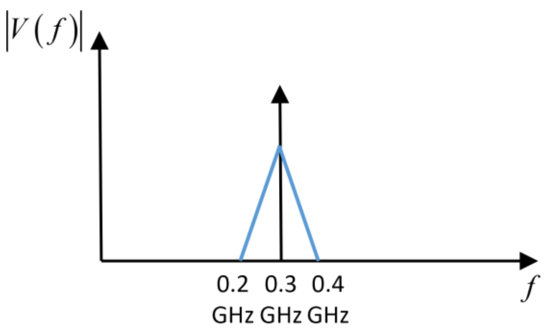
Solution

1) The block scheme of the single-stage heterodyne receiver is:



2) As the local oscillator frequency falls between the two signals, without a proper filter before the mixer, the two signals will be overlapped at intermediate (IF) frequency. Therefore, the RF filter will need to filter out the dashed line signal: the simplest filter to be used is a low-pass filter with cut-off frequency at 10.3 GHz (for example).

3) The IF spectrum is:



Given this spectrum, the best IF filter will be a pass-band filter with cut-off frequencies at 0.2 GHz and 0.4 GHz.

4) When an analogue signal is digitalized, the sampling frequency f_s must be chosen according to the Nyquist-Shannon sampling theorem, which states that $f_s \geq 2f_{max}$, where f_{max} is the maximum frequency of the signal. Here, f_{max} = 0.4 GHz, so the minimum sample rate to properly digitalize the signal is $R = f_s$ = 0.8 GS/s.

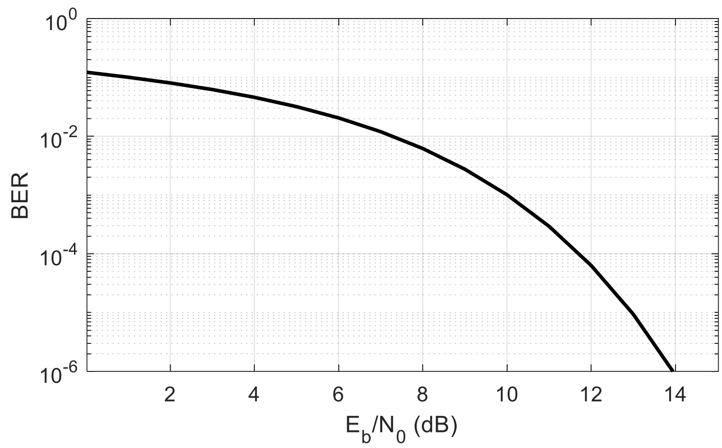
Problem 3

Consider the uplink at 20 GHz from a gateway to a broadcast geostationary satellite. The link has θ = 45° elevation angle and is affected by the tropospheric attenuation A_{dB}, whose statistics are modelled using the following CCDF (probability expressed in percentage values, A_{dB} expressed in dB and representing the value along the zenith):

$$P(A_{dB}) = 100e^{-2.3A_{dB}} \ (\%)$$

The satellite antenna points at the gateway, positioned in an area with brightness temperature T_G = 242 K. Calculate the data rate R with BER lower than 10^{-6} achieved for 99.999% of the time, using 8PSK modulation. Use the following data:

- the directivity of the ground antenna is D_R = 34 dB and its efficiency is η_R = 0.6
- the directivity of the satellite antenna is D_S = 24 dB and its efficiency is η_S = 0.7
- assume that both antennas are optimally pointed
- the power transmitted by the gateway is P_T = 1 kW
- the distance between the ground station and the satellite is H = 40000 km
- the receiver LNA equivalent noise temperature is T_{LNA} = 160 K
- assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
- mean radiating temperature T_{mr} = 295 K.



Solution

1) The wavelength is $\lambda = c/f$ = 0.01 m. The gains of the two antennas are:

$G_R = \eta_R D_R$ = 1507.1 (converted to linear scale)

$G_S = \eta_S D_S$ = 175.8 (converted to linear scale)

The zenithal atmospheric attenuation is obtained by inverting the CCDF after setting P = 0.001%:

$A_{dB} = 5$ dB

The slant path attenuation (in linear scale) is:

$A = 10^{-A_{dB}/(10\sin(\theta))} = 0.196$

The received power is:

$$P_R = P_T G_S f_S \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R A = 4.624 \cdot 10^{-14} \text{ W}$$

where $f_S = f_R = 1$ (antennas optimally pointed). The noise power depends on the total system equivalent noise temperature:

$T_{sys} \approx T_A + T_{LNA}$

The equivalent antenna noise (sky noise) is given by:

$T_A = T_G A + T_{mr} (1-A) = 284.6$ K

Thus $T_{sys} \approx 444.6$ K

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K).

To achieve BER < 10^{-6} → E_b/N_0 = 14 dB = 25.12. Using 8PSK, each symbol carries 3 bits, so the ideal bandwidth is $B = R/3$. Therefore:

$$SNR = \frac{E_b R}{N_0 B} = 3\frac{E_b}{N_0} = 75.36 = 18.77 \text{ dB}$$

By inverting the last equation, the maximum bandwidth can be determined.

$$B = \frac{P_R}{kT_{sys}SNR} = 100 \text{ kHz}$$

As a result, R = 300 kbit/s.

Problem 4

The Lunar Communication and Navigation System (LCNS) will provide satellite-based navigation and communication services to users on the Moon surface. Assuming that the system will operate employing the same PRN sequences used in the GPS:

1. Determine the radio frequency bandwidth of the satellite signal necessary to guarantee a ranging error lower than 0.6 m. To this end, consider that the PRN phase error is $p_\varepsilon = 20\%$ of the chip length and neglect all other sources of errors (e.g. due to precise orbit determination).
2. Indicate a suitable carrier frequency,
3. Determine the PRN sequence length (number of chips and duration) to guarantee an autocorrelation isolation of at least -30.2 dB.
4. Indicate a reasonable data rate for the system to guarantee a satisfactory signal acquisition.

Solution

1) The bandwidth B depends on the chip duration T, and the latter is associated with the ranging error ε as follows:

$$\varepsilon = cTp_\varepsilon$$

Imposing $\varepsilon = 0.6$ m → $T = 10$ ns

The radio frequency bandwidth is given by:

$$B = \frac{2}{T} = 200 \text{ MHz}$$

2) Given B, let us consider that the bandwidth can be up to 10% of the carrier frequency f → $f \geq 2$ GHz

3) The autocorrelation isolation is defined as:

$$I = 10\log_{10}(1/N)$$

where N is the chip number of the PRN sequence. Setting $I = -30.2$ dB → $N = 1050$. Thus, the duration of the PRN sequence is $D = NT = 10.5$ μs.

4) Given the duration of the PRN sequence D, for a satisfactory signal acquisition, the bit duration T_b should be at least 10 times the PRN sequence duration (in the GPS, it is 20 times). Therefore:

$$T_b = 10D = 100.5 \text{ μs}$$

The bit rate is:

$$R = 1/T_b = 9.524 \text{ kb/s}$$