Satellite Communication and Positioning Systems – Prof. L. Luini
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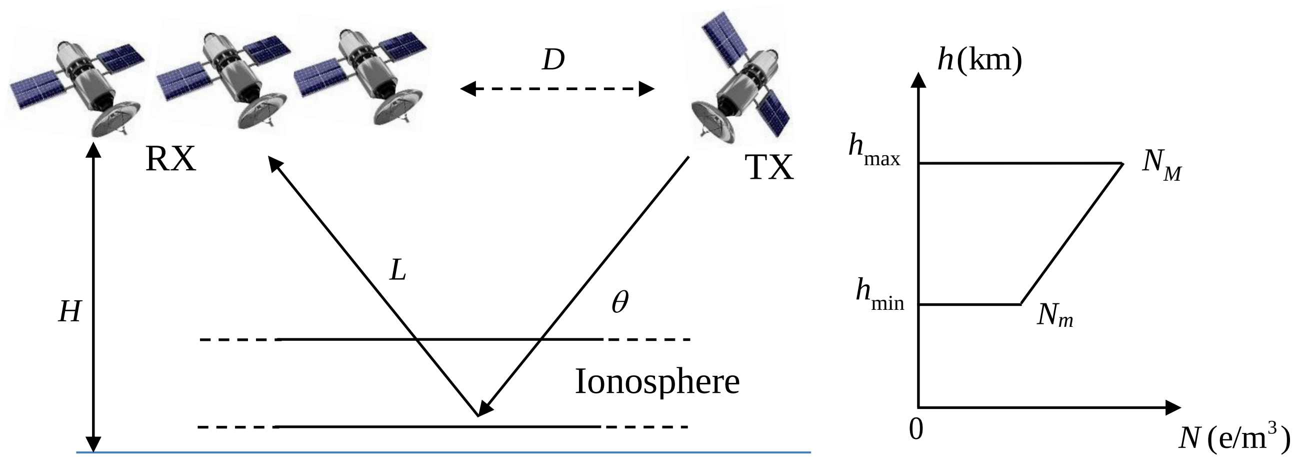
Problem 1

A space-borne pulsed radar system is designed to sound the ionosphere. The radar works at a fixed angle $\theta = 40°$, but it can change the carrier frequency f. An array of receiving satellites is deployed to catch the signal reflected by the ionosphere (see left side of the figure below). The electron content profile is depicted on the right side ($N_M = 4\times10^{12}$ e/m^3, $N_m = 4\times10^{11}$ e/m^3, $h_{\max} = 350$ km, $h_{\min} = 100$ km)

1. Determine the carrier frequency f_M to correctly identify N_M (i.e. reflection at $h_{\max}$).
2. Determine the carrier frequency f_m to correctly identify N_m (i.e. reflection at $h_{\min}$).
3. Calculate the propagation time τ between the TX and RX when $f = 0.9\, f_M$.
4. Calculate the distance D between the TX and RX at when $f = 1.1\, f_M$.

Assumptions: flat Earth, no tropospheric effects, real reflection height equal to the virtual one, altitude of the satellites $H = 800$ km.



Solution

1) f_M can be identified as follows:

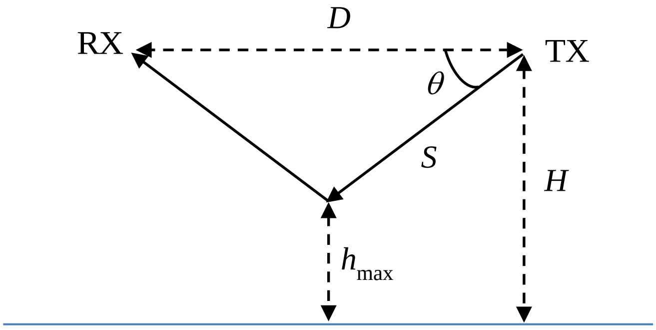
$$\cos(\theta) = \sqrt{1 - \left(\frac{9\sqrt{N_{\mathrm{M}}}}{f}\right)^2} \quad \Rightarrow \quad f_{\mathrm{M}} = 28\ \mathrm{MHz}$$

2) Reflection at $h_{\min}$ is not possible: for frequencies below f_M, the wave will be reflected. For frequencies higher than f_M, the wave will cross the ionosphere and will be reflected at the ground.

3) Working at f = 0.9 f_M = 25.2 MHz, the wave will be reflected at $h_{\max}$, so the propagation will occur at light speed c. Therefore:

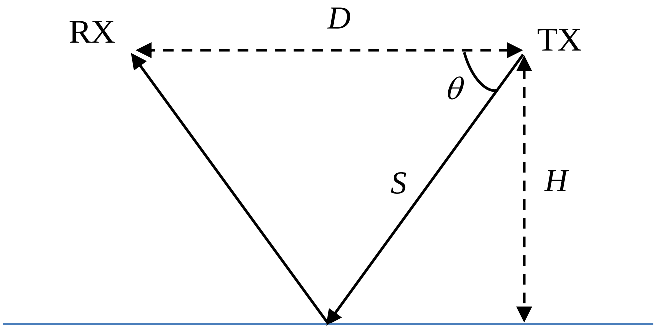
$$S = \frac{(H - h_{\max})}{\sin(\theta)} = 700\ \mathrm{km}$$

$$\tau = \frac{2S}{c} = 4.7\ \mathrm{ms}$$



4) Working at f = 1.1 f_M = 30.8 MHz, the wave will be reflected at the ground, so distance D can be calculated as:

$$D = \frac{2H}{\tan(\theta)} = 1907\ \mathrm{km}$$



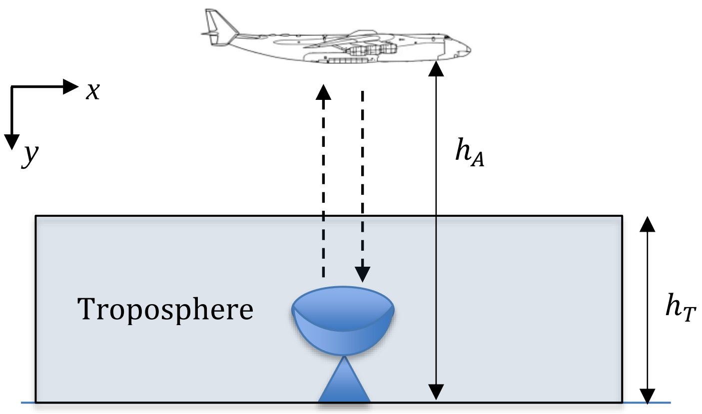
Problem 2

A pulsed radar with zenithal pointing (transmit power P_T = 1 kW), working with carrier frequency f = 80 GHz and with antenna gain G = 40 dB, illuminates an aircraft flying at altitude h_A = 10 km. As depicted in the figure below, the wave crosses an isotropic layer (precipitating particles), whose propagation constants are:

$\gamma_z = \alpha_z + j\,\beta_z = 4\times10^{-4} + j1675.5$ 1/m
$\gamma_x = \alpha_x + j\,\beta_x = 4\times10^{-4} + j15080$ 1/m

For this scenario:

1) Which polarization among the following ones allows improving the radar accuracy in estimating the aircraft distance and its backscatter section σ? Linear along x, linear along z, circular.
2) Working with the polarization chosen at point 1, calculate the radar backscatter σ and the troposphere height h_T, knowing that the power received by the radar in clear sky conditions and under precipitation is 2.126×10^{-13} W and 1.75×10^{-15} W, respectively.



Solution

1) From the propagation constants, it emerges that the two linear polarizations are subject to the same attenuation, but to a different delay: the z-component will travel at light speed c (in fact, $v_z = \omega/\beta_z = c$), while the x-component will travel at a lower speed → $v_x = \omega/\beta_x = 0.33\times10^8$ m/s. Therefore, considering that the radar estimates the aircraft distance from the propagation time (which is affected by the propagation velocity), it is better to use the linear polarization along z.

2) The backscatter σ can be obtained from the power received in clear sky conditions. In this case, the power density reaching the aircraft is:

$$S_A = \frac{P_T}{4\pi h_A^2} G$$

The power received back by the radar is:

$$P_R = \frac{S_A \sigma}{4\pi h_A^2} G \frac{\lambda^2}{4\pi}$$

Inverting the equation to solve for the backscatter yields $\sigma = 3$ m^2.
Under precipitation, the equations become:

$$S_A = \frac{P_T}{4\pi h_A^2} G A_R$$

and

$$P_R = \frac{S_A \sigma_C}{4\pi h_A^2} A_R G \frac{\lambda^2}{4\pi}$$

Solving for A_R yields A_R = 0.0907. Recalling that:

$A_R = e^{-2\alpha_z h_T}$,

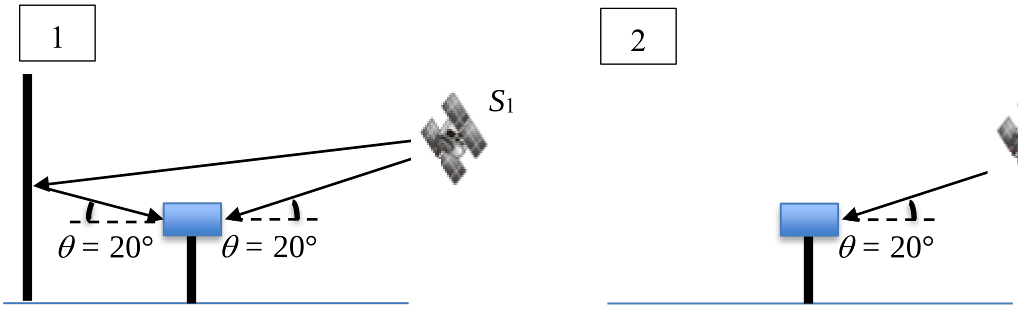
we get → h_T = 3000 m.

Problem 3

Consider the two scenarios below: the same GNSS received (RX) is positioned close to a building causing a reflection (case 1) or it is installed in an open field (case 2).

1) Considering exactly the same atmospheric conditions between the two scenarios, which GNSS RX is expected to provide a more accurate positioning? Answer the following question making reference to the chosen scenario.
2) Describe what is necessary (e.g. type of GNSS RX, minimum number of acquired satellites, …) for the RX to be used to monitor the ionosphere.

Assumption: perfect orbit determination for the GNSS satellites; perfect synchronization of the satellites; the troposphere is horizontally homogeneous and the zenithal tropospheric delay is d_T = 4 m; the ionosphere is not horizontally homogeneous; typical patch antenna for the RX; perfect C/A code synchronization.



Solution

1) Considering scenario 1, the ray undergoing one reflection is practically rejected by the RX because that ray has a LHCP and the typical GNSS RX antenna receives only RHCP waves. Therefore, from the accuracy point of view, the two scenarios are fully equivalent. For simplicity, let us anyway make reference to scenario 2 from now on.

2) Considering the case in which the GNSS RX has not yet acquired a sufficient number of satellites to provide the PVT solution, as well as the assumptions above, the pseudorange, for a generic satellite, is given by:

$$\rho = L + d^I + \frac{d^T}{\sin(\theta)} + d^C$$

where d^I is the zenithal ionospheric error (changing for each satellite path), $d^T = 4$ m is the zenithal tropospheric error and d^C is the RX clock bias. Two possible solutions can be devised to use the RX to map the ionosphere:

- The obvious one is to use a two-frequency receiver: in this case, each satellite will provide information on the TEC along the path.
- Using a single-frequency RX is also possible, but knowledge of the receiver position is required (i.e. L in the equation above). Also, a minimum of 4 satellites is required to remove the RX clock bias. In this case, for each satellites, the pseudorange becomes:

  $$\rho = L + d^I + \frac{d^T}{\sin(\theta)}$$

  As a result, d^I (from which the TEC can be easily derived) is obtained as:

  $$d^I = \rho - L - \frac{d^T}{\sin(\theta)}$$

Problem 4

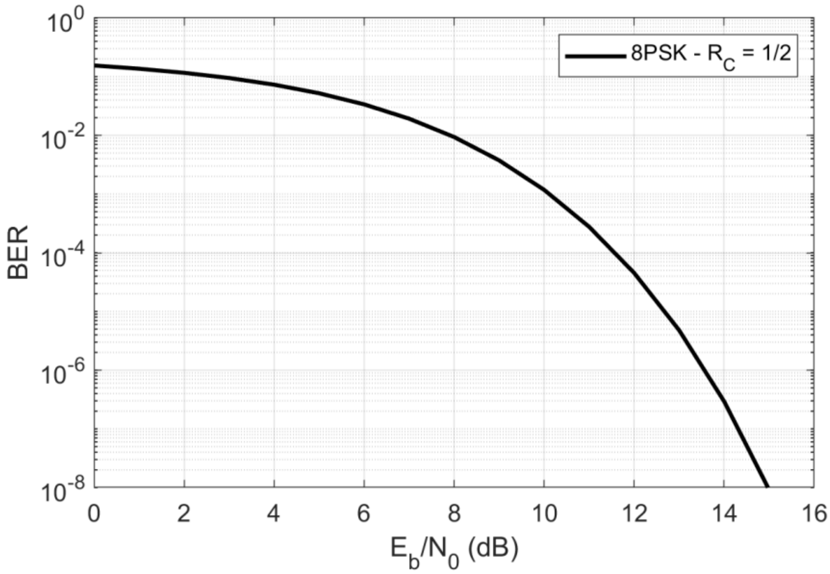
A geostationary satellite operates at a frequency of 19 GHz and provides a communication link to a ground station with altitude h_S = 1 km a.m.s.l (elevation angle ϕ = 60°). The slant path distance between the satellite and the ground station is D = 38000 km. The satellite transmits with an effective isotropic radiated power (EIRP) of 90 dBm.

Consider the following data:

- Zenithal tropospheric attenuation profile (α in dB/km and h in km a.m.s.l.):

  $\alpha = 3.65e^{-0.1h}$ for $0 \le h \le 5$ km
  $\alpha = 0$ for $h > 5$ km
- Mean radiating temperature T_{mr} = 290 K
- Ground station antenna gain G_G = 43 dB
- Receiver noise temperature T_R = 150 K
- Data rate R = 12 Mb/s
- 8PSK modulation with information coding (code rate R_C = 1/2)
- Gound antenna optimally pointed to the satellite, but satellite mispointed by 16° with respect to the ground station
- Radiation pattern of the satellite antenna $f_T = [\cos(\theta)]^2$, being θ the boreside off angle

Determine the link bit error rate.



Solution

First, let us calculate the slant path total attenuation:

$$A_T^{dB} = \frac{\int_{h_S}^{5} \alpha(h)dh}{\sin(\phi)} = 12.57 \text{ dB} \rightarrow A_T = 0.0533$$

Let us now calculate the SNR:

$$SNR = \frac{EIRP f_T (\lambda/4\pi D)^2 G_G A_T}{k[T_R + T_{mr}(1 - A_T) + T_C A_T]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = [\cos(\theta)]^2 = 0.924$, $T_C = 2.73$ K, $EIRP = 10^6$ W. The bandwidth can be calculated considering the data rate, the code and the modulation order. The coded information rate is $R_{code} = R/R_C = 24$ Mbit/s. As the modulation is 8PSK (3 bit per symbol), the receiver bandwidth can be set to $B = R_{code}/3 = 8$ MHz. Therefore SNR = 23.8 and:

$$SNR = \frac{E_b R}{N_0 B} \rightarrow \frac{E_b}{N_0} = SNR\,\frac{B}{R} = 12 \text{ dB}$$

Looking at the figure, the BER is approximately 4×10^{-5}.