Satellite Communication and Positioning Systems – Prof. L. Luini,
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SURNAME AND NAME ______________________________

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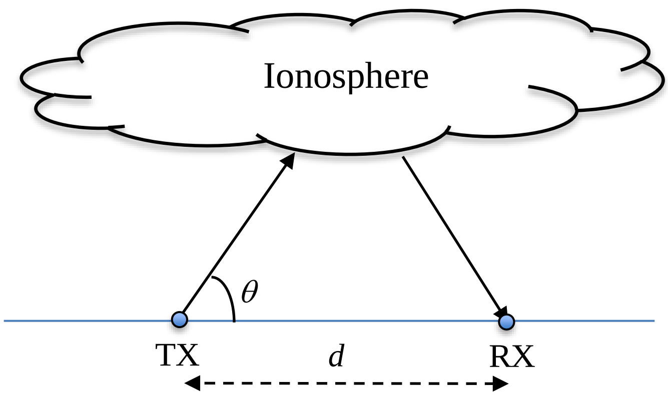
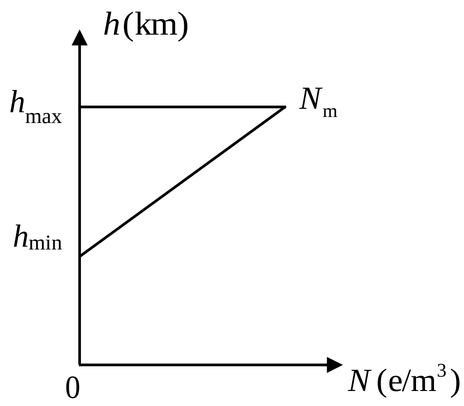
Problem 1

Making reference to the figure below, the ionosphere is modelled with the sketched electron density profile, where $N_m = 4\times10^{12}$ e/m^3, h_{min} = 100 km and h_{max} = 400 km. The distance between TX and RX is d = 600 km and the elevation angle θ = 50°.

1) Calculate the link frequency f.
2) Keeping the same operational frequency f found at point 1), what happens if the elevation angle is increased to θ_2 = 60°?

Assume that the virtual reflection height is 1.2 of the height at which the wave is actually reflected.





Solution

1) The distance d depends on the virtual reflection height as follows:

$$d = \frac{2h_V}{\tan\theta}$$

Therefore:

$$h_V = d\,\frac{\tan\theta}{2} = 357.5 \text{ km}$$

The real reflection height is:

$$h_R = \frac{h_V}{1.2} = 297.9 \text{ km}$$

The trend of the electron content with height is:

$$N(h) = \frac{N_m}{h_{\text{max}} - h_{\text{min}}}(h - h_{\text{min}}) \quad \text{with } h \text{ in km}$$

The N value at height hR is:

$$N(h_R) = 2.6 \times 10^{12} \text{ e/m}^3$$

The link frequency is:

$$f = \sqrt{\frac{81 N(h_R)}{1 - [\cos(\theta)]^2}} \approx 19.1 \text{ MHz}$$

2) The maximum frequency guaranteeing reflection for the new elevation angle θ_2 is:

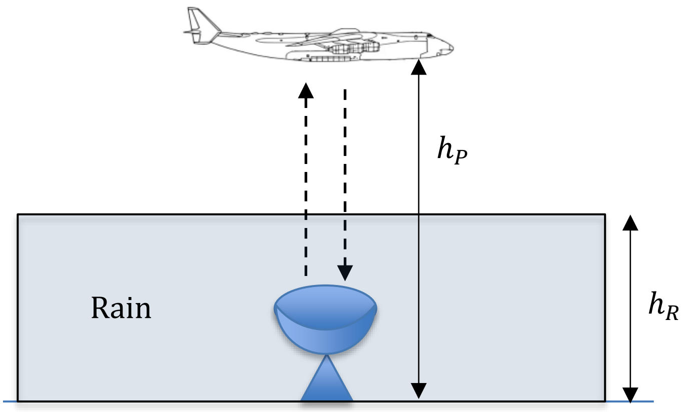
$$f_{\text{max}} = \sqrt{\frac{81 N_m}{1 - [\cos(\theta_2)]^2}} \approx 20.8 \text{ MHz}$$

As $f < f_{\text{max}}$, the wave is still completely reflected by the ionosphere.

Problem 2

A radar with zenithal pointing, working at f = 10 GHz, illuminates an aircraft flying at altitude h_P = 10000 m. As depicted in the figure below, a rain layer (propagation constant in the rain layer $\gamma_R = 1.7\times10^{-4}+j\ 1.886\times10^{3}$ 1/m; rain height h_R = 5000 m) affects the radar. Calculate:

1) The time for the pulse to reach back the radar after reflection on the aircraft.
2) The backscatter section of the airplane, knowing that: the radar transmit power is P_T = 1 kW; the received power is P_R = 0.15 nW; the antenna gain is G = 50 dB.



Solution

1) The wave propagates with different phase velocity in the rain layer and in air. For the former:

$$v_R = \frac{\omega}{\beta_R} = 0.33 \times 10^8 \text{ m/s}$$

while, for the latter, we can assume:

$$v_A = c = 3 \times 10^8 \text{ m/s}$$

The two-way propagation time is given by:

$$\tau = 2\left(\int_0^{h_R} \frac{ds}{v_R} + \int_{h_R}^{h_P} \frac{ds}{v_A}\right) = 2\left(\frac{h_R}{v_R} + \frac{h_P - h_R}{v_A}\right) = 3.33 \times 10^{-4} \text{ s}$$

2) First, the rain attenuation is given by:

$$A_R = e^{-2\alpha_R h_R} = 0.1813$$

The power density reaching the aircraft is:

$$S_A = \frac{P_T}{4\pi h_P^2} G A_R = 0.0144 \text{ W/m}^2$$

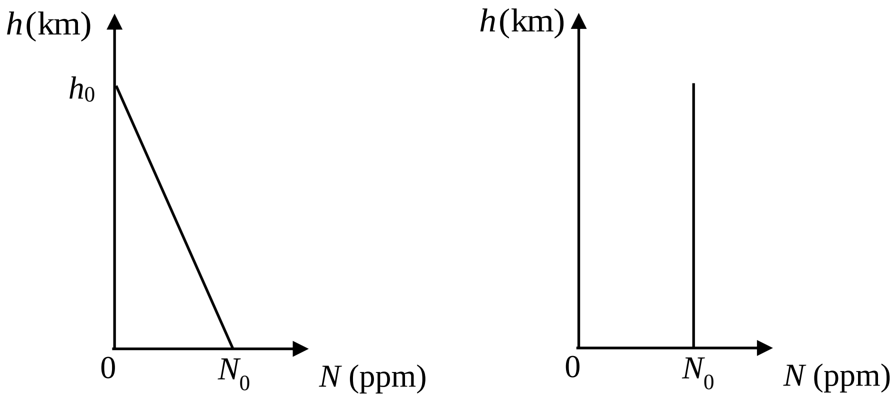
The power received back by the radar is:

$$P_R = \frac{S_A \sigma}{4\pi h_P^2} A_R G \frac{\lambda^2}{4\pi}$$

Solving for the backscatter section → $\sigma \approx 10$ m^2.

Problem 3

Two ground stations operating at f = 15 GHz and f = 30 GHz aim at communicating with a GEO satellite at elevation angle θ = 45°. For each ground station, and for each refractivity profiles depicted below, state if the elevation angle to be used needs to be higher or lower than 45° to correctly point at the satellite (N_0 = 1000 ppm, h_0 = 8 km).



Solution

The vertical gradient of the refractivity will induce a change in the propagation direction (ray bending). This effect is frequency independent, therefore the results will be the same for both ground stations.

The curvature radius of the EM wave depends on the refractivity gradient as follows:

$$\rho = -\frac{10^6}{dN/dh}$$

<u>Profile on the left</u>

The refractivity profile is:

$$N(h) = -\frac{N_0}{h_0}h + N_0 \quad \rightarrow \quad \frac{dN}{dh} = -\frac{N_0}{h_0} = -125\ \text{km}^{-1}$$

Therefore, the curvature radius is:

$$\rho_1 = 8000\ \text{km}$$

ρ_1 is positive, so the ray is bent towards the ground: the actual elevation angle will need to be higher than 45°.

<u>Profile on the right</u>

The refractivity profile is:

$$N(h) = N_0 \quad \rightarrow \quad \frac{dN}{dh} = 0\ \text{km}^{-1}$$

Therefore, the curvature radius is:

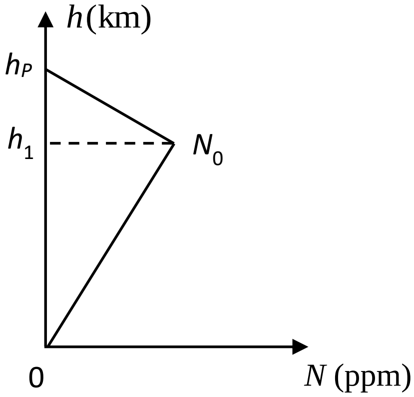
$$\rho_2 \rightarrow \infty$$

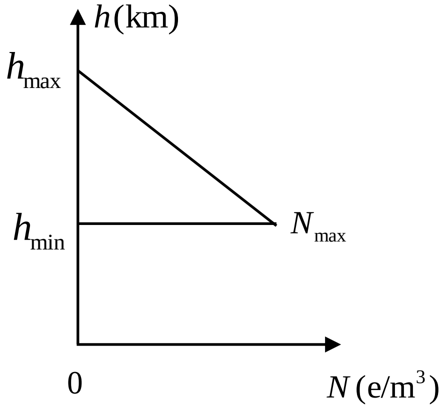
ρ_2 indicates that the EM ray will not be curved, so the actual elevation angle will need to be exactly 45°.

Problem 4

A dual-frequency military receiver, correctly providing the PVT solution, tracks, among the others, a GNSS satellites at distance r = 26248763 m and at elevation angle θ = 40°. The wave crosses an ionospheric layer (horizontal homogeneous) with the vertical electron content profile sketched on the right side of the figure below ($h_{\max}$ = 300 km, $h_{\min}$ = 80 km, $N_{\max} = 4\times10^{12}$ e/m^3) and the vertical refractivity profile (horizontal homogeneous, N_0 = 400, h_1 = 20 km, h_P = 30 km) sketched on the left side of the figure below. For this scenario:

1) Knowing that the L2 P(Y)-code prompt code correlator value is C = 0.7, calculate the L2 range error affecting the receiver for that satellite.





2) The receiver calculates the following DOP matrix:

$$R = \begin{bmatrix} 0.23 & -0.04 & -0.22 & -0.14 \\ -0.04 & 0.19 & 0.08 & 0.04 \\ -0.22 & 0.08 & 2.51 & 1.94 \\ -0.14 & 0.04 & 1.94 & 0.97 \end{bmatrix}$$

The receiver is used to land an aircraft. Calculate the positioning error value that is provided for 95.45% of the time. To this aim, considering that the user-equivalent range error is modelled using the following Probability Density Function ($d\rho$ expressed in m):

$$P(d\rho) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{d\rho^2}{72}}$$

Solution

1) The pseudorange is affected by different error sources. The ionospheric delay can be neglected as we deal with a two-frequency receiver. Also the receiver clock bias error can be neglected, as the receiver is correctly providing the PVT solution. As for the zenithal tropospheric delay, it is given by:

$$\tau = T - T_0 = \frac{10^{-6}}{c}\int_0^{h_P} N dh = \frac{10^{-6}}{c}\left[\frac{N_0 h_1}{2} + \frac{N_0(h_P - h_1)}{2}\right] = 2\times10^{-8}\ \text{s}$$

As a result the tropospheric delay on the pseurorange is:

$$d^T = c\,\frac{\tau}{\sin(\theta)} = 9.3\ \text{m}$$

Considering all the contributions affecting the pseudorange in this case, we obtain:

$$\rho = r + d^T + d^{P(Y)}$$

where $d^{P(Y)}$ is the error due to the correlator

$$d^{P(Y)} = c\,T_{P(Y)}(1 - C) = 8.8\ \text{m}$$

Therefore, the range error is:

$$\Delta\rho = d^T + d^{P(Y)} = 18.1\ \text{m}$$

2) To answer the question, let us first calculate PDOP (3D error important for aircrafts) as:
PDOP $= \sqrt{R_{11} + R_{22} + R_{33}} = 1.71$
From the Gaussian distribution, the UERE is σ = 6 m.
The desired 3D positioning error is given by:
$\sigma_{3D} = \text{TDOP } 2\sigma = 20.5$ m