Satellite Communication and Positioning Systems – Prof. L. Luini,
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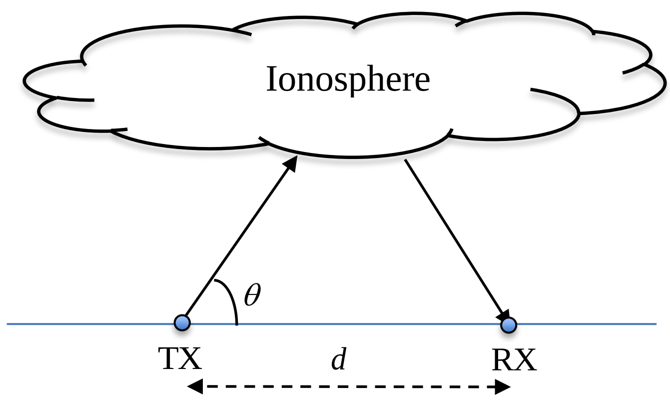
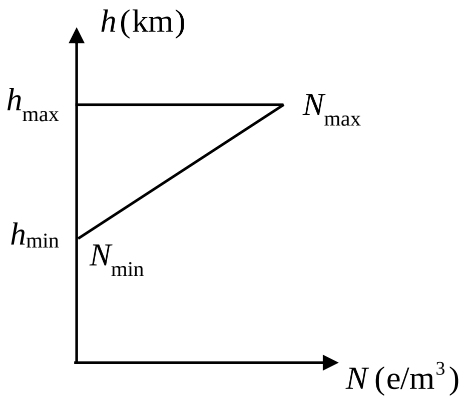
Problem 1

Making reference to the figure below, the ionosphere is modelled with the sketched electron density profile, where N_{max} = 4×10^{12} e/m^3, N_{min} = 4×10^{10} e/m^3, h_{min} = 100 km and h_{max} = 300 km. The elevation angle θ is 40° and the distance between TX and RX is d = 286 km.
For this scenario:
1) Determine the TX operational frequency f for the wave to reach RX.
2) Keeping the same elevation angle, what happens if the operational frequency is f_1 = 1 MHz?
3) Keeping the same elevation angle and the frequency determined at point 1), what happens if N_{max} changes to 10^{11} e/m^3.

Assume that the virtual reflection height is 1.2 of the height at which the wave is actually reflected.



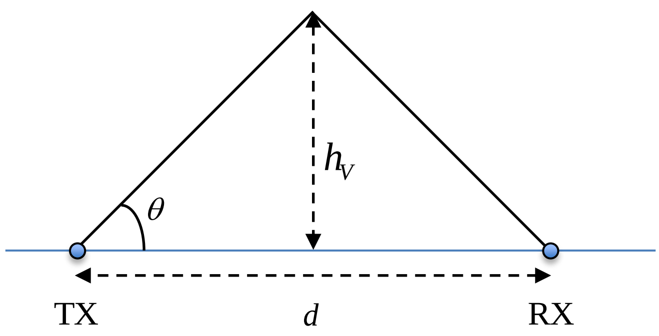


Solution
1) Exploiting the concept of virtual reflection height (see figure below), h_V is given by:

$$h_V = \frac{d}{2}\text{tg}(\theta) = 120\text{ km}$$

from which, the actual reflection height is:

$$h = \frac{h_V}{1.2} = 100\text{ km} = h_{\text{min}}$$



For the reflection to occur at h_{min}, the operational frequency is:

$$f = \sqrt{\frac{81 N_{\mathrm{min}}}{1 - [\cos(\theta)]^2}} \approx 2.8 \text{ MHz}$$

2) For any frequency lower than f (as in the case of f_1), given that elevation angle, the wave will be totally reflected at h_{min}, so RX will still be reached.

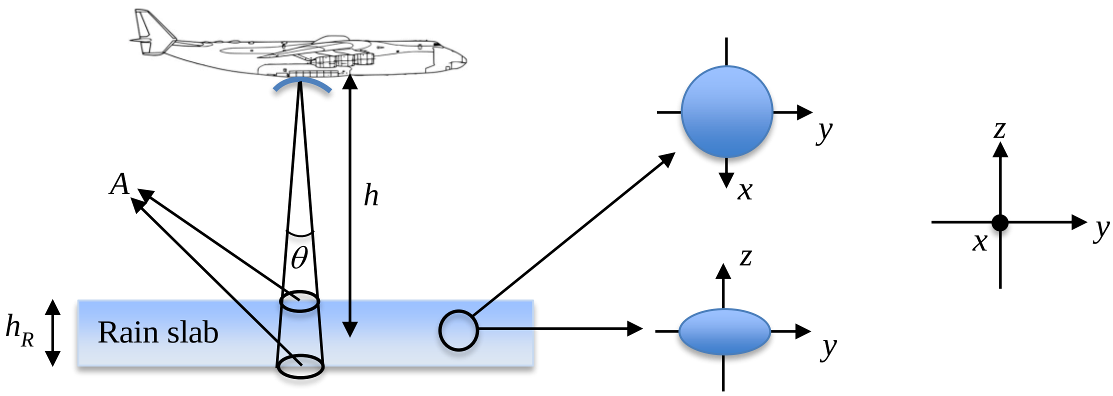
3) As the reflection occurs at h_{min}, the change in N_{max} will not affect the ionospheric link.

Problem 2

Making reference to the figure below, a pulsed radar onboard an airplane, operating with carrier frequency f = 4 GHz and pointed zenithally, is used to measure precipitation. The beam illuminates a volume V filled with rain at distance h = 10 km. The area of the volume is A = 314 m^2 and its height is h_R = 100 m. The rain drops, whose density is N = 150 drops/m^3, are oblate spheroids, all equi-oriented with major axis parallel to the ground. All drops have the same dimension and same backscatter section, i.e. σ = 2 mm^2.

1) Determine the radar transmit power P_T to guarantee that the power received back by the radar, P_R, is higher than 1 pW.
2) What is the best polarization to be used?

Consider the following data: radar antenna gain G = 30 dB; assume no atmospheric attenuation and neglect the cosmic background noise.



Solution

1) First, let us calculate the power density reaching the rain volume:

$$S = \frac{P_T}{4\pi h^2} G f \ \mathrm{W/m^2}$$

where $G = 10^3$, $f = 1$ (radar pointing to the volume).

The power reirradiated by a single rain drop is (with gain = 1 according to the definition of backscatter section), is:

$$P_d = S\sigma$$

Considering all the drops in the volume and under the assumption of Wide Sense Stationary Uncorrelated Scatterers (based on which we can sum the power reirradiated by the single drops), we obtain:

$$P_t = NAh_R S\sigma$$

Finally, the power received by the radar is:

$$P_R = \frac{P_t}{4\pi h^2} A_E = \frac{P_t}{4\pi h^2} G \frac{\lambda^2}{4\pi}$$

Combining all the equations and rearranging the terms:

$$P_R = \frac{NAh_R \sigma P_T G^2 \lambda^2}{h^4 (4\pi)^3}$$

The equation above can be inverted to obtain P_T:

$$P_T > \frac{P_R h^4 (4\pi)^3}{NAh_R \sigma G^2 \lambda^2} \approx 374.5 \ \mathrm{W}$$

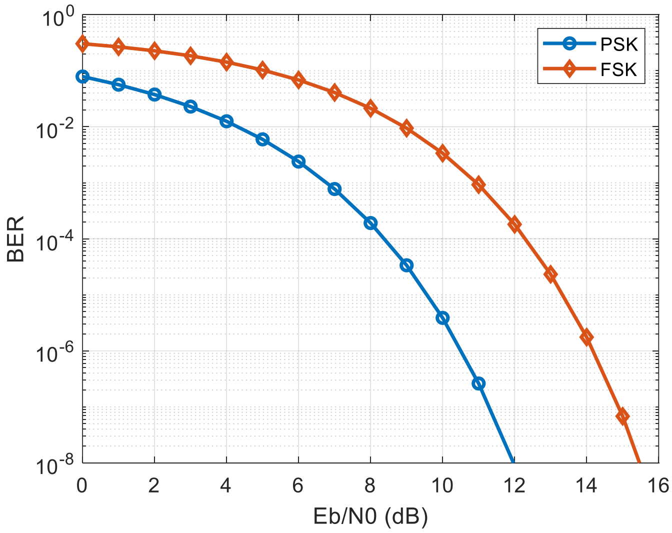
2) There is no preferential polarization to be used as the drops have circular section as seen from the impinging wave.

Problem 3

Consider the downlink from a spacecraft to a ground station (zenithal pointing). The link operating frequency is f = 20 GHz. The atmospheric attenuation is only due to gases and clouds;

1) In this condition, select the suitable digital modulation (PSK or FSK) to guarantee a BER lower than 10^{-6} (see graphs below). To this aim, consider the following data:

- the directivity of the both antennas is D = 42 dB and their efficiency is 0.6
- both antennas are optimally pointed
- the power transmitted by the satellite is P_T = 79 W
- the distance between the ground station and the satellite is H = 40000 km
- the receiver LNA equivalent noise temperature is T_{LNA} = 150 K
- the antenna equivalent noise temperature is T_A = 90 K and the mean radiating temperature of the medium is T_{mr} = 120 K
- antennas are parabolic reflectors with Cassegrain configuration
- disregard the background cosmic noise
- the system bandwidth is B = 30 MHz
- the data rate R is equal to the bandwidth.



2) Draw a sample trend in time of the radiofrequency signal for the chosen modulation, considering the following data stream → [0 1 0 0]

Solution

1) The wavelength is $\lambda = c/f = 0.015$ m. The gain of the two antennas is:

$$G = D\eta \approx 9509.3 = 39.8 \text{ dB}$$

The tropospheric attenuation can be inferred from T_A and T_{mr} as:

$$A_{RF} = 1 - \frac{T_A}{T_{mr}} = 0.25 \rightarrow A_{RF} = 6 \text{ dB}$$

Considering all the terms, the received power is:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R A_{RF} \approx 1.6 \text{ pW}$$

being both f_T and f_R equal to 1.

The noise power depends on the total system equivalent noise temperature (no impact from the transmission line/waveguide, given the Cassegrain configuration):

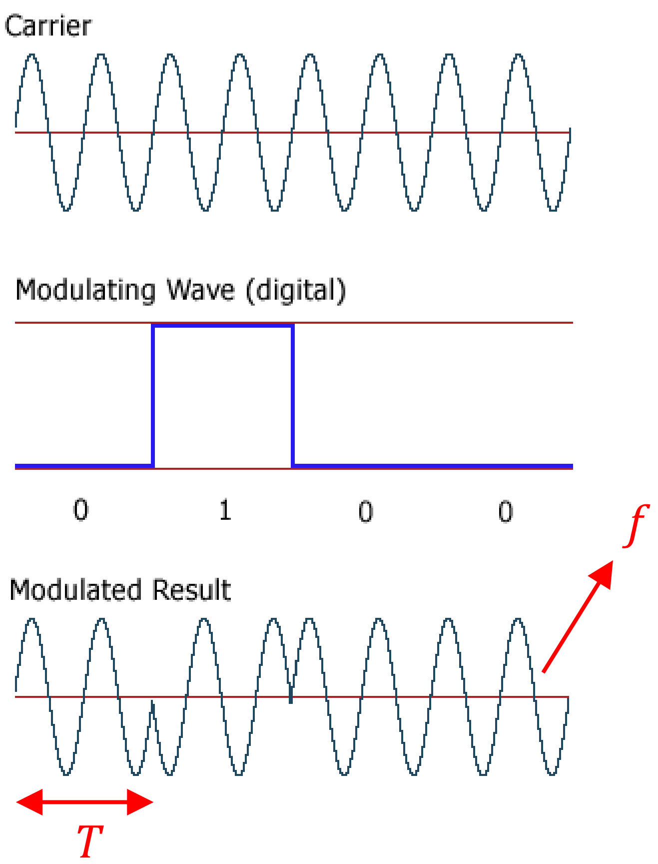
$$T_{sys} = T_A + T_{LNA} = 240 \text{ K}$$

The SNR is therefore:

$$SNR = \frac{P_R}{kT_{sys}B} = \frac{E_b R}{N_0 B} = \frac{E_b}{N_0} = 16 = 12 \text{ dB}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K) and assuming $R = B$. Considering the two modulations in the picture, PSK can guarantee the desired BER (BER = 10^{-8}), while FSK does not satisfy the constraint (BER > 10^{-4}).

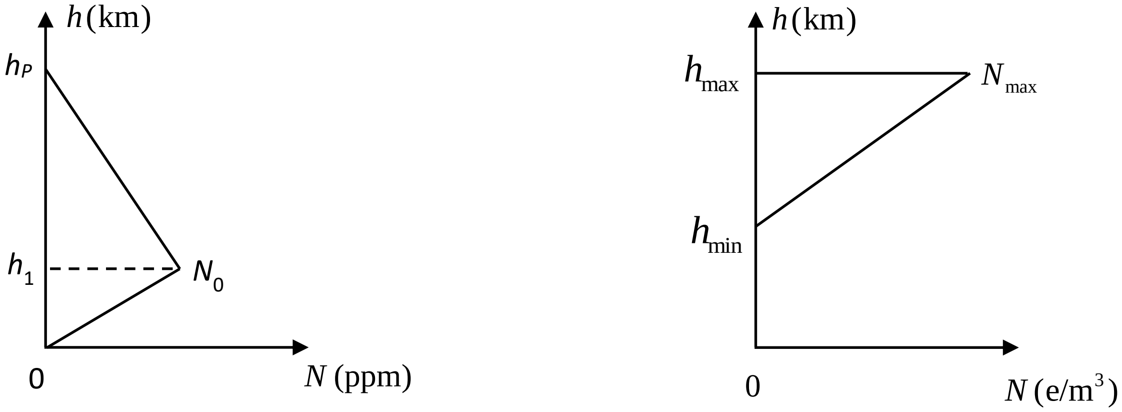
2) As the data rate is R = 30 Mbit/s, the duration of each bit $T = 1/R$ = 34 ns. Therefore, considering PSK and the given data stream, the radiofrequency signal is (for example):



Problem 4

A receiver of the IGS (International GNSS Service) network (i.e. whose position is known) tracks a GNSS satellites at distance r = 26248763 and at elevation angle θ = 20°. The pseudorange value measured by the receiver is ρ = 26248870.3 m. The wave crosses a ionospheric layer (horizontal homogeneous) with the vertical electron content profile sketched on the right side of the figure below ($h_{\max}$ = 350 km, $h_{\min}$ = 80 km, $N_{\max}$ = 3×10^{12} e/m^3) and the vertical refractivity profile (horizontal homogeneous, N_0 = 400, h_1 = 10 km, h_P = 35 km) sketched on the left side of the figure below. For this scenario:

1) Assuming perfect synchronization of the clock onboard the satellite and that the clock bias of the receiver is t_B = 30 ns, determine the value of the L1 C/A prompt code correlator.



2) Assume that the receiver now tracks 8 satellites, including the one at point 1), which provides the following DOP matrix:

$$R = \begin{bmatrix} 0.23 & -0.04 & -0.22 & -0.14 \\ -0.04 & 0.19 & 0.08 & 0.04 \\ -0.22 & 0.08 & 2.51 & 1.94 \\ -0.14 & 0.04 & 1.94 & 0.97 \end{bmatrix}$$

The receiver is used to synchronize an external device to UTC time. Calculate the synchronization error value that is provided for 99.73% of the time. To this aim, consider that 1-sigma UERE, equal for all satellites, to be σ = 6 m.

Solution

1) The pseudorange is affected by different error sources. The ionospheric delay, depends on the TEC, which can be calculated, in the zenithal direction, as:

$$\mathrm{TEC}_z = N_{\max}\frac{h_{\max} - h_{\min}}{2} = 40.5\ \mathrm{TECU}$$

The ionospheric delay for the satellite is obtained by scaling the zenithal one using the elevation angle:

$$d^I = c\,dt^I = \frac{40.3}{f^2}\frac{\mathrm{TEC}_z}{\sin(\theta)} = 19.2\ \mathrm{m}$$

As for the zenithal tropospheric delay, it is given by:

$$\tau = T - T_0 = \frac{10^{-6}}{c}\int_0^{h_P} N dh$$

The integral can be easily calculated as:

$$\int_0^{h_P} N dh = \frac{N_0 h_1}{2} + \frac{N_0(h_P - h_1)}{2} = 7\times10^6\ \mathrm{ppm/m}$$

As a result the tropospheric delay on the pseurorange is:

$d^T = c\,\dfrac{\tau}{\sin(\theta)} = 20.5$ m

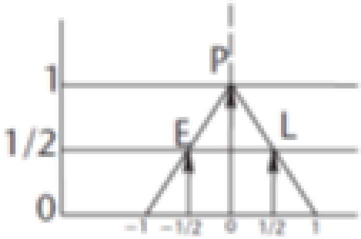
Considering all the contributions affecting the pseudorange, we obtain:

$\rho = r + d^I + d^T + d^{C/A} + d^C$

where $d^{C/A}$ is the error due to the correlator and $d^C = ct_B = 9$ m is the error due to the clock bias t_B. Therefore

$d^{C/A} = \rho - r - d^I - d^T - d^C = 58.7$ m

Considering the typical correlation function:



the error due to the correlator is given by:

$d^{C/A} = c\,T_{C/A}(1 - C)$

Therefore, the correlation value is:

$C = 1 - \dfrac{d^{C/A}}{c\,T_{C/A}} = 0.8$

2) To answer the question, let us first calculate TDOP as:

$\text{TDOP} = \frac{\sqrt{R_{44}}}{c} = 3.3$ ns

The desired synchronization error is given by:

$\sigma_t = \text{TDOP}\ 3\sigma = 59.1$ ns