Satellite Communication and Positioning Systems – Prof. L. Luini,
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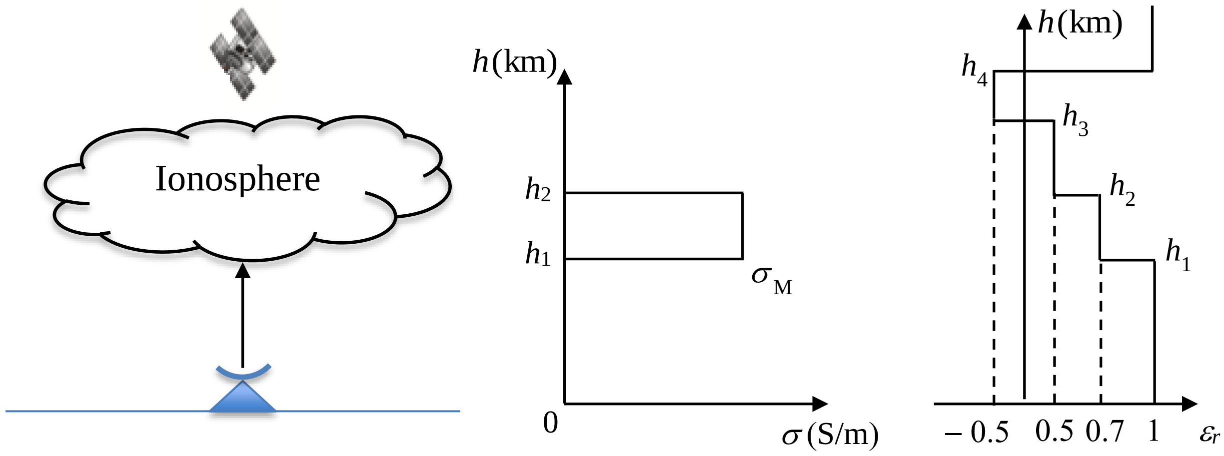
Problem 1

Making reference to the figure below, a ground station transmits an EM signal to a satellite (zenithal pointing). The figure also reports the vertical profile of the equivalent conductivity σ of the ionosphere ($\sigma_M = 1.03\times10^{-7}$ S/m, h_1 = 50 km, h_2 = 100 km), as well as the vertical profile of its equivalent relative permittivity ε_r (h_3 = 150 km, h_4 = 200 km). Both profiles are associated to the link frequency f = 9 MHz. For this scenario:

1) Determine if the wave reaches the satellite.
2) Calculate the peak electron content of the ionospheric profile.
3) Calculate the total path attenuation impairing the wave propagation.

Assume: if required, virtual reflection height $h_V = h_R$ (where h_R is the real reflection height); no tropospheric effects.



Solution

1) Looking at the vertical profile of the ionosphere, between h_3 and h_4, ε_r becomes negative (-0.5). This condition corresponds to having an evanescent wave, i.e. total reflection. As a result, at frequency f, the wave will not reach the satellite.

2) Recalling the expression of ε_r in the ionosphere (when there is no attenuation, which is the case for between h_3 and h_4, as $\sigma = 0$):

$$\varepsilon_r = 1 - \left(\frac{f_P}{f}\right)^2 = 1 - \left(\frac{9\sqrt{N}}{f}\right)^2$$

From the expression above, the higher the value of N, the more negative ε_r. The lowest value of ε_r is found between h_3 and h_4, i.e. where the highest value of N lies. Setting $N = N_{\max}$, and inverting the equation above:

$$N_{\max} = \frac{f^2(1-\varepsilon_r)}{81} = 1.5 \times 10^{12}\ \mathrm{e/m^3}$$

3) The wave travels from the ground up to h_3, where it is reflected back to ground. Given the profiles in the figure, the ionosphere attenuates only between h_1 and h_2, as $\sigma = 0$ elsewhere along the profile. The specific attenuation α is obtained from the propagation constant γ:

$$\gamma = \sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_r\varepsilon_0)} = 2.32 \times 10^{-5} + j0.158\ \ 1/\mathrm{m}$$

where $\sigma = \sigma_M$ and $\varepsilon_r = 0.7$.

Thus:

$$\alpha = 2.32 \times 10^{-5}\ \mathrm{Np/m} \ \rightarrow \ \alpha_{dB} = 0.2014\ \mathrm{dB/km}$$

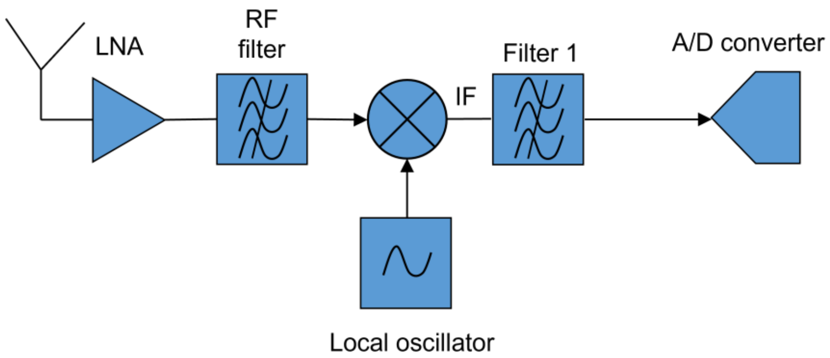
Therefore the path attenuation (one way) is:

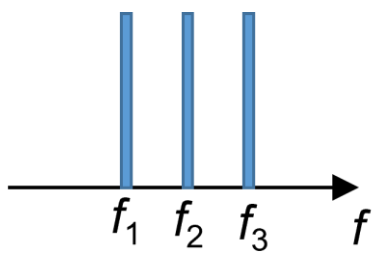
$$A = \alpha_{dB}(h_2 - h_1) = 10\ \mathrm{dB}$$

Problem 2

Consider the heterodyne receiver depicted below (left side), which aims at receiving the RF signal with carrier frequency f_2 = 30 GHz, to be digitalized at the end of the receiver chain. As indicated in the picture below (right side), the RF spectrum is occupied by multiple signals, all having the same bandwidth B = 100 MHz, with other carriers being f_1 = 28 GHz and f_3 = 32 GHz. The local oscillator frequency is f_{LO} = 29 GHz.

1) Assuming that the RF filter can be either a low-pass or a high-pass filter, indicate which one should be chosen and explain why. Finally, propose a suitable cutoff frequency for such a filter.
2) Calculate the optimum band of Filter 1 to maximize the SNR as input to the A/D converter.
3) Indicate the minimum sampling frequency of the A/D converter.
4) Calculate the exact SNR as input to the A/D converter knowing that:
    - the signal power as input to the LNA is P_1 = 1 pW;
    - the equivalent noise temperature as input to the LNA is T_{IN} = 250 K;
    - the equivalent noise temperatures of the LNA is T_{LNA} = 200 K and its gain is G_{LNA} = 30 dB;
    - that the other mixer and the RF filter are ideal, i.e. they do not introduce any loss/amplification/noise;
    - Filter 1 do not introduce any loss/amplification, but its equivalent noise temperatures is T_{F1} = 1000 K.





Solution

1) When down converting signals from RF to intermediate frequency (IF), image signals represent a problem. In this case, as the oscillator frequency is f_{LO} = 29 GHz, the image signal is the one with carrier f_1. As a result, the RF filter should be a high-pass filter, with cutoff frequency ideally around 29 GHz.

2) After down conversion, the carrier frequency f_2 is converted to $f_{IF} = f_2 - f_{LO}$ = 1 GHz. The optimum Filter 1 will have a lower cut-off frequency of $f_{min} = f_{IF}-B/2$ = 0.95 GHz and $f_{max} = f_{IF}+B/2$ = 1.05 GHz.

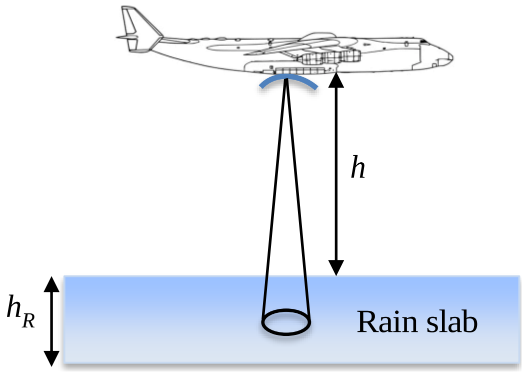
3) From the Nyquist theorem, the minimum sampling frequency of the A/D converter is $f_S = 2f_{max}$ = 2.1 GS/s.

4) The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_1}{kT_{sys}B} = \frac{P_1}{k(T_{IN} + T_{LNA} + T_{F1}/G_{LNA})B} = \frac{1\text{ pW}}{6.22 \times 10^{-13}} = 1.61 = 2.06\text{ dB}$$

Problem 3

Making reference to the figure below, a pulsed radar onboard an airplane, operating with carrier frequency f = 10 GHz and pointed zenithally, is used to measure precipitation. The beam illuminates an area filled with rain at distance h = 8 km from the top of the rain slab. The height of the rain slab is h_R = 2 km. The rain drops density is N = 100 drops/m^3, they all have the same dimension and same extinction section, i.e. C = 3 mm^2. The power transmitted by the radar is P_T = 370 W and the radar antenna gain is G = 35 dB. Calculate if the radar can measure the precipitation at position h_R /2 from the ground, knowing that the sensitivity of the radar receiver is P_R = 1 pW and that, at any position in the rain slab, the total radar cross section due to rain is σ = 6 m^2.



Solution

First, let us calculate the power density reaching the point of interest in the rain volume:

$S = \frac{P_T}{4\pi(h+h_R/2)^2} G A_R f = 8.516 \times 10^{-4}$ W/m^{2}v

where G = 3162, f = 1 (radar pointing to the volume).

A_R takes into account the attenuation induced by rain: in fact, as the wave penetrates into the rain slab, it gets partially reflected (see σ) and partially attenuated. The specific attenuation due to rain is calculated as:

$\alpha = \frac{1}{2} NC = 1.5 \times 10^{-4}$ Np/m

The path attenuation due to rain (from the beginning of the slab to the point of interest) is:

$A_R = e^{-2\alpha \frac{h_R}{2}} = 0.74$

The power reirradiated by the ensemble of rain drops is quantified by the total radar cross section due to rain σ (with gain = 1 according to the definition of radar cross section):

$P_t = S\sigma = 0.0051$ W

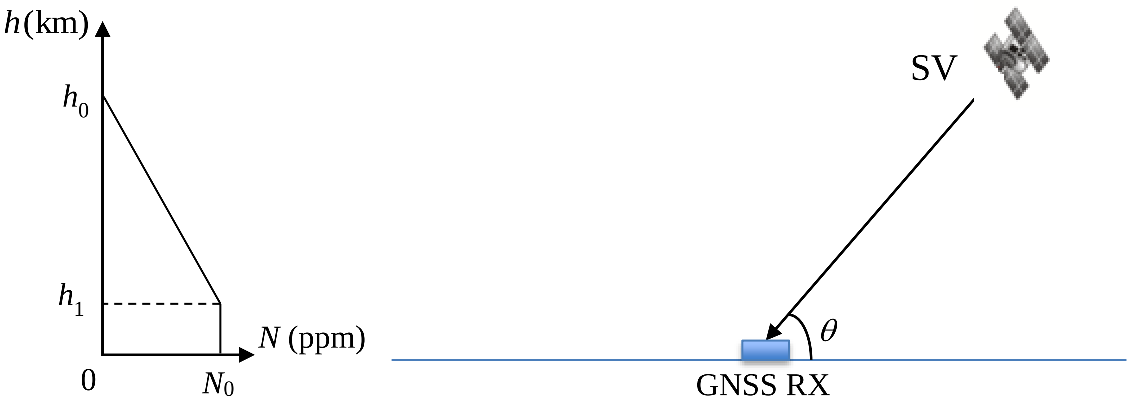
Thus, the power received by the radar is given by:

$$P_R = \frac{P_t}{4\pi(h+h_R/2)^2} A_R A_E = \frac{P_t}{4\pi(h+h_R/2)^2} A_R G \frac{\lambda^2}{4\pi} = 8.42 \times 10^{-13} \text{ W}$$

As P_R < 1 pW, the radar cannot measure the precipitation at the point of interest.

Problem 4

The figure below shows a dual-frequency GNSS RX tracking a GPS satellite. The position of the RX is known, and so is the line-of-sight distance to the satellite, L = 35182756 m. The refractivity vertical profile is shown on right side (N_0 = 700 ppm, h_0 = 10 km and h_1 = 2 km), while the zenithal TEC value is 40 TECU. The RX is affected by a code synchronization bias: the normalized early-minus discriminator value is -0.175 chips (L1 C/A code). The measured pseudorange is ρ = 35182713 m. Calculate the link elevation angle θ. Assume that there is perfect clock synchronization and that the atmosphere is horizontally homogeneous.



Solution

Considering the assumptions, the pseudorange is given by:

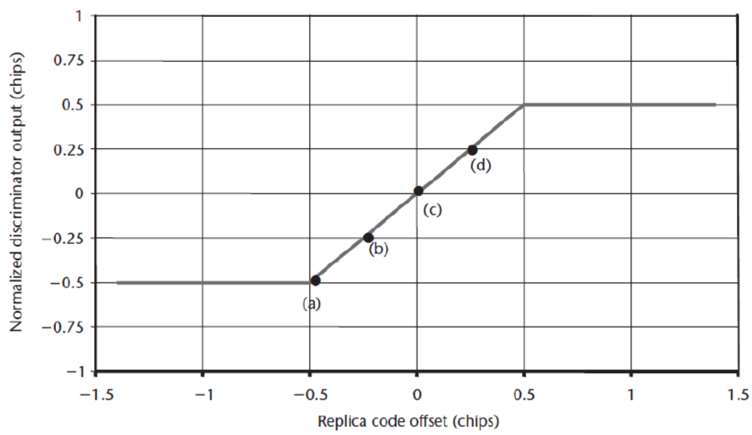
$$\rho = L + \frac{d^I}{\sin(\theta)} + \frac{d^T}{\sin(\theta)} + d^{C/A}$$

where:

$d^I = 0$ m, as a dual-frequency receiver is employed.

$d^T = 10^{-6}\left[h_1 N_0 + \frac{N_0(h_0 - h_1)}{2}\right] = 4.2$ m is the zenithal tropospheric error (given the profile in the figure).

$d^{C/A} = cT_{C/A}d_{C/A} = -51$ m ($T_{C/A} = 0.9775$ µs is the C/A code chip duration) is the code bias phase error. This is derived by the simple relationship between the normalized early-minus discriminator value and the prompt code offset, recalled here below for convenience. Note that, in this case, the code phase error contribution to the pseudorange is negative, as the prompt code is in advance.



As a result, by solving the pseudorange equation above for the elevation angle → θ = 30°.