Satellite Communication and Positioning Systems – Prof. L. Luini,
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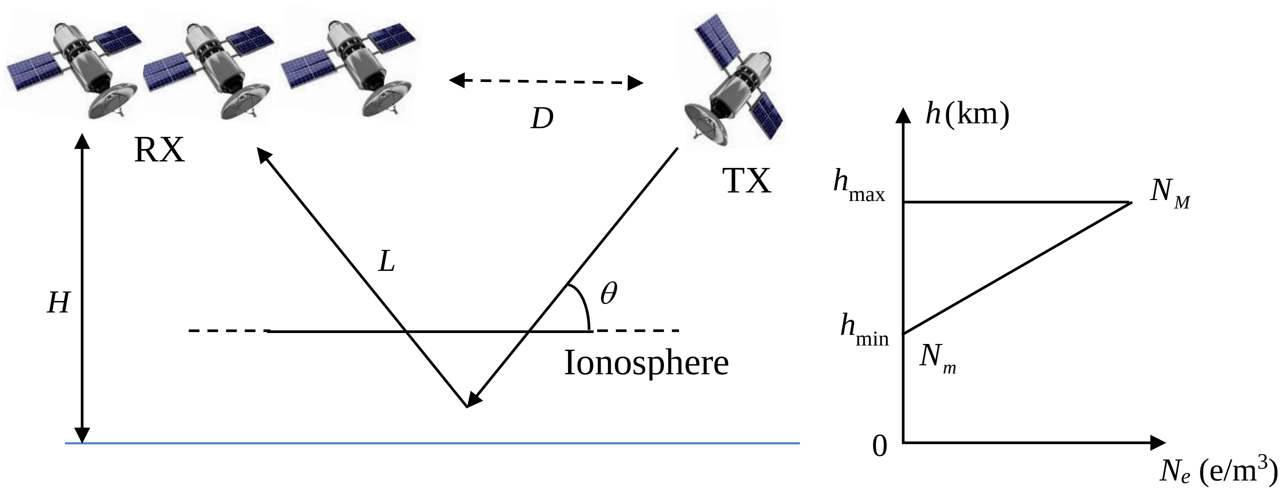
Problem 1

Making reference to the figure below, a bistatic space-borne radar system is designed to measure the height of the ionosphere (θ = 40°). The electron content profile is shown in the figure below (right side), where $h_{\max}$ = 400 km, $h_{\min}$ = 100 km, $N_M = 2\times10^{12}$ e/m^3, $N_m = 2\times10^{10}$ e/m^3. The radar can select different carrier frequencies, and the receiver consists of an array of satellites in formation flight, at increasing distance from the transmitter. For all satellites, the height above the ground is H = 800 km.

1) Determine the value of the radar operational frequency f_1 to correctly measure the height of the peak electron content of the provided profile. Calculate the pulse propagation time t_1 from TX to RX.
2) Calculate the propagation time from TX to RX when the radar operational frequency becomes $f_2 = 1.1\, f_1$.

Assume: flat Earth; virtual reflection height h_V equal to the real reflection height h_R; no tropospheric effects.



Solution

1) For the wave to be reflected at $h_{\max}$, the target radar operational frequency is:

$$f_1 = \sqrt{\frac{81 N_M}{1-[\cos(\theta)]^2}} \approx 19.8 \text{ MHz}$$

For any frequency lower than f_1, the wave will be reflected at h_{max}.

Above the ionosphere, free space propagation can be assumed. Let us first calculate L as:

$$L = \frac{H - h_{\max}}{\sin(\theta)} = 622.3 \text{ km}$$

Therefore, the pulse propagation time from TX to RX is:

$$t_1 = \frac{2L}{c} = 4.1 \text{ ms}$$

where c is the speed of light.

2) When the radar operational frequency is $f_2 = 1.1\, f_1 = 21.8$ GHz, the wave will fully cross the ionosphere and will be reflected from the ground. Therefore L will increase and, in addition, the pulse propagation time will be longer due to the additional ionospheric delay. The latter is given by:

$$dt_I = \frac{40.3}{c f_2^2} \frac{\text{TEC}}{\sin(\theta)} \approx 0.133 \text{ ms}$$

where:

$$\text{TEC} = \frac{(h_{\max} - h_{\min})(N_M - N_m)}{2} + (h_{\max} - h_{\min}) N_m = 30.3 \text{ TECU}$$

The new value for L is:

$$L_2 = \frac{H}{\sin(\theta)} = 1244.6 \text{ km}$$

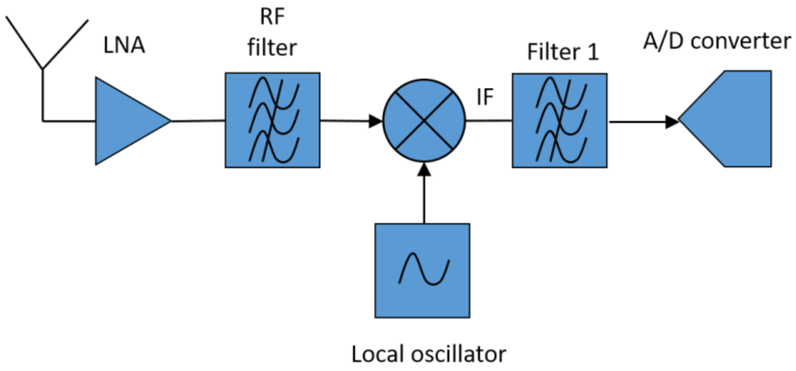
As a result, the pulse propagation time becomes:

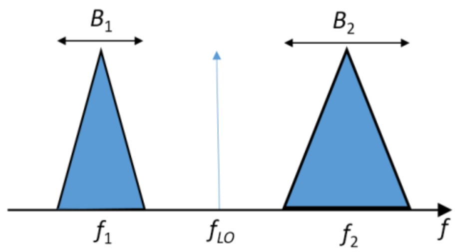
$$t_2 = \frac{2L_2}{c} + 2 dt_I = 8.6 \text{ ms}$$

Problem 2

Consider the heterodyne receiver depicted below (left side), where the local oscillator frequency is f_{LO} = 20 GHz. The RF spectrum is occupied by two signals: one centered about f_1 = 18 GHz with bandwidth B_1 = 1 GHz, and the other one centered about f_2 = 22 GHz with bandwidth B_2 = 2 GHz.

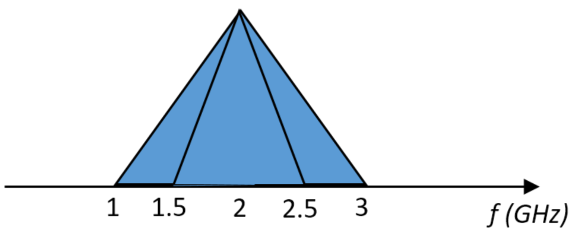
1) Show the spectrum occupation at intermediate frequency IF just after the mixer, when the RF filter is an ideal band-pass filter with bandwidth B_{RF} = 6 GHz, centered around f_{LO}.
2) Propose a different RF filter (the least expensive one) to correctly receive the signal centered about f_2 (define the filter type and its bandwidth) and show the spectrum in this case. From this point on, consider the new RF filter.
3) Define the ideal Filter 1 to maximize as much as possible the SNR as input to the A/D converter.
4) Define a suitable sampling frequency for the A/D converter.
5) Determine the symbol rate at the output of the A/D converter, considering a 10-level quantization of the samples and 4-PSK modulation.





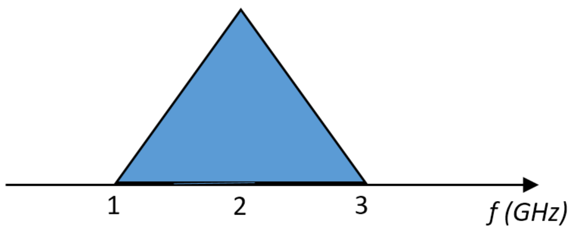
Solution

1) Considering the ideal bandpass RF filter, and that the local oscillator frequency will act on both signals, the spectrum occupation at IF will be:



The two signals will completely overlap at IF.

2) The RF filter will need to filter out the signal at f_1 to correctly receive the one at f_2. The least expensive solution is a high-pass filter with cut-off frequency around f_{LO}. The new IF spectrum occupation is:



3) The ideal Filter 1 will be lossless, with a bandwidth of 2 GHz centered about 2 GHz at IF.

4) The minimum sampling frequency for the A/D converter is $f_S = 2\,f_{max} = 6$ GS/s. To be on the safe side, $f_S = 6.1$ GS/s.

5) Considering 10 levels for the signal quantization, we need to use 4 bits to represent each sample (up to $2^4 = 16$ levels). As a result, the data rate will be $R = 6.1$ GS/s × 4 bits/S = 24.4 Gbit/s. Considering the 4-PSK modulation (2 bits per symbol), the symbol rate is $R_S = R/2 = 12.2$ Gbaud/s.

Problem 3

Consider a zenithal link (elevation angle θ = 90°) from a LEO satellite to a ground station (parabolic antenna with Cassegrain configuration), operating at f = 19 GHz, in which the signal crosses a uniform ice cloud (of thickness h = 4 km) consisting of equioriented ice needles. The specific attenuation of the ice cloud $\alpha_V = \alpha_H = \alpha$ = 0.05 dB/km (V and H are associated to the vertical and horizontal wave polarization, respectively) is constant and uniform through the whole cloud and, in addition, the ice needles cause a differential phase shift (between H and V) equal to 180°/km. Knowing that the satellite transmits a left-end circular polarization (LHCP):

1) What is the polarization in front of the receiver?

2) Calculate the signal-to-noise ratio (SNR) at the receiver.

3) Complete point 1 and 2 above, but considering that cloud thickness reduces to h = 1 km.

Assumptions:

- antennas optimally pointed
- the antenna on the ground receives LHCP waves

Additional data:

- mean radiating temperature T_{mr} = -5 °C
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 15 dB
- power transmitted by the satellite: P_T = 80 W
- altitude of the LEO satellite: H = 400 km
- bandwidth of the receiver: B = 5 MHz
- internal noise temperature of the receiver: T_R = 300 K

Solution

1) A LHCP wave consists of two orthogonal linear components with the same amplitude and a differential phase shift of 90°. The ice cloud, overall, causes the same attenuation on both components:

$$A_{dB} = \alpha h = 0.2 \text{ dB} \rightarrow A = 10^{-\frac{A_{dB}}{10}} = 0.955$$

which means that at the receiver the two components will have the same amplitude.

The cloud also causes a total differential phase shift of:

$$\Delta\phi = 180° h = 720°$$

Therefore at the receiver the two linear components will still produce a LHCP wave (720° corresponds to 2 full differential phase rotations).

2) The signal-to-noise ratio (SNR) is:

$$SNR = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi H\right)^2 G_R f_R A}{k\left(T_R + T_A\right)B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_R = f_T = 1$ (antenna optimally pointed). $T_A = T_{mr}$ (1-A) + A T_C = 14.7 K. Considering $\lambda = c/f$ = 0.0158 m, and using the data available → SNR = 15.4 dB.

3) If the cloud thickness reduces to h = 1 km, the differential phase shift between the H and V components of the wave becomes 90°+180° = 270° = -90° → the wave in front of the receiver has polarization RHCP. As the receiver antenna has LHCP, the received power will be very low, and so will the SNR.

Problem 4

We want to design a new navigation and communication system for the Moon, based on CDMA and consisting of several non-geosynchronous satellites deployed on different orbital planes.

1) Determine the approximate radiofrequency (RF) bandwidth necessary for the PRN code (rectangular chips) to achieve a ranging error (only due to code alignment) lower than 15 cm (assume 10% error on the code alignment).
2) In the light of the results of point 1, propose a suitable carrier frequency.
3) For the satellite deployment, two solutions are considered: 1) half of the satellite on the equatorial orbit and the other half on the polar orbit; 2) satellites deployed of 3 different orbits with diversified inclination. Which solution will guarantee the best positioning accuracy? Justify the answer.
4) Considering the PRN code duration to be N_C = 4000 chips, propose a suitable data rate R for the communication system.

Solution

1) The ranging error ε_C due to code alignment is tightly linked to the chip duration T_C; specifically:

$$\varepsilon_C = 0.1cT_C$$

The approximate bandwidth necessary to support such chip duration (at RF) is:

$$B_C = \frac{2}{T_C}$$

Therefore:

$$\varepsilon_C = 0.1c\frac{2}{B_C} < 0.15 \text{ m}$$

As a result:

$$B_C > 0.1c\frac{2}{0.15} = 400 \text{ MHz} = 0.4 \text{ GHz}$$

2) The carrier frequency f must be high enough to support the spreading signal whose bandwidth has been calculated in point 1. Considering that, the modulating signal bandwidth can reach up to 20% of the carrier, the minimum carrier frequency is:

$$0.2\, f > B_C$$

Therefore → $f > 2$ GHz.

3) Solution 2 is the best choice in terms of global satellite visibility, but it is also the best one in terms of positioning accuracy: indeed, such a solution will provide lower Dilution Of Precision (DOP) values, i.e. lower positioning errors, if compared to solution 2.

4) Considering N_C = 4000 chips, the PRN code duration is:

$$D_C = N_C T_C = 2 \times 10^{-5} \text{ s}$$

In a combined navigation and positioning system, the code acquisition is impaired if the bit duration T_B is of the same order of the PRN code duration. As a rule of thumb, let us consider that the bit duration is at least 10 times longer than the PRN code duration (other values can be selected, such as 20 – GPS system value), i.e.:

$$T_B = \frac{1}{R} \geq 10D_C = 2 \times 10^{-4} \text{ s}$$

Therefore:

$R \leq 5$ kb/s.