Satellite Communication and Positioning Systems – Prof. L. Luini,
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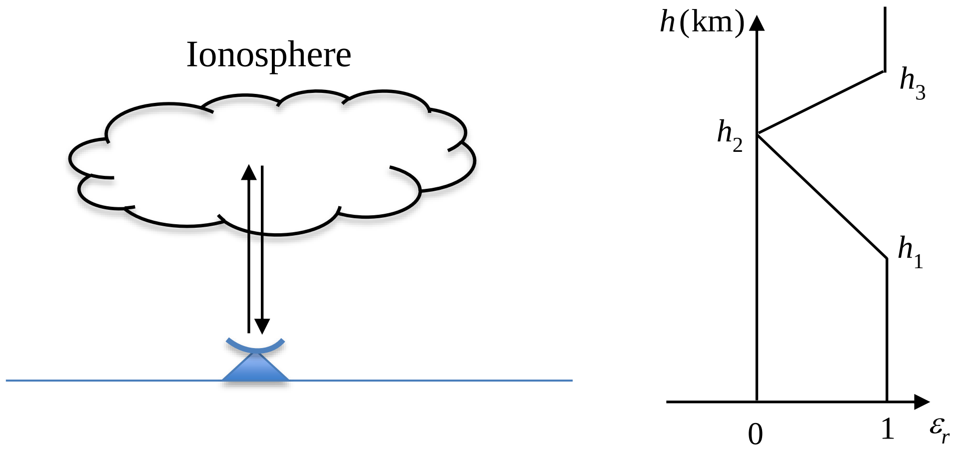
Problem 1

Making reference to the figure below, a monostatic ground-based pulsed radar system, operating at f = 18 MHz and pointing zenithally, aims at monitoring the ionosphere, i.e. the peak electron content and its value. The trend of ε_r in the ionosphere is depicted in the figure below on the right side (h_1 = 100 km). Knowing that the radar pulse round trip time is τ = 2.33169 ms, determine the value and position of peak electron content $N_{\max}$.

Assumption: neglect tropospheric effects.



Solution

The peak electron content value can be obtained from the following expression:

$$\cos(\theta) = \sqrt{1 - \left(\frac{9\sqrt{N_{\max}}}{f}\right)^2} = \sqrt{\varepsilon_r}$$

Setting θ = 90° and inverting the expression → $N_{\max}$ = 4×10^{12} e/m^3. $N_{\max}$ obviously corresponds to the lowest value of ε_r; also, being ε_r = 0 (for zenithal pointing) indicates that the wave is totally reflected exactly where $N_{\max}$ lies, i.e. at h_2.

The time required for the radar pulse to reach h_2 from the ground is $t = \tau/2 = 1.165845$ ms. Such time can be expressed as:

$$t = \frac{h_2}{c} + \frac{40.3}{cf^2}\mathrm{TEC} = \frac{h_2}{c} + \frac{40.3}{cf^2}\frac{(h_2 - h_1)N_{\mathrm{max}}}{2}$$

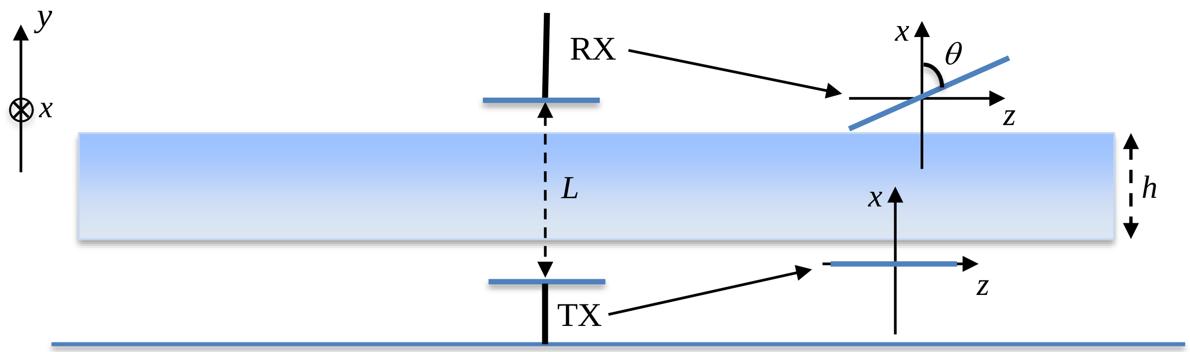
Solving for the only unknown, i.e. $h_2 \rightarrow h_2 = 300$ km.

Problem 2

An Earth-space link, with path length L = 400 km and operating at f = 90 GHz, crosses a homogeneous cloud with thickness h = 3 km. The transmitter (TX) uses a linear horizontal antenna, while the receiver (RX) linear horizontal antenna is tilted by an angle of $\theta = 60°$ (see sketch below) due to problems with the satellite attitude control. For this link:

1) Determine the polarization of the wave in front of the receiver RX (specify: LH or RH for a circular/elliptical, tilt angle for a linear).
2) Calculate the power density in front of the RX antenna.
3) Calculate the power received by RX.

Assume: EIRP = 36 dBW; effective area of the RX antenna, A_{RX} = 2 m^2; specific attenuation due to clouds, α_c = 2 dB/km; RX pointing at the Earth center and TX pointing at zenith; no additional losses at TX and RX.



Solution

1) As clouds consist of small spherical droplets, which are isotropic, the polarization transmitted by TX is unaffected by the presence of clouds.

2) The power density in front of the RX antenna is given by:

$$S_{RX} = \frac{EIRP}{4\pi L^2} f_T L_{TX} A_C \approx 0.5 \text{ nW/m}^2$$

where, given the assumptions: $f_T = L_{TX} = 1$. A_C is the cloud attenuation in linear scale. Such value in dB is obtained as: $A = \alpha_C h = 6$ dB → $A_C = 0.2512$.

3) The power received by RX needs to account for the RX antenna tilt:

$$P_{RX} = S_{RX} A_{RX} f_R L_{RX} [\cos(90 - \theta)]^2 \approx 0.75 \text{ nW}$$

Problem 3

In the context of satellite communication and positioning systems:

1) Discuss the different strategies employed to share the channel resources (e.g. frequency) among different users and/or satellites.
2) Which one is typically used in GNSSs? Why? What limits the number of users/satellites with such a technique?

Solution:

1) Three different strategies can be used, also in combination:

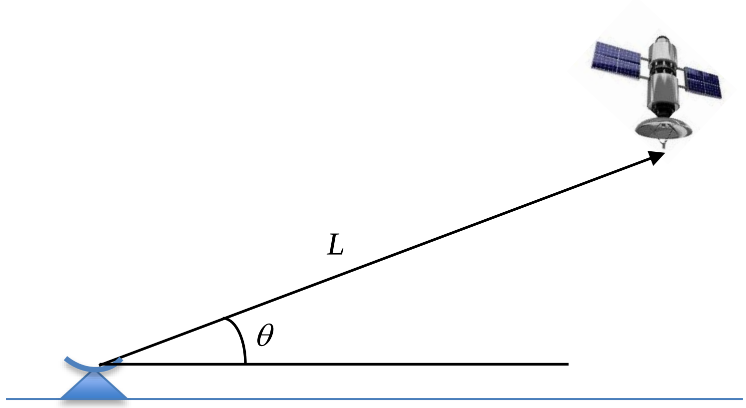
- TDMA (Time Division Multiple Access): each user is associated to a given time slot within each time frame, and they exploit the whole bandwidth allocated to the system.
- FDMA (Frequency Division Multiple Access): each user is associated to a portion of the whole frequency band allocated to the system, and they exploit completely each time frame.
- CDMA (Code Division Multiple Access): each user is associated to a code that needs to be as orthogonal as possible to the codes of the other users; in this case, each user can exploit the whole bandwidth allocated to the system, as well as completely each time frame.

2) The CDMA is the technique used in GNSSs for two main reasons: it allows identifying what satellites are acquired by the receiver and the pseudo random code is used by the receiver for satellite ranging (calculation of the pseudorange). The number of users/satellites allowable with CDMA is limited by the fact that each user will increase the equivalent noise power perceived by the other users: the higher the number of users (codes), the lower the SNR.

Problem 4

Consider the uplink to a LEO satellite (pointing at the Earth center) from a ground station, operating at f = 30 GHz, whose antenna is pointed zenithally. Determine the uplink yearly availability to guarantee a minimum signal-to-noise ratio (SNR) of 6 dB at the satellite. The CCDF of the zenithal tropospheric attenuation is given by:

$$P\left(A_T^{dB}\right) = 100 e^{-1.15 A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in \%})$$



Additional assumptions and data:

- elevation angle θ = 30°
- power transmitted from the ground P_T = 378 W
- radiation patter of the antennas (circular symmetry): $f = [\cos(\phi)]^2$
- equivalent noise temperature emitted by the ground T_B = 200 K
- mean radiating temperature T_{mr} = 285 K
- gain of the antennas (on board the satellite and on the ground) $G_T = G_R$ = 30 dB
- distance to the satellite L = 2400 km
- bandwidth of the receiver B = 2 MHz
- internal noise temperature of the receiver T_R = 310 K
- no additional losses in the transmitter and the receiver

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_S}{k[T_R + T_A]B}$$

where A_S is the slant path attenuation in linear scale, $f_T = f_R = [\cos(90°-\theta)]^2 = 0.25$, k is the Boltzmann's constant (1.38×10^{-23} J/K) and T_A is the equivalent antenna noise temperature. For this scenario, as the satellite points at the ground, T_A is therefore calculated as:

$$T_A = A_S T_B + T_{mr}(1 - A_S)$$

Therefore, imposing $SNR_{\min}$ = 6 dB = 3.981:

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_S}{k[T_R + A_S T_B + T_{mr}(1 - A_S)]B} = 3.981$$

Inverting the equation above to solve for A_S:

$$A_S = \frac{SNR_{min}(BkT_R + T_{mr}Bk)}{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R - SNR_{min}(BkT_B - Bk)} = 0.0254$$

In dB:

$$A_S^{dB} = -10\log_{10}(A_S) = 15.948 \text{ dB}$$

The zenithal attenuation is:

$$A_T^{dB} = A_S^{dB} \sin(\theta) = 7.974 \text{ dB}$$

Using the CCDF expression, such an attenuation corresponds to an outage probability of roughly 0.01% in a year, i.e. to a yearly availability of 99.99%.