Satellite Communication and Positioning Systems – Prof. L. Luini,
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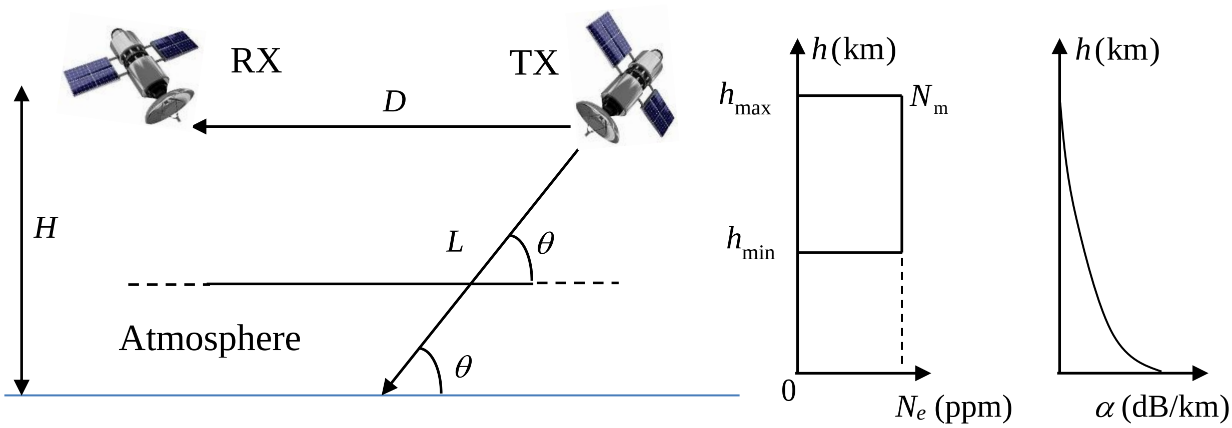
Problem 1

The figure below shows a bistatic radar system (carrier frequency f_1 = 1 GHz), consisting of two LEO satellites (same orbit, satellite height above the ground H = 500 km, elevation angle θ = 45°). The radar extracts information on the atmosphere by measuring the difference between the power received along the direct path and the downlink to the ground station. Making reference to the right, where the electron content (N_e) profile ($h_{\min}$ = 100 km, $h_{\max}$ = 400 km, N_m = 5×10^{12} e/m^3, N homogeneous horizontally) and the tropospheric specific attenuation profile ($\alpha = 0.01e^{-h/50}$, where h is in km and α, horizontally homogeneous, is in dB/km) are shown, and to the left side, where the simplified geometry is reported:

1) Calculate the difference of the power received along the two paths.
2) Will the difference between the power received along the two paths decrease or increase (compared to point 1) if the frequency changes to f_2 = 25 MHz?

Assume: transmit power P_T = 100 W; gain of all antennas (circular parabolic reflectors) G = 45 dB; antenna radiation pattern $f = [\cos(\phi)]^2$, where ϕ is the angle between the specific direction and the antenna axis; perfect pointing for the downlink, on both ends.



Solution

1) The power received along the direct path (free space) is simply given by:

$P_{R1} = P_T G_T f_T (\lambda/4\pi D)^2 G_R f_R$

where:

$$D = \frac{2H}{\mathrm{tg}(\theta)}$$

$$G_T = G_R = G$$

$$f_T = f_R = \cos(\theta)$$

Therefore → $P_{R1} = 14.2$ µW

Regarding the second path, the power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l$$

where $L = H/\sin(\theta)$ and A_l is the tropospheric attenuation (the ionospheric one can be neglected given the carrier frequency in the GHz range). The zenithal tropospheric attenuation can be calculated as:

$$A_Z = \int_0^H \alpha dh = \alpha_0 \int_H^{H_S} e^{-\frac{h}{h_0}} dh = -\alpha_0 h_0 \left[e^{-\frac{h}{h_0}} \right]_0^H$$

As $H >> h_0 = 50$ km:

$$A_Z = -\alpha_0 h_0 \left[e^{-\frac{h}{h_0}} \right]_0^{\infty} = \alpha_0 h_0 = 0.5 \text{ dB}$$

The slant path attenuation in linear scale is:

$$A_l = 10^{-(A_Z/\sin(\theta))/10} = \alpha_0 h_0 = 0.85 \text{ dB}$$

The power received on the ground is:

$$P_{R2} = S A_{RX} = \frac{P_T}{4\pi L^2} G A_l \frac{\lambda^2}{4\pi} G = 96.9 \text{ µW}$$

The differential power is:

$$\Delta P_R = P_{R2} - P_{R1} = 82.6 \text{ µW}$$

2) Using the second frequency (range of tens of MHz), it is worth checking if the wave can actually cross the ionosphere. For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as by:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{m}}}{f_2} \right)^2} \quad \Rightarrow \quad \theta_{min} = 53.6°$$

As $\theta < \theta_{min}$, the wave will be totally reflected and it will not be received by RX.

Problem 2

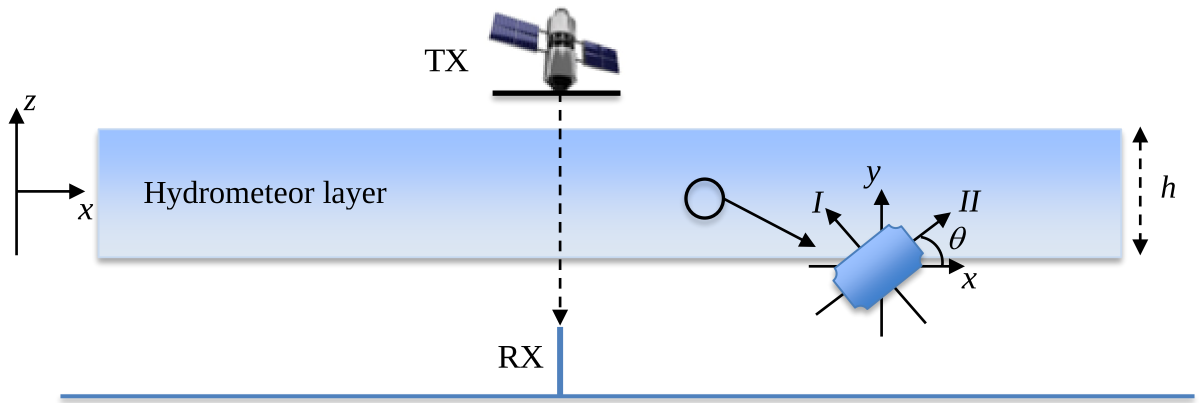
A plane EM wave ($|\vec{E}_{TX}| = 10$ V/m) at 30 GHz propagates across a layer of melting hydrometeors (layer thickness h = 1 km, θ = 45° tilt angle), which is characterized by the following propagation constants (see sketch below):

$\gamma_I = 0.6413 + j628676$ 1/km

$\gamma_{II} = 0.6413 + j628674.43$ 1/km

Both the transmitter (TX) and the receiver (RX) employ linear antennas; the TX antenna is horizontal linear; as for the RX antenna, see point 2). For this link:

1) Determine the wave polarization in front of the receiver RX (no need to specify the tilt angle for a linear polarization, nor the rotation direction for a circular/elliptical polarization).
2) Based on point 1, determine the best antenna to maximize the received power.



Solution

1) As is clear from the propagation constants, the hydrometeor slab induces the same attenuation on both I and II components of the wave; on the other hand, the differential phase shift is:

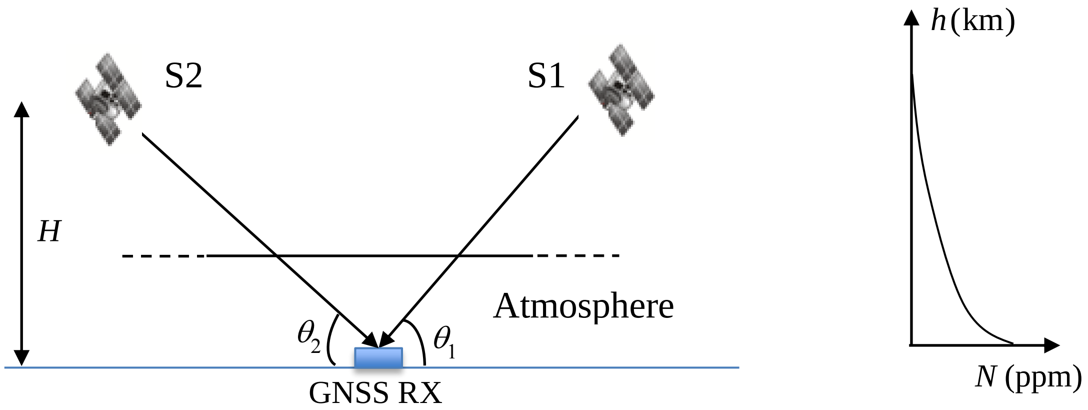
$\Delta\beta = \beta_{II} - \beta_I = -1.57 = -\pi/2$ rad/km

The TX antenna emits a vertical polarization, so the amplitude of components I and II are equal and given by $|\vec{E}_{TX}|\cos(45°) = 7.07$ V/m. Therefore, in front of RX, the two components I and II will have: 1) same amplitude (attenuated by the slab); 2) a differential phase shift of $\pi/2$ rad/km → the polarization will be circular.

2) Given the circular polarization determined at point 1, a circular polarized antenna is the best choice.

Problem 3

The figure below shows a dual-frequency GNSS RX (positioned at average mean sea level altitude) tracking two GPS satellites at different elevation angles (θ_1 = 45°, θ_2 = 30°, satellites height above the ground H = 20180 km). Making reference to the right, where the refractivity profile ($N = N_0 e^{-h/40000}$ ppm, where h is in m and N is homogeneous horizontally) are shown, and to the left side, where the simplified geometry is reported (flat Earth): calculate the value of N_0 and the receiver clock bias, knowing that the pseudorange for S1 and S2 measured by the GNSS RX are ρ_1 = 28538868 m and ρ_2 = 40360058 m, respectively. To this aim, assume perfect code synchronization.



Solution

The two slant paths can be easily calculated as:

$$L_1 = \frac{H}{\sin(\theta_1)} \approx 28538829 \text{ m}$$

$$L_2 = \frac{H}{\sin(\theta_2)} \approx 40360000 \text{ m}$$

The pseudorange is given by:

$$\rho = L + \frac{d^I}{\sin(\theta)} + \frac{d^T}{\sin(\theta)} + d^{C/A} + ct_B$$

Assuming perfect code synchronization → $d^{C/A} = 0$ m; also, for a dual-frequency receiver → $d^I = 0$ m.

$$\rho = L + \frac{d^T}{\sin(\theta)} + ct_B$$

Given the exponential trend of the refractivity profile, the zenital tropospheric delay can be calculated as:

$$d^T = -h_0 N_0 10^{-6} \left[e^{-\frac{h}{h_0}} \right]_0^H$$

Considering the satellite height $H >> h_0$ = 40 km:

$d^T = h_0 N_0 10^{-6}$

We have two equations with two unknowns (d^T and t_B). Taking the difference of ρ_2 and ρ_1:

$$\rho_2 - \rho_1 = (L_2 - L_1) + d^T \left[\frac{1}{\sin(\theta_2)} - \frac{1}{\sin(\theta_1)}\right]$$

as the clock bias is the same for both pseudoranges. As a result:

$$d^T = \frac{(\rho_2 - \rho_1) - (L_2 - L_1)}{\left[\frac{1}{\sin(\theta_2)} - \frac{1}{\sin(\theta_1)}\right]} = 32 \text{ m}$$

Therefore:

$$N_0 = \frac{d^T}{h_0 10^{-6}} = 800 \text{ ppm}$$

Finally:

$$\rho_1 = L_1 + \frac{d^T}{\sin(\theta_1)} + c t_B$$

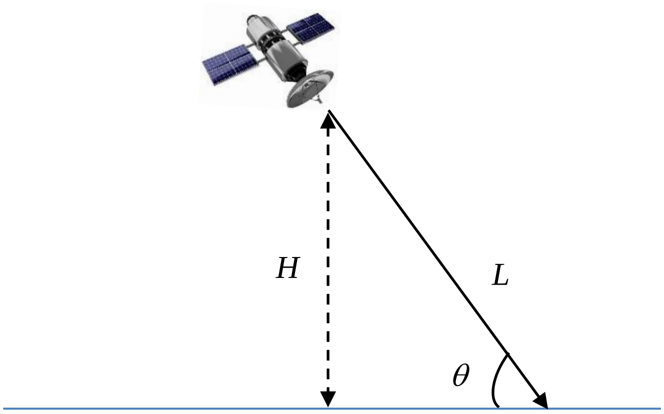
$$t_B = \frac{1}{c}\left[\rho_1 - L_1 - \frac{d^T}{\sin(\theta_1)}\right] = -20 \text{ ns}$$

Problem 4

Consider a satellite link, implementing adaptive coding and modulation, which adapts the data rate depending on the link signal to noise ratio (SNR). The link elevation angle is θ = 45° and the link operates at f = 20 GHz. The Complementary Cumulative Distribution Function (CCDF) of the zenithal tropospheric attenuation is given by:

$$P(A_{dB}^{Z}) = 100e^{-0.545\,A_{dB}^{Z}} \quad (A_{dB}^{Z} \text{ in dB and } P \text{ in \%})$$

Determine the yearly time for which 10 Mbit/s can be guaranteed to the user.



15 dB < SNR ≤ 20 dB	D = 20 Mbit/s
10 dB < SNR ≤ 15 dB	D = 10 Mbit/s
SNR ≤ 10 dB	D = 1 Mbit/s

Additional assumptions and data:

- both antennas pointed optimally
- disregard the cosmic background radiation
- power transmitted by each satellite P_T = 100 W
- mean radiating temperature T_{mr} = 290 K
- gain of both antennas G = 35 dB
- satellite altitude H = 600 km
- bandwidth of the receiver: B = 100 MHz
- internal noise temperature of the receiver: T_R = 300 K

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A}{k[T_R + T_{mr}(1-A)]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = f_R = 1$, $L = H/\sin(\theta)$, $G_R = G_T = G$ (same antenna features). The target data rate is obtained for 10 dB < SNR ≤ 15 dB: to calculate the yearly time for which 10 Mbit/s can be guaranteed to the user, the threshold SNR$_{\min}$ = 10 dB is used. Therefore, the above equation can be solved for A (slant path attenuation in linear scale):

$$A = \frac{SNR_{min} kB(T_R + T_{mr})}{P_T G^2 (\lambda/4\pi L)^2 + SNR_{min} kBT_{mr}} = 0.0041$$

The slant path attenuation in dB is:

$$A_{dB} = -10\log_{10}(A) = 23.9 \text{ dB}$$

The zenithal path attenuation in dB is:

$A_{dB}^{Z} = A_{dB}\sin(\theta) = 16.9\ \text{dB}$

Using such a value in the CCDF of the tropospheric attenuation:

P = 0.01%, which corresponds to approximately 0.89 hours in a year. Therefore, 10 Mbit/s can be guaranteed for 99.99% of the yearly time, i.e. always but 0.89 hours (53 minutes) in a year.