Satellite Communication and Positioning Systems – Prof. L. Luini,
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SURNAME AND NAME ______________________________

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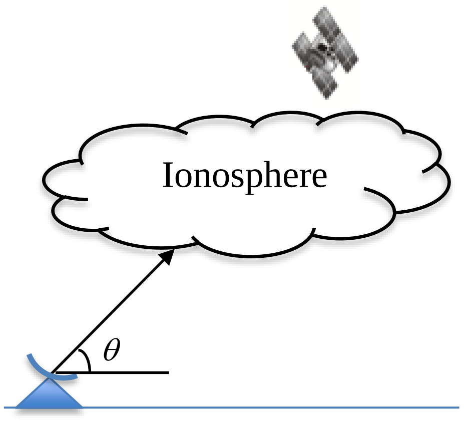
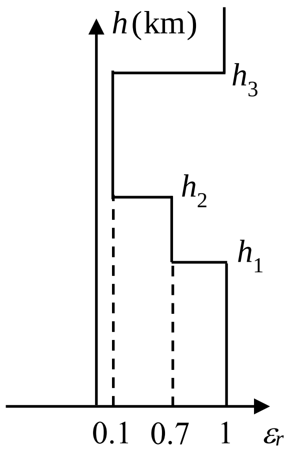
Problem 1

Making reference to the figure below, a ground station transmits an EM signal to a satellite with elevation angle $\theta = 60°$. The figure also reports the vertical profile of the relative permittivity ε_r in the ionosphere (h_1 = 100 km, h_2 = 150 km and h_3 = 200 km), associated to the link frequency f = 12 MHz. For this scenario:

1) 3 Draw the vertical profile of the total electron content N.
2) 3 Determine if the wave reaches the satellite.
3) 4 Determine the change in the elevation angle necessary to modify the condition at point 2 (from crossing to reflection; from reflection to crossing).





Solution

1) Inverting the usual relationship and using the values in the profile:

$$\sqrt{\varepsilon_r} = \sqrt{1 - \left(\frac{f_P}{f}\right)^2} = \sqrt{1 - \frac{81N}{f^2}}$$

we obtain → $N_1 = 5.3\times10^{11}$ e/m^3 (value between h_1 and h_2) and $N_2 = 1.6\times10^{12}$ e/m^3 (value between h_2 and h_3).

2) The condition to obtain total reflection is

$\cos(\theta) = \sqrt{\varepsilon_r}$

For the given elevation angle, $\cos(\theta) = 0.5$. As this value lies between $\sqrt{\varepsilon_{r1}} = 0.3162$ and $\sqrt{\varepsilon_{r2}} = 0.8367$, the wave is totally reflected at h_2.

3) For the wave to cross the ionosphere, it is necessary to increase the elevation angle:

$\cos(\theta') = \sqrt{0.1} = 0.3162 \rightarrow \theta' = 71.57°$

For any elevation angle higher that θ', the wave will cross the ionosphere.

Problem 2

A signal, whose baseband spectrum is indicated in the first figure below, is transmitted with circular polarization from the ground to a satellite, with carrier frequency f_C = 1 GHz.

1) 3 For that carrier frequency, which atmospheric effect(s) need to be taken into account?

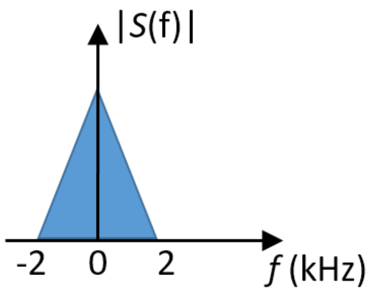
Before transmission, the signal is digitalized.

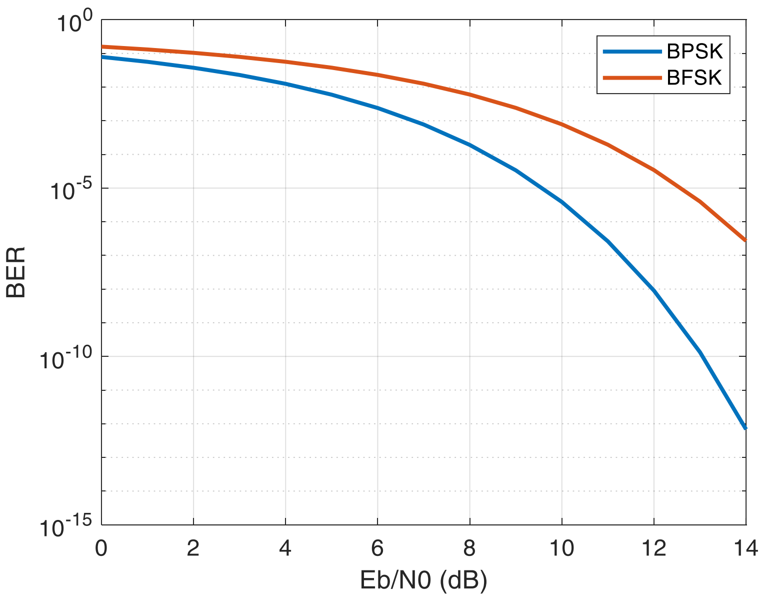
2) 3 Indicate a suitable sampling frequency to be used.

The receiving system is a heterodyne receiver characterized by a system equivalent noise temperature T_{sys} = 1000 K. The power received by the antenna is P_R = 1 fW.

3) 4 Considering the second figure below, which modulation(s) can be used to guarantee an error free communication (BER < 10^{-6})?

Assume that the signal bandwidth is the same also at radio frequency, regardless of the modulation type.





Solution

1) At that carrier frequency, the ionosphere becomes almost transparent, and the troposphere has quite limited effects as well. The only effect to be taken into account is delay, induced both by the electron content and by the tropospheric gasses. Given the circular polarization, also depolarization can be neglected.

2) The maximum frequency component of the baseband signal if f_M = 2 kHz: the minimum sampling frequency will be $f_S = 2f_M$ = 4 kb/s.

3) Given the specifications, the SNR will be:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B} = 12.58 \text{ dB}$$

Using the curves in the graph, only the BPSK can guarantee an error free communication.

Problem 3

Making reference to the figure below, a satellite, equipped with a linear antenna oriented along *x*, transmits a signal to a ground station, whose receiving antenna is linear and oriented along *z*. The distance between the satellite and the ground station is d = 600 km and the link operates at f = 20 GHz. The signal crosses the layer consisting of anisotropic particles (h = 400 m, h_S = 598 km), all oriented as indicated in the figure below. The propagation constants associated to the two principal axes of the particles are:

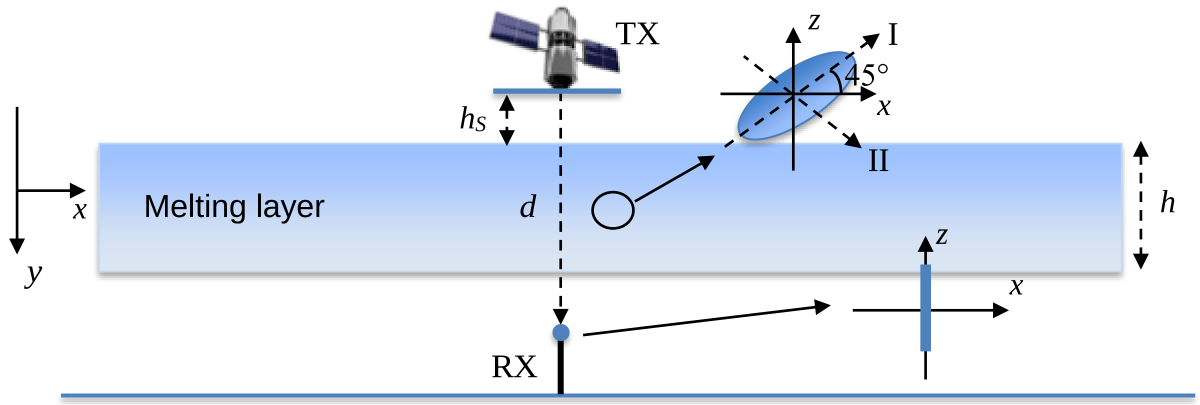
γ_I =3.4×10^{-4}+j1623 1/m

γ_{II} =2.4×10^{-4}+j1082 1/m

The absolute value of the electric field emitted by the satellite antenna is $E_0 = 100$ V/m.

For this link, calculate the power received at RX.

Additional data: disregard additional attenuation due to the atmosphere; the effective area of the receiver antenna A_E = 0.004 m^2. Assume plane wave propagation. Assume free space propagation outside the melting layer.



Solution

The power received by the RX antenna will depend on the *z* electric field component at RX. The propagation along the two principal axes of the particle will be different, according to the associated propagation constants. First, the *x* component of the electric field is projected along directions I and II:

$$E_I(y=0) = E_0(0)\cos(45°) = E_0(0)/\sqrt{2}$$
$$E_{II}(y=0) = E_0(0)\sin(45°) = E_0(0)/\sqrt{2}$$

Assuming plane wave propagation, these two components will propagate in the same way from the satellite to the upper limit of the melting layer and from its lower limit to RX, while they will propagate differently in the melting layer. Thus, the two components reaching RX will be:

$$E_I(y=d) = E_0(0)/\sqrt{2}\ e^{-j\beta_0 h_S} e^{-\gamma_I h} e^{-j\beta_0(d-h_S-h)}$$
$$E_{II}(y=d) = E_0(0)/\sqrt{2}\ e^{-j\beta_0 h_S} e^{-\gamma_{II} h} e^{-j\beta_0(d-h_S-h)}$$

Finally, the field along *z* is obtained as:

$$E_z(y=d) = E_I(y=d)\sin(45°) - E_{II}(y=d)\cos(45°) = -64 + j9.5 \quad \text{V/m} \ \rightarrow \ |E_z(y=d)| = 26.4 \text{ V/m}$$

Note that, for the calculation of the absolute value of E_z, in turn needed to calculate the power density reaching the RX antenna, the propagation terms in "free space" involving β_0 can be neglected (no additional depolarization is introduced).

The power density reaching RX is therefore:

$$S_{RX} = \frac{1}{2}\frac{|E_z(y = d)|^2}{\eta_0} \approx 0.9249 \text{ W}$$

Finally, the received power is:

$$P_{RX} = S_{RX} A_E = 3.7 \text{ mW}$$

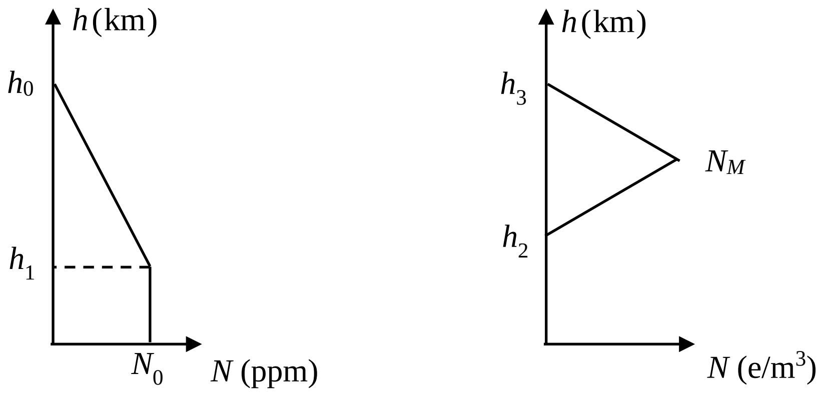
Problem 4

We want to design a single-frequency GNSS receiver for the Earth that needs to give a maximum SV range error of 30 m while providing the PVT solution and operating at an altitude of h_S = 500 m. The equivalent noise temperature of the target receiver is $T_R = 1200$ K. The error due to the code correlator is associated to the SNR before dispreading as follows:

$$d^{C/A} = -0.205SNR + 20.59 \quad \text{(SNR in linear scale, } d^{C/A} \text{ in m)}$$

The system is conceived to work correctly considering: the reference refractivity profile modeled as sketched in the figure below (left side), where N_0 = 700 ppm, h_1 = 1 km and h_0 = 9.5 km; the electron content profile reported in the figure below (right side), where $N_M = 2\times10^{12}$ e/m^3, h_2 = 50 km and h_3 = 400 km; the target elevation angle θ = 20°; the satellite distance L = 22000 km, the carrier frequency f = 30 GHz, the rain rate R = 5 mm/h (α = 0.9311 and k = 0.2347, rain height h_R = 2 km). Determine the chip rate (BPSK-R) to guarantee the target range error.

Assumption: disregard the gaseous attenuation, the mean radiating temperature is T_{mr} = 270 K, the transmit power P_T = 150, the antennas of the transmitter and receiver have gain G = 32 dB, perfect pointing.



Solution

Under the assumption that the GNSS receiver is correctly providing the PVT solution, there is no clock bias. The pseudorange is therefore affected by the following error sources:

$$\rho = r + d^{C/A} + d^I + d^T$$

The slant tropospheric delay can be calculated as:

$$d^T = \frac{10^{-6}}{\sin(\theta)} \int_{h_S}^{h_0} N\,dh = \frac{10^{-6}}{\sin(\theta)} \left[(h_1 - h_S)N_0 + \frac{(h_0 - h_1)N_0}{2} \right] = 9.72 \text{ m}$$

The ionospheric delay can be neglected because of the high operational frequency.

$d^{C/A}$ depends on the SNR.

The frequency is f = 30 GHz, so the wavelength is $\lambda = c/f$ = 0.01 m. Such a high frequency is subject to rain attenuation, which amounts to:

$$A_{dB} = kR^{\alpha} \frac{h_R}{\sin(\theta)} = 6.14 \text{ dB} \rightarrow A = 0.243$$

The received power can be thus calculated as:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi L} \right)^2 G_R f_R A = 1.2 \times 10^{-13} \text{ W}$$

The system noise power depends on the antenna noise temperature T_A and on the receiver noise temperature T_R:

$$T^{sys} \approx T_C A_{lin} + T_{mr}(1 - A_{lin}) + T_R = 1405\ \mathrm{K}$$

Before despreading, the bandwidth to accommodate the whole GNSS signal is quite large and it depends on the chipping rate T_C as:

$$B = \frac{2}{T_C}$$

The noise power is:

$$P_N = kT_{sys}B = kT_{sys}\frac{2}{T_C}$$

As a result:

$$SNR = \frac{P_R T_C}{kT_{sys}2}$$

The target range error is: $d = d^{C/A} + d^T = 30\ \mathrm{m} \ \rightarrow\ d^{C/A} = 20.28\ \mathrm{m}$.

Imposing:

$$d^{C/A} = -0.205 SNR + 20.59 < 20.28$$

$$-0.205\frac{P_R T_C}{kT_{sys}2} + 20.59 < 20.28$$

$$\frac{P_R T_C}{kT_{sys}2} > 1.5122$$

$$T_C > \frac{1.5122\ kT_{sys}2}{P_R} = 489.3\ \mathrm{ns} \ \rightarrow\ B = \frac{2}{T_C} \approx 4.09\ \mathrm{MHz}$$