Satellite Communication and Positioning Systems – Prof. L. Luini,
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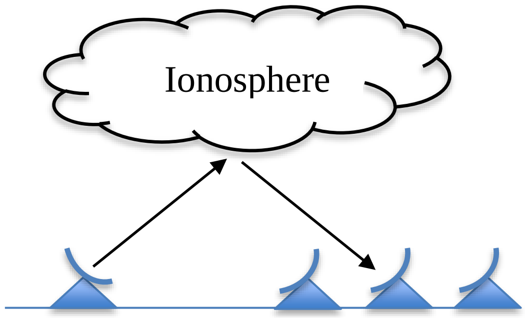
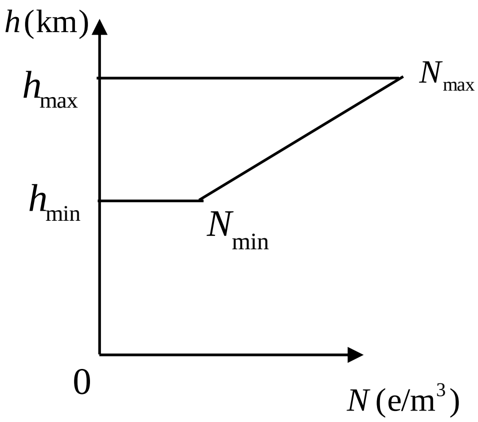
Problem 1

Making reference to the figure below, a bistatic ground-based radar system aims at measuring the extent of the ionosphere, as well as the minimum and maximum value of the electron content. The transmitter elevation angle is θ = 40°. The electron content profile is shown in the figure below (right side), where h_{max} = 400 km, h_{min} = 100 km. The radar operates by gradually switching the carrier frequency from 5 MHz to 50 MHz, and the receiver consists of an array of terminals at increasing distance from the transmitter.

1) Determine the value of N_{min} knowing that the radar starts to receive the reflected signal at f_1 = 10 MHz and determine the distance d_1 between the transmitter and the terminal receiving the signal.
2) Determine the value of N_{max} knowing that the radar no longer receives the reflected signal for frequencies higher than f_2 = 30 MHz and determine the distance d_2 between the transmitter and the terminal receiving the signal.
3) Calculate the propagation time of the EM pulse from the transmitter to the receiver for the conditions at point 1)
4) Calculate the propagation time of the EM pulse from the transmitter to the receiver for the conditions at point 2)

Assume: virtual reflection height h_V = 1.1h_R (where h_R is the real reflection height), no tropospheric effects.





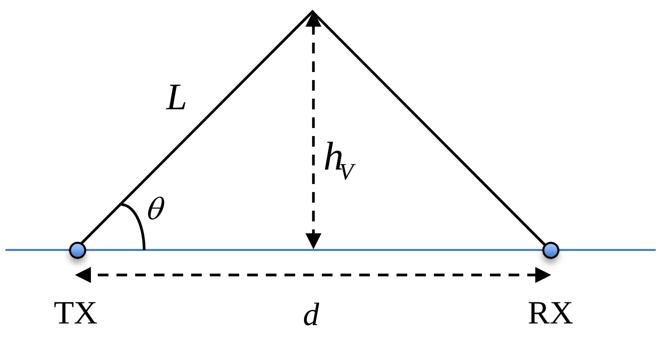
Solution

1) The radar starts to receive some signal when the first reflection at the base of the ionosphere occurs, due to N_{min}. It is obtained as:

$$N_{\text{min}} = \frac{f_1^2(1 - [\cos(\theta)]^2)}{81} \approx 5.1 \times 10^{11}\ \text{e/m}^3$$

Exploiting the concept of virtual reflection height, and making reference to the figure below, d_1 is given by:

$$d_1 = \frac{2h_{V1}}{\text{tg}(\theta)} = \frac{2.2\ h_{\text{min}}}{\text{tg}(\theta)} = 262.2\ \text{km}$$



2) The radar no longer receives the reflected signal when the wave crosses the ionosphere, which occurs at 30 MHz. Using the same equations above, but with f_2:

$$N_{\text{max}} = \frac{f_2^2(1 - [\cos(\theta)]^2)}{81} \approx 4.6 \times 10^{12}\ \text{e/m}^3$$

and

$$d_2 = \frac{2h_{V2}}{\text{tg}(\theta)} = \frac{2.2\ h_{\text{max}}}{\text{tg}(\theta)} = 1048.7\ \text{km}$$

3) Assuming to neglect the effect of the troposphere, the propagation time in this case is simply:

$$t_1 = \frac{2L_1}{c} = \frac{2(h_{V1}/\sin(\theta))}{c} = 0.0011\ \text{s}$$

4) As the signal crosses partially the ionosphere, it will be delayed by it. In this case, the propagation time is:

$$t_2 = \frac{2L_2}{c} + \frac{40.3}{cf_2^2} 2 \frac{TEC}{\sin(\theta)}$$

The zenithal TEC is calculated as:

$$TEC = \frac{(h_{\text{max}} - h_{\text{min}})(N_{\text{max}} - N_{\text{min}})}{2} + N_{\text{min}}(h_{\text{max}} - h_{\text{min}}) = 7.65 \times 10^{17}\ \text{e/m}^2$$

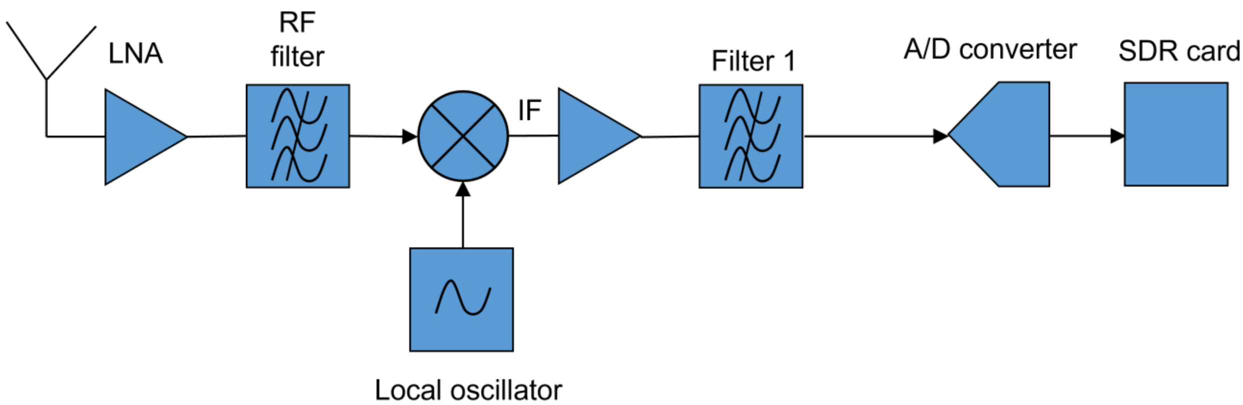
Therefore:

$$t_2 = 0.0049\ \text{s}$$

Problem 2

Consider the heterodyne receiver depicted below, which aims at receiving an RF signal, whose carrier frequency is f_{RF} = 20 GHz, to be converted from analog to digital at the end of the receiver chain. The signal bandwidth is B = 20 MHz. The A/D converter sampling frequency if f_S = 80 MHz. The LNA amplifier gain is G = 30 dB and its noise figure is NF = 2 dB. The voltage at the input of the LNA is P_1 = 10 nW. Equivalent noise temperature at the input of the LNA is T_1 = 350 K.

1) Determine the minimum and maximum local oscillator frequency f_{LO} for the signal to be properly sampled by the A/D converter.
2) Calculate the ideal filter bandwidth of filter 1 (specify minimum and maximum frequencies of the pass band).
3) Determine the minimum gain of the amplifier after the mixer to obtain a minimum voltage at the input of the A/D converter equal to P_2 = 100 μW (assume a loss of 1 dB for each filter, and no loss nor gain for the active mixer).
4) Calculate the SNR in input to the A/D converter.



Solution

1) The A/D converter has a fixed sampling frequency of 80 MHz. The sampling theorem states that:
$f_S > 2f_{\max}$
where f_{max} is the maximum frequency of the whole signal to be sampled. The local oscillator frequency f_{LO} needs to be chosen such that the signal is shifted down at intermediate frequency at the same time having a maximum frequency component equal to f_{max}. Given the sampling frequency, from the equation above → f_{max} = 40 MHz. Considering that the bandwidth of the signal is 20 MHz, the IF frequency must be lower than or equal to f_{IF} = 30 MHz.
When the RF signal is mixed with the local oscillator, we have:
$f_{IF} = f_{RF} - f_{LO}$
Therefore:
$f_{LO} = f_{RF} - f_{IF}$ = 19.97 GHz
The above equation is valid for $f_{LO} < f_{RF}$. If $f_{RF} > f_{LO}$:
$f_{IF} = f_{LO} - f_{RF}$
Therefore:
$f_{LO} = f_{IF} + f_{RF}$ = 20.03 GHz
To sum up → 19.97 GHz < f_{LO} < 20.03 GHz.

2) Given the calculations above, the ideal filter band spans from 20 MHz to 40 MHz.

3) The total gain required to amplify the signal along the chain is:

$G_T = 10\log_{10}\left(\frac{P_1}{P_2}\right) = 40\ \text{dB}$

The total gain is given by:

$G_T = G_{LNA} - G_{F-RF} - G_{MIX} + G_A - G_{F-IF}$

The loss of both filters is 1 dB, the LNA gain is 30 dB, while the loss due to the mixer is set to 0 dB (see the data). Therefore:

$G_A = 12$ dB

4) Given the high gain of the LNA, the contributions to the noise power due to the elements after the LNA can be neglected. Therefore, the SNR in input to the A/D converter is given by:

$$SNR = \frac{P_1}{P_N} = \frac{P_1}{k(T_1 + T_{LNA})B} = 28.45\ \text{dB}$$

Problem 3

Consider a link to a MEO satellite of the O3b constellation, which transmits a telemetry beacon signal centred about f =19.8 GHz.

1) Which wave polarization is likely used for such a beacon signal?

A ground station is equipped with a steerable vertically polarized antenna, which features a closed-loop tracking system to point at the MEO satellite in an optimal way. Such a system is based on the received power and it requires a minimum SNR = 15 dB for an accurate pointing to the satellite.

2) Determine the minimum elevation angle guaranteeing the proper tracking of the satellite in clear sky conditions. To this aim, assume:

- that the LNA noise temperature is T_R = 200 K;
- to neglect the cosmic background temperature;
- that the mean radiating temperature of is T_{mr} = 20 °C;
- to use a parabolic antenna with Cassegrain configuration with gain G_R = 50 dB;
- that the equivalent antenna noise temperature measured at 20° (elevation peak of the satellite) is T_A = 170 K.
- that the MEO satellite features an isoflux system, i.e. the power density reaching the ground is S = 1.25 pW/m^2, regardless of the link elevation angle and not considering any atmospheric attenuation.
- that the system bandwidth is B = 1 MHz.
- the atmosphere is horizontally homogeneous.

Solution

1) The best polarization to be used for non-geosynchronous satellites is the circular one (LHCP or RHCP), mainly for geometrical reasons.

2) First, we need to derive the reference tropospheric attenuation at 20°, as follows:

$$T_A = T_{mr}\left(1 - A_{20°}\right) \;\rightarrow\; A_{20°} = 1 - \frac{T_A}{T_{mr}} = 0.4201$$

As:

$$A_{20°} = 10^{-\frac{A_{90°}^{dB}}{10\sin(20)}} \;\rightarrow\; A_{90°}^{dB} = 1.288 \text{ dB}$$

The system noise temperature is (for the Cassegrain configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}\left(1 - A_\theta\right)$$

where:

$$A_\theta = 10^{-\frac{A_\theta^{dB}}{10\sin(\theta)}}$$

The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B}$$

The received power can be conveniently expressed as follows:

$$P_R = \frac{P_T}{k\pi L^2} G_T A_E A_\theta\, 0.5 = S A_E A_\theta\, 0.5$$

The effective area of the receiving antenna is given by:

$$A_E = \frac{\lambda^2}{4\pi} G_R = 1.8268$$

while the 0.5 takes into account the fact that the antenna is linearly polarized, while the signal is circularly polarized.

Inserting P_R into the SNR equation and solving for A_θ:

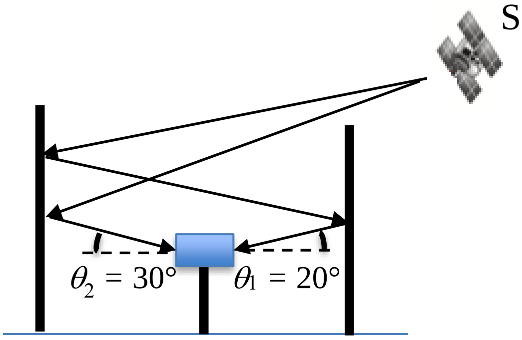
$$A_\theta = \frac{k\ SNR\ B(T_R + T_{mr})}{S\ A_E + k\ SNR\ B\ T_{mr}} = 0.2063 \ \rightarrow \ A_\theta^{dB} = -10\log_{10}(A_\theta) = 6.8553$$

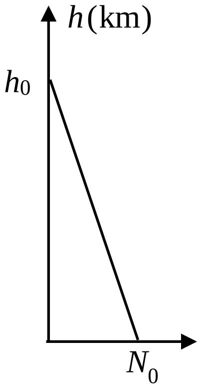
As:

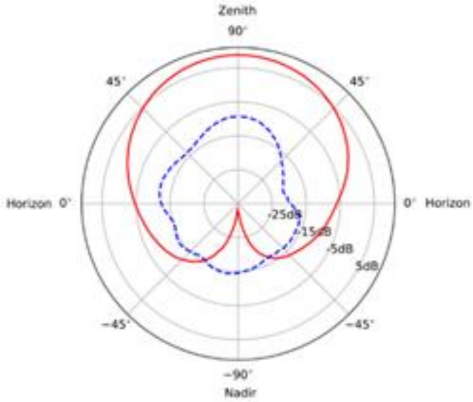
$$A_\theta^{dB} = \frac{A_{90°}^{dB}}{\sin(\theta)} \ \rightarrow \ \theta = 10.83°$$

Problem 4

The figure below shows a single-frequency GNSS RX in an urban canyon tracking a GPS satellite, with the signal reaching the RX through the two paths depicted in the figure. The position of the RX, which is devoted to assessing the impact of multipath in the urban canyon, is known, and so is the line-of-sight distance to the satellite, $L = 35182756$ m. The refractivity vertical profile is shown on right side (N_0 = 700 ppm and h_0 = 10 km), while the zenithal TEC value is 40 TECU. In this adverse receiving conditions, the RX is affected by a code synchronization bias: the L1 C/A code correlation value is C = 2/3. Calculate the pseudorange error component induced by multipath. Assume: perfect clock synchronization; the pseudorange measured by the RX is ρ_1 = 35182903 m ρ_2 = 35182890 m; the atmosphere is horizontally homogeneous; the radiation patter of the RX is depicted on the right side (red curve → RHCP, dashed blue curve → LHCP)







Solution

Considering the assumptions, the pseudorange is given by:

$$\rho = L + \frac{d^I}{\sin(\theta)} + \frac{d^T}{\sin(\theta)} + d^{C/A} + d^{MP}$$

where:

$d^I = 40.3\mathrm{TEC}/f_{L1}^2$ (f_{L1} = 1575.42 MHz) is the zenithal ionospheric error.

$d^T = \frac{h_0 N_0 10^{-6}}{2}$ is the zenithal tropospheric error (given the profile in the figure).

$d^{C/A} = cT_{C/A}(1-C)$ ($T_{C/A} = 0.9775$ μs is the C/A code chip duration) is the code bias phase error.

d^{MP} is the unknown multipath error.

It is worth pointing out that the ray undergoing one reflection is practically rejected by the RX because that ray has a LHCP and the antenna radiation pattern has an associated LHCP gain that is very low (≈ -15 dB), regardless of the elevation angle. The only ray received by the RX is the one undergoing 2 reflections, i.e. with RHCP. Therefore, in the equation above, the $\theta = \theta_1$.

Thus, inverting the equation:

$d^{MP} = 20$ m