Satellite Communication and Positioning Systems – Prof. L. Luini,
June 27th, 2022

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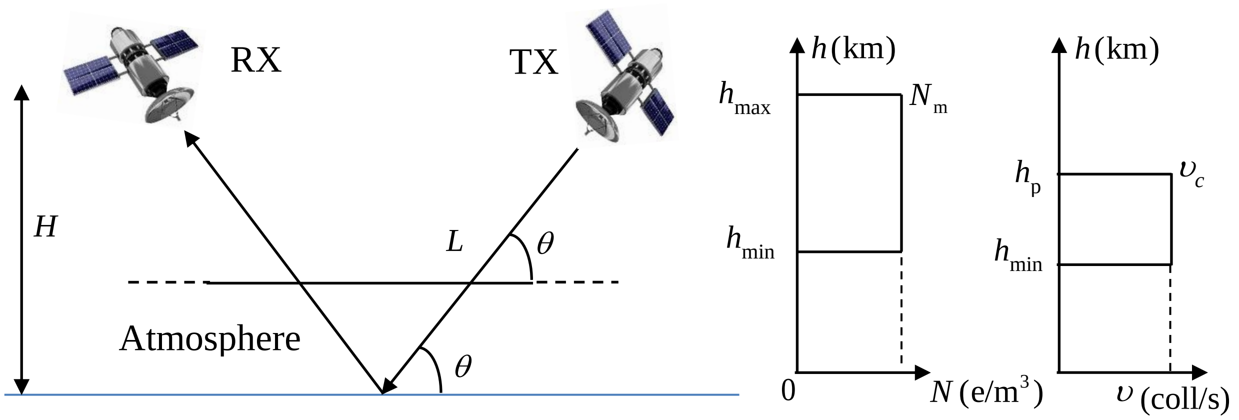
Problem 1

The figure below shows a bistatic radar system consisting of two LEO satellites (same orbit, satellite height above the ground H = 500 km). The radar extracts information on the ground measuring the power received at RX by reflection. Making reference to the right, where the electron content profile ($h_{\min}$ = 50 km, $h_{\max}$ = 400 km, $N_m = 4\times10^{12}$ e/m^3, N homogeneous horizontally) and the collisions frequency profile (h_p = 80 km, $\upsilon_c = 10^4$ coll/s, υ homogeneous horizontally) are shown, and to the left side, where the simplified geometry is reported:

1) Determine the minimum value of θ for the system to avoid reflection from the atmosphere, when the operational frequency is f = 21 MHz.
2) Considering to work with θ calculated at point 1, determine which type of target is on the ground, according to the associated backscatter section, σ: if $\sigma < 100$ m^2 → rural area; if $\sigma \geq 100$ m^2 → urban area.

Additional data: the gain of TX antenna is G = 45 dB, the transmit power is P_T = 100 W, the power density reaching RX is $S_{RX} = 10^{-18}$ W/m^2.



Solution

1) For the wave to avoid total reflection due to the atmosphere (ionosphere at 21 MHz), the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_{\mathrm{m}}}}{f_1}\right)^2} \quad \Rightarrow \quad \theta_{min} = 59°$$

2) To calculate correctly the link budget, first it is necessary to consider the attenuation induced by the ionosphere. The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{N_{\mathrm{max}} e^2 \nu_C}{m(\nu_C^2 + \omega^2)} = 6.5 \cdot 10^{-8} \text{ S/m}$$

where $m = 9 \cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6 \cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{N_{\mathrm{max}} e^2}{m\varepsilon_0} = 1.285 \cdot 10^{16} \text{ rad}^2/\text{s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu_C^2 + \omega^2} = 0.26$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 2.4 \cdot 10^{-5} + j0.2252 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between h_{min} and h_p, and that the zenith attenuation can be scaled to the slant path using the cosecant law. Therefore:

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.21 \text{ dB/km}$$

$$A_{IONO} = \alpha_{dB}\left(h_p - h_{\mathrm{min}}\right)/\sin(\theta) \approx 7.3 \text{ dB} \rightarrow A_l = 0.186$$

Working at f = 21 MHz, the tropospheric effects can be neglected, but not the ionospheric ones. The power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l = 1.37 \cdot 10^{-7} \text{ W/m}^2$$

where $L = H/\sin(\theta) = 583.3$ km. The power density reaching RX is:

$$S_{\mathrm{RX}} = \frac{S\sigma}{4\pi L^2} A_l$$

Combining both equations:

$$S_{\mathrm{RX}} = \frac{P_T}{4\pi L^2} G A_l \frac{\sigma}{4\pi L^2} A_l = P_T G \sigma \left(\frac{A_l}{4\pi L^2}\right)^2$$

Inverting the equation to solve for σ:

$$\sigma = \frac{S_{\mathrm{RX}}}{G P_T}\left(\frac{4\pi L^2}{A_l}\right)^2 = 167.7 \text{ m}^2 \rightarrow \text{urban area}$$

Problem 2 (GNSS)

An aircraft, flying at h = 5 km, is equipped with two two-frequency GNSS receivers tracking only 3 GPS satellites, all at 45°. The equivalent noise temperature of RX A and RX B are $T_A^R = 2500$ K and $T_B^R = 300$ K. The clock bias of RX A and RX B are $d_A^C = 6$ m and $d_B^C = 10$ m, respectively. The error due to the code correlator is associated to the SNR as follows (consider the L1 C/A code):

$$d^{C/A} = 0.15 SNR^{1.6} \quad \text{(SNR in dB, } d^{C/A} \text{ in m)}$$

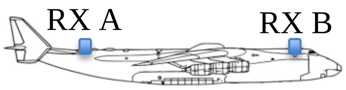
Moreover, RX A features an antenna designed for waves with RHCP, while RX B features a horizontal linear antenna (radiation patter of both antennas → red line in the figure).

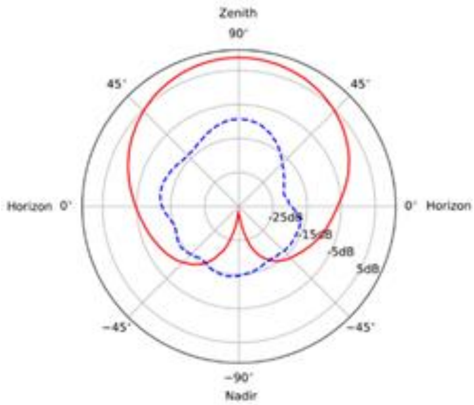
1. (NOT REQUIRED FOR THOSE TAKING THE FULL EXAM) Determine which receiver should be best used to fly the aircraft.

Assume that: the zenithal atmospheric attenuation at H = 5 km is A_Z = 0.5 dB, the mean radiating temperature is T_{mr} = 260 K, the distance between the receivers and the satellites is L = 20000 km, disregard the cosmic background equivalent noise temperature.

2. (REQUIRED FOR BOTH THOSE TAKING THE FULL EXAM AND THOSE TAKING ONLY THE GNSS PART) How much does the pseudorange error increase when the aircraft is landing compared to the initial position at H = 5 km? Consider the following refractivity profile:

$$N = N_0 e^{-\frac{h}{h_0}} = 800 e^{-h/25000} \quad \text{(h in m)}$$





Solution

1) The two receivers are at the same height, so they are subject to the same atmospheric impairments, i.e. attenuation and delay. The pseudorange is affected by different error sources:

$\rho = r + d^{C/A} + d^I + d^T + d^C$

The best receiver to be used is the one showing the lowest pseudorange. The ionospheric delay d^I is cancelled by both receivers. The tropospheric delay d^T is the same for both receivers. The different clock biases are given (they cannot be cancelled as only 3 satellites are used for positioning) and the correlator bias will depend on the SNR of each receiver.

L1 frequency is f = 1575.42 MHz, so the wavelength is $\lambda = c/f$ = 0.1904 m. The received power can be calculated as:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi L}\right)^2 G_R f_R A_{ATM}$$

Making reference to the specifications of GPS satellites, we can assume P_T = 21.9 W and $G_T f_T$ = 13.4 dB = 21.9 (worst case). The slant path atmospheric attenuation is:

$A = A_Z/\sin(45°) = 0.7$ dB → $A_{lin} = 0.85$

The gain of the receiver antenna can be derived from the radiation pattern figure: at 45°, the receiver gain is $G_R = 5$ dB = 3.16. This value also includes f_R.

The received power for RX A is calculated directly from the equation above, yielding:

$P_R^A = 7.4 \times 10^{-16}$ W

For RX B, we need to subtract 3 dB (i.e. adding approximately 0.5 to the link budget equation above), as RX B features a linear antenna, receiving approximately half of the RHCP wave power. Therefore:

$P_R^B = 3.7 \times 10^{-16}$ W

The noise power depends on the antenna noise temperature T_A and on the receiver noise temperature T_R:

$T^{sys} \approx T_{mr}(1 - A_{lin}) + T_R$

Therefore:

$T_A^{sys} = 2539$ K

$T_B^{sys} = 339$ K

After despreading, the bandwidth for the calculation of the noise power is $B = 50$ Hz. The noise power is calculated as:

$P_N = kT_{sys}B$

As a result:

$SNR_A = 26.25$ dB → $d_A^{C/A} = 27.97$ m

$SNR_B = 31.99$ dB → $d_B^{C/A} = 38.37$ m

Calculating the differential pseudorange (considering only the terms that are not common):

$\Delta\rho = \rho_B - \rho_A = d_B^{C/A} + d_B^C - \left(d_A^{C/A} + d_A^C\right) = 14.4$ m

RX A is the one to choose.

2) The pseudorange error while the aircraft is landing will increase due to the increase in the tropospheric delay τ, which, for the given profile, can be calculated as:

$$\tau = -\frac{h_0 N_0 10^{-6}}{c}\left[e^{-\frac{h}{h_0}}\right]_H^{H_S}$$

Considering the satellite high $H_S >> h_0$, the tropospheric delay is:

$d_1^T = h_0 N_0 10^{-6} e^{-\frac{5000\text{ m}}{h_0}} = 16.3$ m

$d_2^T = h_0 N_0 10^{-6} e^{-\frac{0\text{ m}}{h_0}} = 20$ m

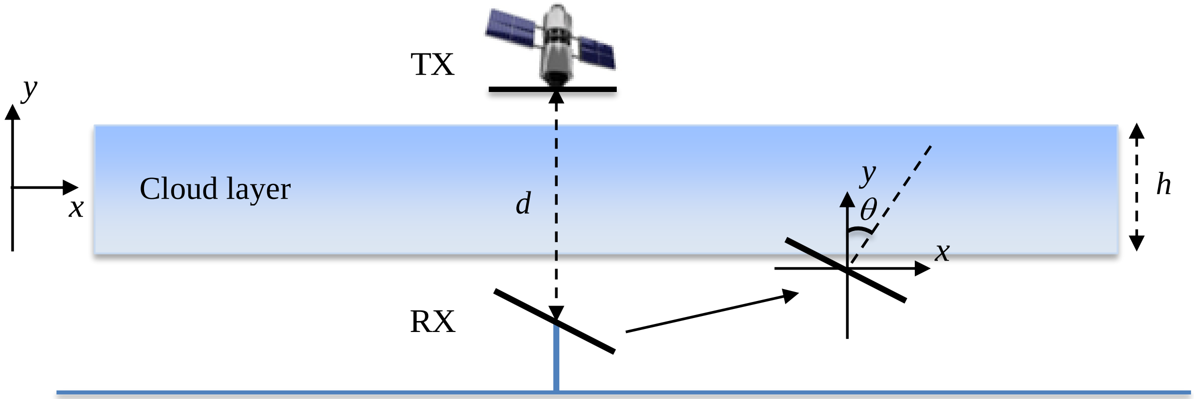
These values need to be devided by the sin(θ) to account for the slant path to the satellite. Considering this aspect, the increase in the pseudorange is 5.2 m.

Problem 3

An Earth-space link, with path length d = 400 km and operating at f = 40 GHz, crosses clouds, consisting of spherical droplets of liquid water. The transmitter (TX) uses a horizontal antenna, while the receiver (RX) antenna is tilted by an angle of θ = 30° (see sketch below) due to strong winds. For this link:

1) Determine the polarization of the wave in front of the RX antenna (specify: LH or RH for a circular/elliptical, tilt angle for a linear).
2) Calculate the power received by the RX antenna?

Additional data: TX and RX antennas are identical; disregard the attenuation due to gases; transmit power, P_T = 100 W; effective area of the antennas, A_E = 0.001 m^2; cloud attenuation, A_f = 3 dB, radiation pattern of both antennas, $f = \cos(\theta)$.



Solution

1) As fog consists of small spherical droplets, which are isotropic, and the TX antenna is horizontal, the polarization reaching RX will still be horizontal.

2) To calculate the power received at RX, we must consider the antenna tilt angle and the attenuation due to fog. The received power is:

$$P_{RX} = P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_R \approx 4.8 \text{ pW}$$

where

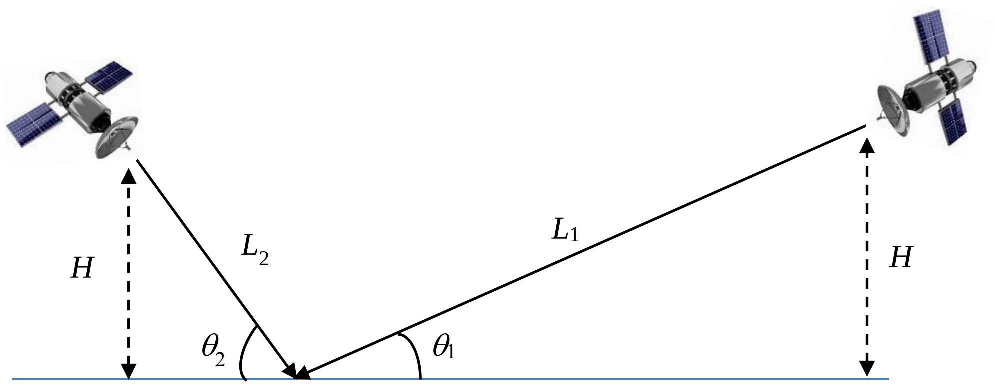
$$G_T = G_R = \frac{4\pi}{\lambda^2} A_E = 223.4$$

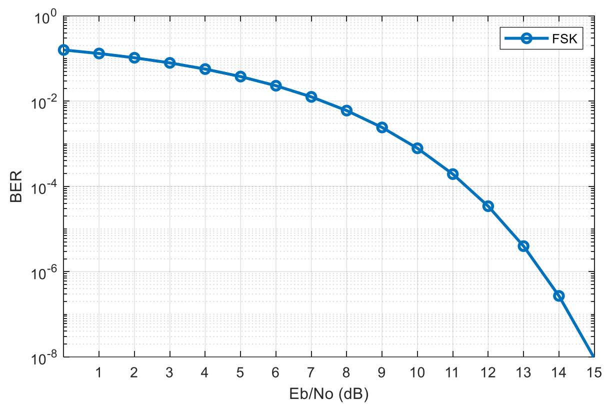
and

$$f_R = \cos(30°) = 0.866$$

Problem 4

Consider a ground station, implementing orbital diversity (i.e. which always selects the satellite with the best SNR): it can be potentially served by two satellites. Satellite 1 (elevation θ_1 = 20°) operates at f_1 = 20 GHz , while satellite 2 (elevation θ_2 = 40°) operates at f_2 = 30 GHz. The ground station is affected by a zenithal rain attenuation at 20 GHz, $A_{R1}^Z = 2.95$ dB (rain rate is constant, both horizontally and vertically). Considering FSK modulation, determine if the ground station can achieve a BER lower than 10^{-6} using either of the satellites (specify which one).





Additional assumptions and data: use the simplified geometry depicted above (flat Earth); typical frequency scaling for rain attenuation from 20 GHz to 30 GHz; ground station tracking the satellites optimally; both satellites can maintain a perfect pointing to the ground station; power transmitted by each satellite P_T = 2 W; mean radiating temperature T_{mr} = 290 K; LEO satellite antenna and ground antenna: parabolic reflectors with diameter D = 0.5 m and efficiency η = 0.6; altitude of the LEO satellites: H = 800 km; bandwidth of the receiver: B = 100 MHz; internal noise temperature of the receiver: T_R = 350 K.

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_R}{k[T_R + T_{mr}(1 - A_R) + T_C A_R]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = f_R = 1$, $L = H/\sin(\theta)$, $G_R = G_T$ (same antenna features). Depending on the link, several quantities will change. As for rain attenuation A_R:

Zenith: $A_{R1}^Z = 2.95$ dB → Slant: $A_{R1}^S = \frac{A_{R1}^Z}{\sin(\theta_1)} = 8.63$ dB → Linear scale: $A_{R1}^L = 0.1372$

Zenith: $A_{R2}^{Z} = A_{R1}^{Z}\left(\frac{f_2}{f_1}\right)^{1.72} = 5.93\text{ dB}$ → Slant: $A_{R2}^{S} = \frac{A_{R2}^{Z}}{\sin(\theta_2)} = 9.2\text{ dB}$ → Linear scale $A_{R2}^{L} = 0.1197$

As for the antenna gains, the effective areas of all antennas, at both frequencies, is:

$$A_E = \eta\pi\frac{D^2}{4} = 0.1178\text{ m}^2$$

The gain of the both the satellite and ground antennas at 20 GHz is:

$$G_1 = \frac{4\pi}{\lambda_1^2}A_E = 6588.7 = 38.2\text{ dB}$$

The gain of the both the satellite and ground antennas at 30 GHz is:

$$G_2 = \frac{4\pi}{\lambda_2^2}A_E = 14825 = 41.7\text{ dB}$$

λ_1 and λ_2 are associated to f_1 and f_2, respectively.

Therefore:

$$SNR_1 = \frac{P_T G_1(\lambda_1/4\pi L_1)^2 G_1 A_{R1}^{L}}{k[T_R + T_{mr}(1 - A_{R1}^{L}) + T_C A_{R1}^{L}]B} = 5.73\text{ dB}$$

$$SNR_2 = \frac{P_T G_2(\lambda_2/4\pi L_2)^2 G_2 A_{R2}^{L}}{k[T_R + T_{mr}(1 - A_{R2}^{L}) + T_C A_{R2}^{L}]B} = 14.1\text{ dB}$$

The target BER can be achieved by connecting to satellite 2.