Satellite Communication and Positioning Systems – Prof. L. Luini,
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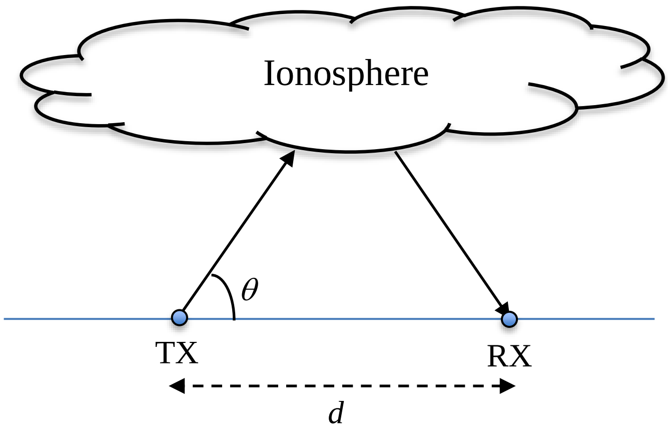
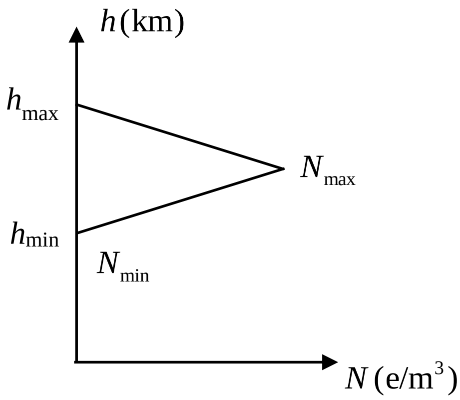
Problem 1

Making reference to the figure below, we want the transmitter TX to reach the user RX at distance d by exploiting the ionosphere (elevation angle $\theta = 60°$). The ionosphere is modelled with the symmetric electron density profile (daytime) sketched in the figure (right side), where $N_{\text{max}} = 6\times10^{12}$ e/m^3, $N_{\text{min}} = 4\times10^{10}$ e/m^3, $h_{\text{min}} = 100$ km and $h_{\text{max}} = 400$ km.

1) Determine the maximum distance d achievable for the TX → RX link.
2) Determine the operational frequency f to achieve the conditions at point 1).
3) Indicate a reasonable margin on f found at point 2) to guarantee the TX → RX link notwithstanding ionospheric variations.
4) Indicate the best polarization to be used for the TX → RX link.

Assume: the virtual reflection height h_V is 1.2 times h_R, the height at which the wave is actually reflected; the Earth is flat; no tropospheric effects to be considered.

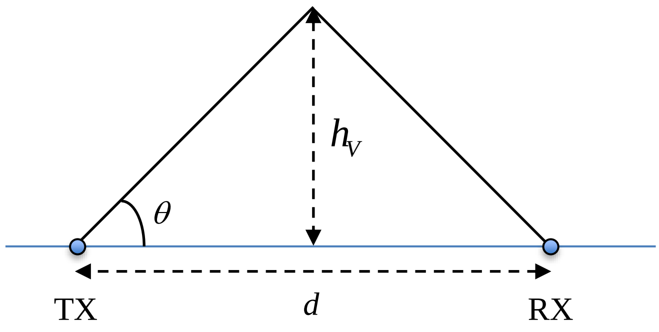




Solution

1) The distance d is maximized if the reflection occurs as high as possible in the ionosphere, i.e. at the height $h_p = 250$ km, correspondent to N_{max}. Considering the figure below, the distance can be found by inverting the following expression:

$h_V = 1.2\, h_p = {}^{d}/_{2}\, \tan\theta \rightarrow\ d = \frac{2.4\, h_p}{\tan\theta} = 346.4$ km



2) The link operational frequency *f* can be determined by inverting the following equation:

$$\cos\theta = \sqrt{1 - \left(\frac{f_C}{f_m}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N_{\max}}}{f_m}\right)^2}$$

Solving for the frequency f_m, we obtain:

$$f_m = \sqrt{\frac{81 N_{\max}}{1 - [\cos(\theta)]^2}} = 25.5 \text{ MHz}$$

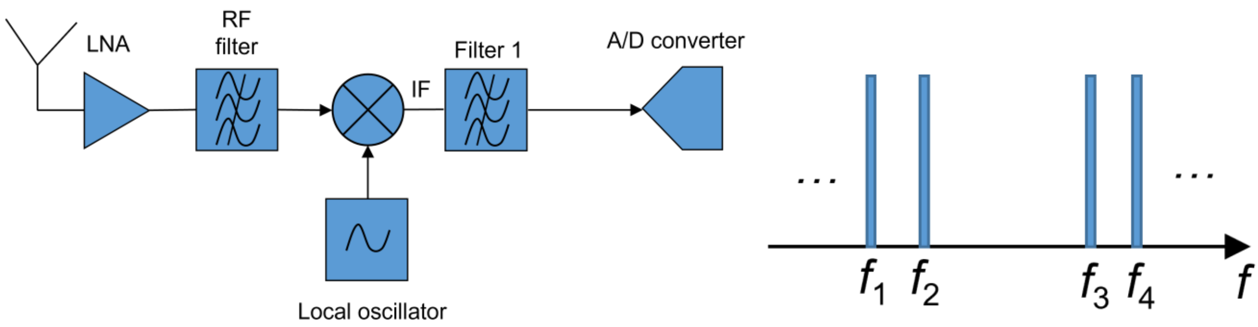
3) During the night, the peak values of the electron content will decrease: it is a good rule of thumb to use reduce by 10% the peak frequency to avoid that the wave crosses the ionosphere at nighttime. Therefore → $f' = 0.9f = 22.95$ MHz.

4) Depolarization in the ionosphere affects linearly polarized waves, but not circularly polarized ones. Therefore, the best polarization is LHCP or RHCP.

Problem 2

Consider the heterodyne receiver depicted below (left side), which aims at receiving the RF signal with carrier frequency f_2 = 20 GHz, to be digitalized at the end of the receiver chain by using an A/D converter with a maximum sampling frequency f_S = 4.2 GS/s. As indicated in the picture below (right side), the RF spectrum is occupied by multiple signals, all having the same bandwidth B = 200 MHz, with the closest carriers being f_1 = 19 GHz and f_3 = 25 GHz (more signals are present as indicated in the figure).

1) Design the RF filter and local oscillator considering the A/D converter features and the aim of reducing as much as possible the RF filter complexity (i.e. selectivity): specifically, provide a suitable f_{LO}, indicate the type of filter to be used for the RF filter and its cutoff(s) frequency(ies).
2) Indicate the ideal bandwidth of Filter 1.
3) Considering that the equivalent noise temperature at the input of the LNA is T_{IN} = 300 K and that the gain of the LNA is G_{LNA} = 30 dB, determine the maximum LNA equivalent noise temperature to achieve a noise power lower than P_N = 1.5 pW in front of the A/D converter.



Solution

1) When down-converting signals from RF to intermediate frequency (IF), image signals represent a problem. The same IF can be obtained using a local oscillator f_{LO} higher or lower than the target carrier frequency f_2. If $f_{LO} < f_2$, the image signals to be rejected using the RF filter are those lower than f_2; if $f_{LO} > f_2$, the image signals to be rejected using the RF filter are those higher than f_2. As a consequence, the RF filter for the case $f_{LO} < f_2$ will need to be much more selective than the one for the $f_{LO} > f_2$ case. Therefore, to meet the requirements expressed at point 1, the $f_{LO} > f_2$ case is to be selected. To reduce the RF filter complexity (low-pass filter) as much as possible, the ideal cutoff frequency is, for example, 22.5 GHz, exactly in the middle of the f_3- f_2 interval (other choices are possible). Regarding the value for the local osc illator frequency, if f_{LO} = 2.5 GHz, then $f_{IF} = f_{LO} – f_2$ = 2.5 GHz, with a maximum frequency of the IF signal equal to f_{max} = 2.6 GHz. However, according to the Nyquist theorem, in this case, the minimum sampling frequency of the A/D converter should be $f_S = 2f_{max}$ = 5.2 GS/s. This exceeds the available specifications for the A/D converter, i.e. f_S = 4.2 GS/s. Using this value as an additional constraint, a possible final optimum design is: f_{LO} = 22 GHz, $f_{IF} = f_{LO} – f_2$ = 2 GHz, f_{max} = 2.1 GHz, $f_S = 2f_{max}$ = 4.2 GS/s.

2) Given the design at point 1, the optimum Filter 1 (bandpass filter) will have a lower cut-off frequency of $f_{min} = f_{IF}$-B/2 = 1.9 GHz and $f_{max} = f_{IF}$+B/2 = 2.1 GHz.

3) The LNA gain value is high enough to consider negligible the additional contributions to noise introduced by the other circuital elements. Therefore the noise power is:

$P_N = k(T_{IN} + T_{LNA})B$
Imposing $P_N < 1$ pW, we obtain → $T_{LNA} < 243.5$ K.

Problem 3

We need to design a link to a deep-space probe orbiting Mars and operating at Ka-band, specifically at f = 26 GHz. The ground station is equipped with a steerable antenna to track the probe.

1) Calculate the reflector antenna diameter of the ground station (Gregorian configuration with efficiency η = 0.5) necessary to guarantee that the probe can be correctly tracked down to an elevation angle θ = 30° for 99.9% of the time in a year, i.e. that the minimum SNR is 5 dB. To this aim, assume:
   - that the atmosphere is stratified;
   - that the ground station LNA noise temperature is T_R = 50 K;
   - to neglect the cosmic background temperature;
   - that the mean radiating temperature of the atmosphere is T_{mr} = 10 °C;
   - that the probe makes use of a parabolic antenna with gain G_T = 45 dB;
   - the transmit power is P_T = 110 W;
   - the probe antenna always points at the ground station;
   - the distance between the probe satellite and the ground station is L = 225000000 km;
   - the receiver bandwidth is B = 1 kHz;
   - that the CCDF of the zenithal atmospheric attenuation A_T is modelled by:

$$P\left(A_Z^{dB}\right) = 100e^{-0.69A_Z^{dB}} \quad (A_Z \text{ in dB and } P \text{ in } \%)$$

2) Calculate the maximum data rate achievable for this channel with the conditions at point 1), considering a negligible bit error rate.

Solution

1) The zenithal attenuation A_Z^{dB} is determined using the CCDF model. 99.9% availability corresponds to P = 0.1% exceedance. Inverting the CCDF formula:

$$A_Z^{dB} = -\frac{1}{0.69}\ln\left(\frac{0.1}{100}\right) \approx 10 \text{ dB}$$

Scaling the zenithal attenuation to the target elevation angle:

$$A_S^{dB} = \frac{A_Z^{dB}}{\sin(\theta)} \approx 20 \text{ dB}$$

which, in linear scale, corresponds to:

$$A_L = 10^{\frac{A_S^{dB}}{10}} \approx 0.01$$

The system noise temperature is (for the Gregorian configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}(1 - A_L) = 330.3 \text{ K}$$

The SNR is given by:

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_L}{k T_{sys} B}$$

where f_R = 1 and f_T = 1.

Inverting the expression above to solve for G_R (by setting SNR = SNR_{min} = 5 dB):

$G_R \approx$ 74 dB

Recalling that:

$$\frac{\eta A_g}{G_R} = \frac{\lambda^2}{4\pi}$$

where A_g is the geometrical area of the antenna:

$$A_g = \left(\frac{D_R}{2}\right)^2 \pi$$

the antenna diameter D_R is obtained:

$D_R \approx 26$ m

This is indeed the dimension of Ka-band deep-space antennas installed at NASA Deep Space Network (DSN) sites (Goldstone, Madrid, Canberra).

2) The reply is given by the Shannon limit:

$$C = B\log_2(1 + SNR_{min}) \approx 2 \text{ kb/s}$$

Problem 4

A two-frequency GNSS receiver, installed at sea level, is correctly providing the PVT solution. The equivalent noise temperature of the receiver is $T_R = 1000$ K. The error due to the code correlator is associated to the SNR after dispreading as follows (consider the L1 C/A code):

$$d^{C/A} = 60SNR^{-0.2} \quad \text{(SNR in dB, } d^{C/A} \text{ in m)}$$

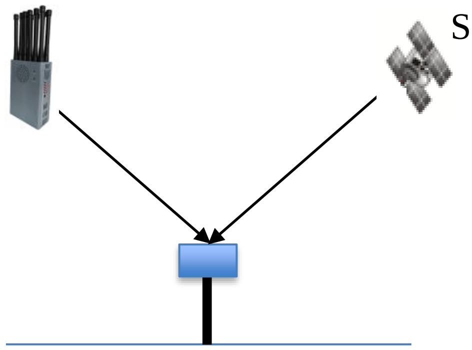
The refractivity profile is modeled as sketched in the figure below, where N_0 = 900 ppm and h_0 = 9.5 km, while the zenithal TEC value is 40 TECU.

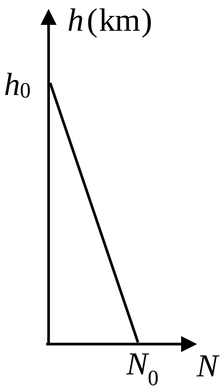
1) Determine the range error for a specific GPS satellite, visible at elevation angle θ = 45°, at L = 20000 km from the receiver.

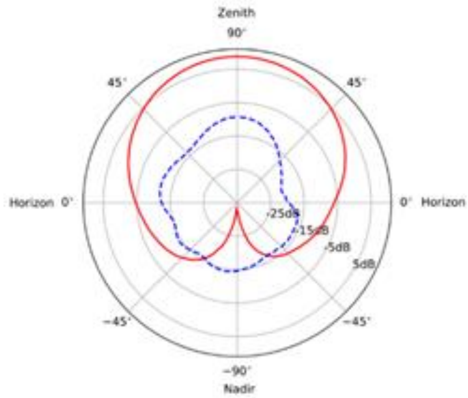
OPTIONAL: at some point, a jammer, also at 45° elevation angle, begins transmitting a wideband signal evenly covering the full L band and reaching the GPS receiver antenna with power spectral density S_J in W/(m^2·Hz).

2) For the same satellite, determine the maximum value of S_J for the range error to be limited below 42 m.

Assume that: the zenithal atmospheric attenuation is A_Z = 0.4 dB, the mean radiating temperature is T_{mr} = 260 K, the distance between the receivers and the satellites is L = 20000 km.







Solution

As the GNSS receiver is correctly providing the PVT solution, there is no clock bias. The pseudorange is therefore affected by the following error sources:

$\rho = r + d^{C/A} + d^I + d^T$

where:

$d^T = \frac{h_0 N_0 10^{-6}}{2\sin(\theta)} = 6.05$ m is the slant tropospheric error (given the profile in the figure).

$d^I \approx 0$ m, as a dual frequency receiver is considered.

$d^{C/A}$ depends on the SNR.

The L1 frequency is f = 1575.42 MHz, so the wavelength is $\lambda = c/f$ = 0.1904 m. The received power can be calculated as:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi L}\right)^2 G_R f_R A_{ATM}$$

Making reference to the specifications of GPS satellites, we can assume P_T = 21.9 W and $G_T f_T$ = 13.4 dB = 21.9 (worst case). The slant path atmospheric attenuation is:

$A = A_Z/\sin(45°) = 0.57$ dB → $A_{lin} = 0.88$

The gain of the receiver antenna can be derived from the radiation pattern figure: at 45°, the receiver gain is G_R = 5 dB = 3.16. This value also includes f_R.

The received power is calculated directly from the equation above, yielding:

$P_R = 7.63 \times 10^{-16}$ W

The system noise power depends on the antenna noise temperature T_A and on the receiver noise temperature T_R:

$T^{sys} \approx T_C A_{lin} + T_{mr}(1 - A_{lin}) + T_R = 1034.1$ K

After despreading, the bandwidth for the calculation of the noise power is B = 50 Hz. The noise power is calculated as:

$P_N = kT_{sys}B = 7.14 \times 10^{-19}$ W

As a result:

SNR = 30.29 dB → $d^{C/A} = 30.3$ m.

The total range error is: $d = d^{C/A} + d^T = 36.4$ m.

2) The jammer will increase the noise power as follows:

$$P'_N = P_N + S_J B A_E = P_N + S_J B G_T f_T \frac{\lambda^2}{4\pi}$$

Considering that d^T is fixed, and setting d = 42 m → $d^{C/A} = 35.95$ m → SNR_m = 12.94 dB = 19.7.

As a result:

$P'_N = \frac{P_R}{SNR_m} = 3.88 \times 10^{-17}$ W

Therefore:

$S_J B G_T f_T \frac{\lambda^2}{4\pi} = P'_N - P_N = 3.8 \times 10^{-17}$ W → $S_J = 1.2 \times 10^{-17} \ \frac{\text{W}}{\text{m}^2\text{Hz}}$.