Satellite Communication and Positioning Systems – Prof. L. Luini,
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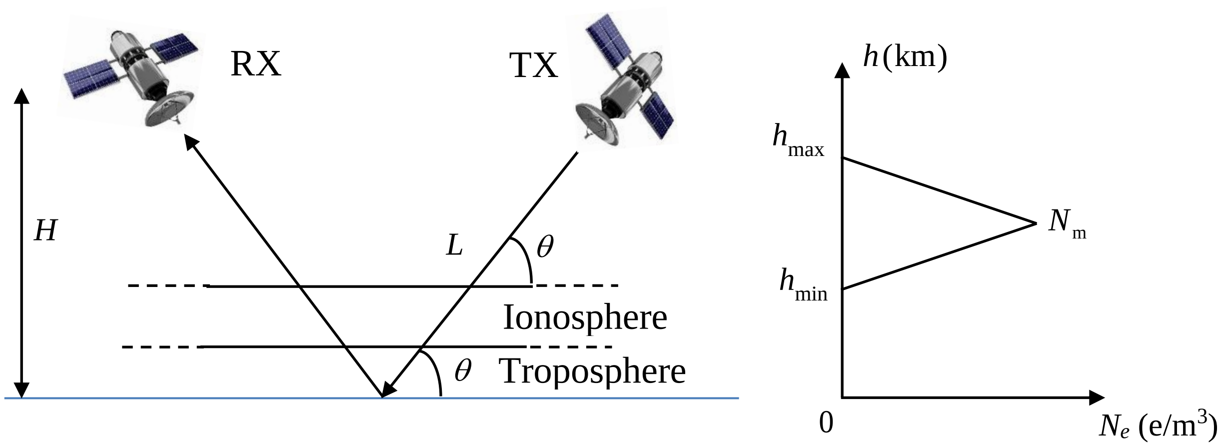
Problem 1

The figure below shows a bistatic radar system (operational frequency f = 51 MHz) consisting of two LEO satellites flying along the same orbit (height above the ground H = 700 km). The radar estimates the ground altitude by measuring the signal propagation time between TX at RX via reflection. To model the ionosphere, consider the symmetric electron content profile sketched below (h_{min} = 50 km, h_{max} = 450 km, N_m = 1×10^{10} e/m^3, N_e homogeneous horizontally); to model the troposphere, consider the following vertical profile for the refractivity N: $N = N_0 e^{-h/h_0}$ (N in ppm, h in km, N_0 = 800 ppm, h_0 = 25 km, N homogeneous horizontally). Taking into account the simplified geometry (left side) below:

1) Determine the minimum value of the angle θ for the system to work properly?
2) Calculate the error in estimating the ground altitude (in m) induced on the radar by the troposphere and to the ionosphere.
3) Assuming no ITU-R constraints, what would be the optimum frequency range for the radar operation (justify your answer)?



Solution

1) For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{m}}}{f}\right)^2} \quad \Rightarrow \quad \theta_{min} \approx 1°$$

2) The error due to the atmosphere is associated to the additional signal delay introduced by the ionosphere and the troposphere. The former can be calculated as:

$$ds_I = \frac{40.3}{f^2}\mathrm{TEC} \approx 30.1 \text{ m}$$

where:

$$\mathrm{TEC} = \frac{(h_{max} - h_{min})N_m}{2} = 2 \times 10^{15}\,\frac{\mathrm{e}}{\mathrm{m}^2}$$

The latter can be calculated as:

$$ds_T = 10^{-6}\left[\int_0^H N dh\right]$$

For an exponential profile, considering that $H >> h_0$:

$$ds_T = h_0 N_0 10^{-6} = 20 \text{ m}$$

The total error in calculating the ground altitude is:

$$ds = ds_T + ds_I = 50.1 \text{ m}$$

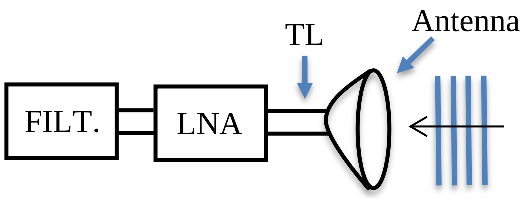
This value is already the zenithal one, i.e. expressing the error on measuring the altitude: theoretically, the delay should be calculated along the slant path and then projected to the zenithal direction, but the approach above inherently assumed that both N_e and N are horizontally homogeneous.

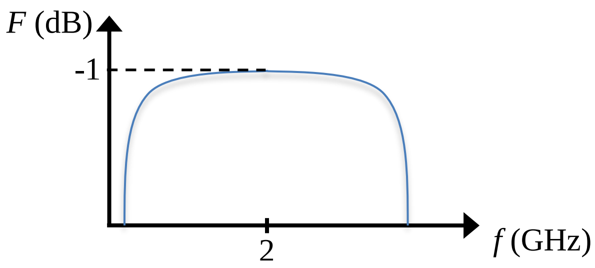
3) As the delay due to the ionosphere is frequency dependent, increasing the frequency will decrease the error. However, beyond 10 GHz, precipitation can cause a strong attenuation, which would impair the link to the point that the SNR at RX becomes too low. Therefore the best frequency range is the X band. For example, selecting f = 9 GHz, the error would reduce to 40 m (basically only due to the troposphere), with no significant impact on the wave amplitude.

Problem 2

The power received by an antenna (f = 2 GHz) is conveyed into a Low Noise Amplifier (LNA), whose gain is G = 20 dB and whose noise figure of the LNA is NF_{LNA} = 3 dB, via a lossless transmission line. For this receiver:

1) Determine the signal power at the output of the filter, knowing that the power at the LNA input port is 5.9 pW: make reference to the receiver chain and to the filter transfer function F in the figure below.
2) Determine the noise power of the system after the filter. To this aim assume: that the antenna noise temperature is T_A = 280 K, the physical temperature of the transmission line T = 300 K, the equivalent noise temperature of the filter T_F = 800 K, the system bandwidth is B = 10 MHz.





Solution

1) After the LNA, the power is:
$P_{FILT} = P_{LNA}G_{lin} = 5.9 \times 10^{-10}$ W
After the filter, the power is:
$P_{OUT} = P_{FILT}F_{lin} = 4.7 \times 10^{-10}$ W

2) The total equivalent noise power of the system is:
$$T_{sys} = T_A + T_{TL} + T_{LNA} + \frac{T_F}{G} = 576.6 \text{ K}$$
where T_{TL}, the noise contribution introduced by the transmission line, is zero, as it is assumed to be lossless, and:
$T_{LNA} = \left(10^{NF_{LNA}/10} - 1\right)290 = 288.6$ K
The noise power is:
$P_N = kT_{sys}B = 7.96 \times 10^{-14}$ W

Problem 3

We need to design a LEO satellite → ground station link operating at f = 15 GHz.

1) What is the best polarization to be used (justify your answer)?

The ground station is equipped with a steerable antenna to track the LEO satellite, and suited to receive the polarization determined at point 1):

2) Calculate the minimum elevation angle θ that can be used to track the LEO satellite to guarantee that the system noise temperature is lower than 520 K for 99.99% of the time in a year. To this aim, assume:

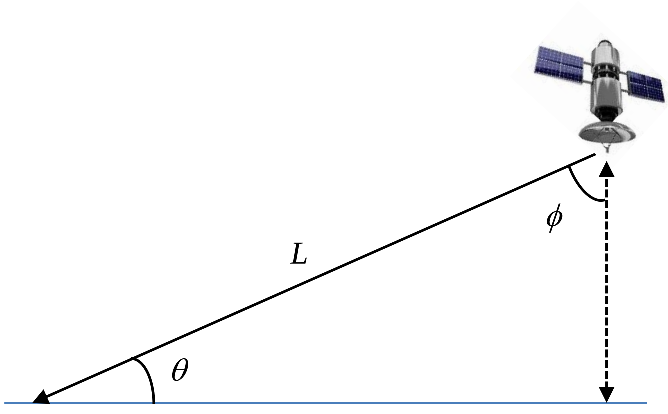
- that the LNA noise temperature is T_R = 240 K;
- to neglect the cosmic background temperature;
- that the mean radiating temperature of is T_{mr} = 10 °C;
- to use a parabolic antenna with Cassegrain configuration with gain G_R = 40 dB;
- that the CCDF of the zenithal atmospheric attenuation A_T is modelled by:

$$P\left(A_T^{dB}\right) = 100e^{-1.15A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in } \%)$$

3) In the conditions of point 2), determine the LEO antenna gain G_T (circular parabolic reflector) to guarantee that the power received at the ground station is P_R = 1 pW. To this aim, assume that (make reference to the figure below):

- the transmit power is P_T = 20 W;
- the LEO antenna always points to the centre of the Earth;
- the radiation pattern of the transmit antenna is $f_T = (\cos\phi)^2$;
- the distance between the LEO satellite and the ground station is L = 1000 km.

4) Considering BPSK modulation, assuming that the data rate is R = 12 Mb/s, calculate the BER that can be guaranteed using the link conditions determined in the previous points. Assume that the bandwidth B is equal to R.



Solution

1) The best polarization to be used for non-geosynchronous satellites is the circular one (LHCP or RHCP), mainly for geometrical reasons.

2) The system noise temperature is (for the Cassegrain configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}\left(1 - 10^{-\frac{A_T^{dB}}{10\sin(\theta)}}\right)$$

where A_T^{dB} is zenithal attenuation to be determined using the CCDF model. 99.99% availability corresponds to P = 0.01% exceedance. Inverting the CCDF formula:

$$A_T^{dB} = -\frac{1}{1.15}\ln\left(\frac{0.01}{100}\right) = 8 \text{ dB}$$

Setting T_{sys} = 520 K and inverting the first equation above to solve for θ:

$$\theta = \sin^{-1}\left\{-\frac{A_T^{dB}}{10\log10\left[1-\frac{T_{sys}-T_R}{T_{mr}}\right]}\right\} = 24.2°$$

3) Let us consider the link budget equation:

$$P_R = P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_T$$

where f_R = 1 and $f_T = [\cos(90-\theta)]^2$ = 0.168. Also:

$$A_T = 10^{-\frac{A_T^{dB}}{10\sin(\theta)}} = 0.011$$

Solving for G_T:

$$G_T \approx 1056 = 30.23 \text{ dB}$$

4) The SNR is:

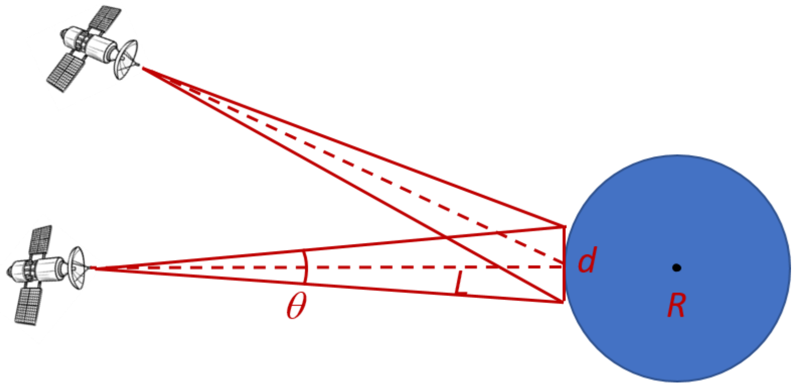
$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B} = \frac{E_b R}{N_0 B} = \frac{E_b}{N_0} = 11.61 = 10.65 \text{ dB}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). Therefore, considering the BER curve above, the link can guarantee a BER lower than 4×10^{-6}.

Problem 4

We want to design a navigation and communication system for the Earth, based on CDMA and consisting of several non-geosynchronous satellites deployed on different orbital planes. Though the system is inherently global, it is designed to cover only a circular region with diameter d = 300 km affected by a maximum (referred to the minimum elevation angle) TEC = 10 TECU. The clock of all satellites are perfectly synchronized, while the user's one is inaccurate.

1) What is the minimum number of satellites required to achieve accurate positioning?
2) Consider a parabolic reflector for the onboard antenna with diameter D = 0.5 m and efficiency η = 0.6: determine the carrier frequency to obtain the onboard antenna gain G_T = 30 dB.
3) Calculate the satellite altitude L such that the main lobe of the onboard antenna covers the target area (i.e. the -3 dB angle is equal to θ in the figure below).
4) Assuming to use 5% of the carrier frequency as the bandwidth available for the system, determine the duration T_C of the (square) PRN chip.
5) Assuming that the PRN code phase can be determined with an accuracy of 10% of the chip length, calculate the pseudorange error due PRN codes.
6) Would the use of two frequencies in this system improve the position accuracy?
7) Assuming that the PRN code consists of 1950 chips, which the data rate can be supported by the system (BPSK modulation): R_1 = 100 kb/s or R_2 = 5 kb/s?



Solution

1) As the user's clock is inaccurate, a minimum of 4 satellites are required.

2) The carrier frequency can be obtained by:

$$G_T = \frac{4\pi}{\lambda^2} A_e = \frac{4\pi}{\lambda^2} \eta \left(\frac{D}{2}\right)^2 \pi$$

By inverting the formula and remembering that $\lambda = c/f$ → f = 7.8 GHz.

3) The value of L can be easily obtained from geometrical considerations and from the HPBW angle of the antenna:

$$HPBW = 70\frac{\lambda}{D} = 5.4°$$

Imposing θ = HPBW and making reference to the figure above:

$$L = \left(\frac{d}{2}\right) \frac{1}{\tan\left(\frac{\theta}{2}\right)} = 3189 \text{ km}$$

4) The bandwidth available for the system is:

$$B = 0.05\, f = 390 \text{ MHz}$$

Considering that square PRN chips, the bandwidth occupation of the PRN code is twice as the chip rate. Therefore, the supported chip rate is $R_C = B/2$ = 195 Mchip/s. The duration of the chip T_C is: $T_C = 1/R_C$ = 5.13 ns

5) The pseudorange error (m) due to PRN codes is:

$$\Delta\rho_C = 0.1cT_C = 0.1c\frac{1}{R_C} = 0.1539 \text{ m}$$

6) The use of two frequencies would allow correcting almost completely the ionospheric error on the pseudorange. Considering the maximum TEC of the region of interest (10 TECU), the maximum ionospheric delay amounts to:

$$\Delta\rho_I = \frac{40.3}{f^2}\text{TEC} = 0.066 \text{ m}$$

The use of two frequencies would complicate significantly the system, but would not increase as much the system accuracy.

7) Considering N = 1950 chips, the PRN code duration is:

$$D_C = NT_C = N\frac{1}{R_C} = 10^{-5} \text{ s}$$

Let us first consider R_1, which is associated to a bit duration of:

$$T_{B1} = \frac{1}{R_1} = 10^{-5} \text{ s}$$

This means that there is a possible bit transition for every PRN code repetition, which would make impossible to achieve code acquisition.

Considering R_2:

$$T_{B2} = \frac{1}{R_2} = 20 \times 10^{-5} \text{ s}$$

In this case, there are 20 code repetitions in each bit, which allows code acquisition (same condition as for the C/A code in the GPS).