Satellite Communication and Positioning Systems – Prof. L. Luini,
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SURNAME AND NAME ______________________________

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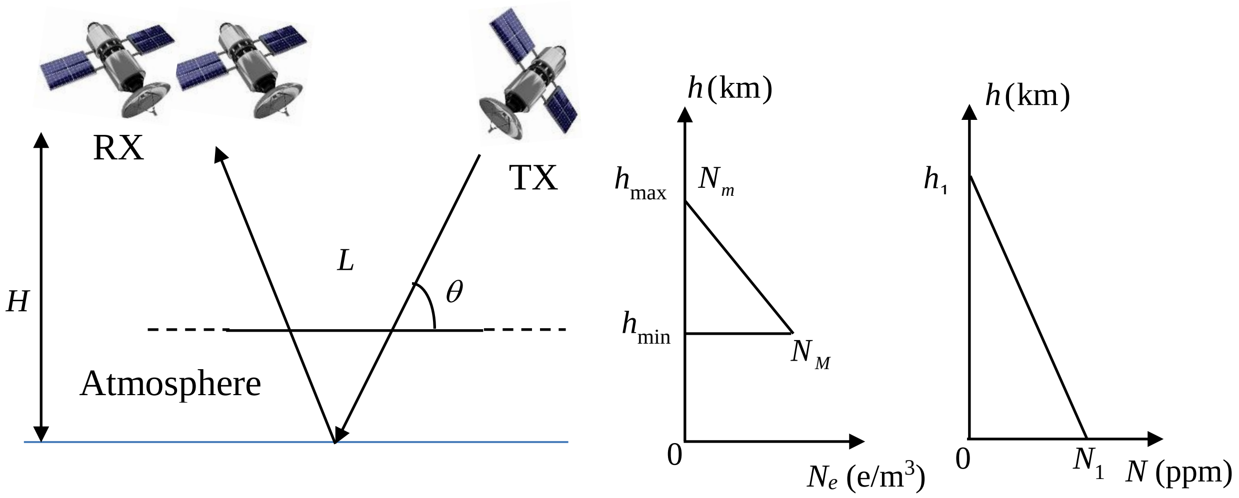
Problem 1

Making reference to the figure below, a bistatic space-borne radar system is designed to measure the ground altitude (θ = 60°). The electron content and the refractivity profiles are shown below, (h_{max} = 350 km, h_{min} = 50 km, $N_M = 4\times10^{12}$ e/m^3, $N_m = 10^{11}$ e/m^3, h_1 = 9 km, N_1 = 200 ppm). The receiver side consists of an array of satellites in formation flight, at increasing distance from the transmitter. For all satellites, the height above the ground is H = 900 km.

1) Determine the range of the operational frequency f for the radar to operate correctly and select the minimum one.
2) Considering the frequency selected at point 1, what is the best polarization to be used for TX (justify the answer)? Accordingly, what is the antenna polarization to be used for RX?
3) Estimate the error, due to the atmosphere, in measuring the ground altitude. Propose another operational frequency to minimize such an error.

Assume: perfect reflection from the Earth surface; rainy conditions in the troposphere.



Solution

1) For the radar to operate correctly, first and foremost, the wave has to cross the ionosphere. Therefore, the minimum operational frequency is:

$$f_{\min} = \sqrt{\frac{81 N_M}{1 - [\cos(\theta)]^2}} \approx 20.8 \text{ MHz}$$

For any frequency higher than f_{min}, the wave will reach the ground. The maximum recommended frequency is f_{max} = 10 GHz, as this is the typical threshold beyond which rain attenuation starts to heavily affect the propagation of EM wave. The selected frequency is $f = f_{min}$.

2) Considering f = 20.8 MHz, ionospheric effects strongly affect the wave propagation. Among them, Faraday rotation will take place. To avoid this detrimental effect, which has an impact only on linearly polarized waves, it is better to select a circular polarization: let us pick RHCP, for example. After reflection on the ground, the wave polarization will be LHCP: this will also need to be the antenna polarization.

3) The error induced on the radar estimates depends on the additional slant atmospheric delay. Regarding the ionospheric one (one way), this is given by:

$$dt_I = \frac{40.3}{c f_2^2} \frac{\text{TEC}}{\sin(\theta)} \approx 0.221 \text{ ms}$$

where:

$$\text{TEC} = \frac{(h_{\max} - h_{\min})(N_M - N_m)}{2} + (h_{\max} - h_{\min}) N_m = 61.5 \text{ TECU}$$

Concerning the tropospheric one (one way):

$$dt_P = \frac{10^{-6}}{\sin(\theta) c} \left[\int_H^{H_S} N dh \right] = \frac{10^{-6}}{\sin(\theta) c} \left(\frac{N_1 h_1}{2} \right) = \frac{10^{-6}}{\sin(\theta) c} \left(\frac{N_1 h_1}{2} \right) = 3.5 \text{ ns}$$

Therefore, the total radar estimation error is:

$$\varepsilon = 2 dt_I c + 2 dt_P c = 132.5 \text{ km}$$

This result shows that this frequency cannot be used to measure the ground altitude using the radar. As the ionosphere is frequency selective, the error can be minimized by selecting $f = f_{max}$ = 10 GHz. In this case:

$$\varepsilon = 2.65 \text{ m}$$

Problem 2

Consider the uplink of the forward link of a single-beam satellite communication system operating at Ka band, specifically at 28 GHz:

1) Determine the maximum bandwidth for the link.
2) Determine the maximum number of users that can be served by the gateway knowing that the system implements FDMA (Frequency Division Multiple Access) and that the channel assigned to each user has bandwidth B_C = 20 MHz.
3) Determine the bit rate (net information rate) for each user, knowing that the link implements 32-PSK with Hamming (code rate R_C = 4/7).

Solution

1) Typically, the maximum bandwidth assigned to a carrier can be around 20% of the carrier itself. In this case, the total bandwidth is:
$B_T = 0.2 f_2 = 5.6$ GHz

2) For a single-beam system, no reuse of the spectrum is considered and therefore the number of users that can be served by the gateway is simply:

$$N = \frac{B_T}{B_C} = 280$$

3) The bit rate can be determined based on B_C and the MODCOD scheme (modulation and coding). The modulation order for 32-PSK is M = 5, i.e. every symbol will store 5 bits. The baud rate is:

$F \approx B_C$ = 20 Mbaud/s

The coded information rate is:

$S = M F$ = 100 Mb/s

The net information rate (bit rate) is:

$R \approx S R_C$ = 57.14 Mb/s

Problem 3

Consider a downlink from a MEO satellite to a ground station (distance to the satellite h = 8000 km, elevation angle = 60°), operating at f = 32 GHz. The signal goes through a uniform rain layer (rain height h_R = 2 km, all rain drops equi-oriented, with minor axis aligned with the zenithal direction). The specific rain attenuation is α_V = 1.15 dB/km and α_H = 1.18 dB/km, for the vertical and horizontal polarizations, respectively.

1) Determine the polarization type in front of the receiving antenna.
2) Calculate the signal-to-noise ratio at the receiver.

Additional data:

- antennas are optimally pointed

Additional data:

- mean radiating temperature T_{mr} = 290 K
- antenna onboard the satellite: helix antenna with gain G_T = 26 dB
- antenna on the ground station: linear horizontal antenna with gain G_R = 10 dB
- power transmitted by the satellite: P_T = 700 W
- bandwidth of the receiver: B = 100 kHz
- LNA noise temperature: T_R = 320 K
- neglect attenuation due to gases and clouds

Solution

1) The helix antenna emits circular polarization and the wave crosses a rain layer with anisotropic particles, as indicated by the difference in the specific rain attenuation. For rain particles, the horizontal and vertical values of the phase constant can be assumed to be equal: the two linear components of the CP wave will be subject to different attenuation levels and therefore the wave polarization in front receiving antenna will be elliptical (same rotation direction as the original CP wave emitted by the satellite).

2) The transmitted wave has circular polarization, but the receiving antenna is linear horizontal, so it will receive only the associated component of the circularly polarized signal. The power associated to each component of a CP wave is half of the total power, so $P_C = P_T/2$. The signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_C G_T f_T \left(\lambda/4\pi h\right)^2 G_R f_R A_H}{kTB}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), T is the total noise temperature (summation of T_R and the antenna noise T_A), $f_R = f_T = 1$ (antenna optimally pointed). A_H is the rain attenuation affecting the horizontal component of the CP wave:

$A_{dB} = \alpha_H h_R/\sin(\theta) = 2.7$ dB $\rightarrow A_H = 0.533$

The antenna equivalent noise temperature is:

$$T_A = T_C A_H + T_{mr}(1 - A_H) = 136.7 \text{ K}$$

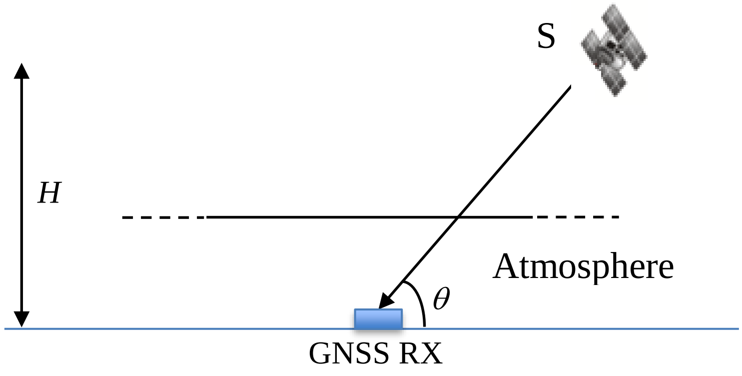
The total equivalent noise temperature is:

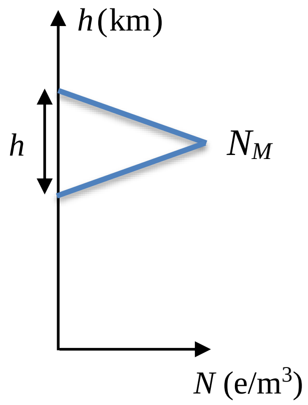
$T = T_A + T_R = 456.7$ K

Therefore: SNR = 10.3 = 10.1 dB

Problem 4

The figure below shows a two-frequency GNSS RX (positioned at average mean sea level altitude) tracking a GPS satellite at elevation angle θ = 30° (satellite height above the ground H = 20000 km). Making reference to the right, where the electron content profile is depicted ($N_M = 10^{12}$ e/m^3) and to the left side, where the simplified geometry is reported (flat Earth assumption): calculate the ionosphere thickness h, knowing that the pseudoranges measured by the GNSS RX are ρ_1 = 40000186.2 m and ρ_2 = 40000083.8 m, for L1 and L2, respectively. To this aim, consider no clock bias at the RX and that the prompt code correlator value is C = 0.6 for both frequency channels.





Solution

The slant path can be easily calculated as:

$$L = \frac{H}{\sin(\theta)} \approx 40000000 \text{ m}$$

Given the problem details, the pseudorange is given by:

$$\rho_i = L + \frac{d_i^I}{\sin(\theta)} + \frac{d^T}{\sin(\theta)} + d_i^C$$

where i indicates the frequency band (1 for L1 and 2 for L2). The errors due to code synchronization are:

$d_1^C = cT^{C/A}(1 - C) = 117.3$ m

$d_2^C = cT^{P(Y)}(1 - C) = 11.7$ m

being $T^{C/A}$ and $T^{P(Y)}$ the chip durations for the C/A code and the P(Y) code, respectively.

Removing such errors from the pseudoranges:

$$\tilde{\rho}_i = L + \frac{d_i^I}{\sin(\theta)} + \frac{d^T}{\sin(\theta)}$$

yields:

$\tilde{\rho}_1 = 40000068.9$ m and $\tilde{\rho}_2 = 40000072.1$ m

Taking the difference of such values:

$$\tilde{\rho}_2 - \tilde{\rho}_1 = \frac{1}{\sin(\theta)}[d_2^I - d_1^I] = \frac{TEC}{\sin(\theta)}\left[\frac{40.3}{f_2^2} - \frac{40.3}{f_1^2}\right]$$

Therefore:

$$TEC = \frac{(\tilde{\rho}_2 - \tilde{\rho}_1)\sin(\theta)}{\left[\frac{40.3}{f_2^2} - \frac{40.3}{f_1^2}\right]} = \frac{N_M h}{2}$$

Finally:

$$h = \frac{2(\tilde{\rho}_2 - \tilde{\rho}_1)\sin(\theta)}{N_M\left[\frac{40.3}{f_2^2} - \frac{40.3}{f_1^2}\right]} = 300\ \mathrm{km}$$