Telecommunication Systems
February 14th, 2019

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SURNAME AND NAME ______________________________

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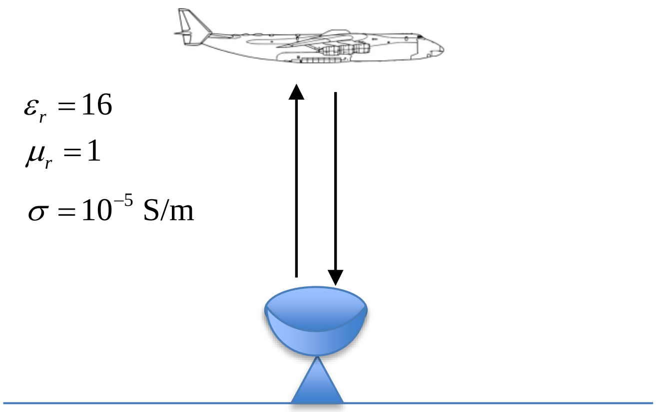
Problem 1

A radar with zenithal pointing, working at f = 5 GHz, illuminates an aircraft with an electromagnetic pulse. Assuming that the radar emits plane waves, calculate:

1) The altitude of the aircraft, given that Δt = 50 μs is the time for the pulse to reach back the radar after reflection on the aircraft. To this aim, assume that the full atmosphere is homogeneous and can be simplistically represented by the following electromagnetic parameters: relative permittivity ε_r = 16, relative permeability μ_r = 1, conductivity $\sigma = 10^{-5}$ S/m.

2) The amplitude of the electric field $\left|\vec{E}_0\right|$ emitted by the radar by knowing that the power density reaching back the radar is S_R = 10 μW/m^2.

Assumptions: the aircraft behaves as a perfect reflector and both the transmitted and reflected waves are plane waves.



Solution

1) First we need to characterize electromagnetically the propagation medium. In this case, the loss tangent is $\frac{\sigma}{\omega\varepsilon}$ << 1. Therefore the medium can be well approximated as a good dielectric. Therefore the attenuation and propagation constants are:

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = 4.7\times10^{-4} \text{ Np/m}$$

and

$$\beta \simeq \omega\sqrt{\mu\varepsilon} = 419.1 \text{ rad/m}$$

As for the intrinsic impedance, we obtain:

$$\eta \simeq \sqrt{\frac{\mu}{\varepsilon}} = 94.2\ \Omega$$

The propagation velocity is therefore:

$$v = \frac{\omega}{\beta} = 7.5\times10^{7} \text{ m/s}$$

As a result, the aircraft altitude is:

$h = v\,\Delta t/2 = 1875$ m

2) The power density emitted by the radar is (assuming plane waves):

$$S_{rad} = \frac{1}{2}\frac{\left|\vec{E}_0\right|^2}{\eta} \text{ W/m}^2$$

The power density reaching back the radar after reflection is (assuming again plane wave propagation):

$$S_R = S_{rad}e^{-2\alpha h}\left|\Gamma\right|^2 e^{-2\alpha h} = S_{rad}e^{-4\alpha h} \text{ W}$$

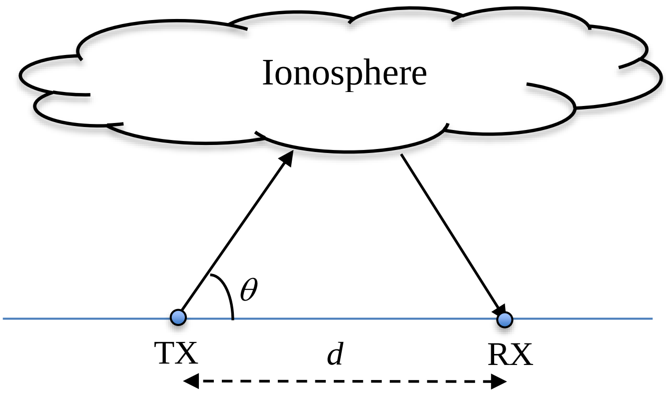
where $\left|\Gamma\right|$ is the absolute value of the reflection coefficient, which is equal to 1 as the aircraft is assumed to be a perfect reflector. Therefore, by knowing S_R:

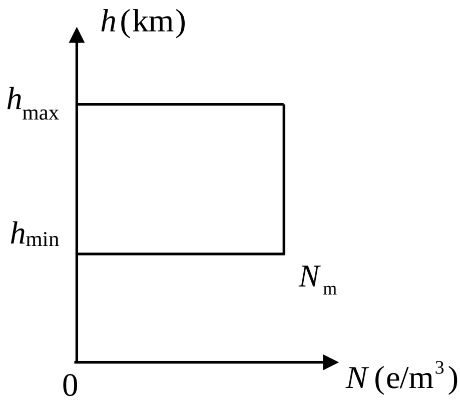
$$\left|\vec{E}_0\right| = \sqrt{\frac{2\eta S_R}{e^{-4\alpha h}}} \simeq 0.254 \text{ V/m}$$

Problem 2

Making reference to the figure below, the ionosphere is modelled with the sketched electron density profile, where $N_m = 2\times10^{12}$ e/m^3, h_{min} = 100 km and h_{max} = 400 km.
For this scenario:
1) Assuming the TX transmits a signal with elevation angle of $\theta = 40°$, determine the maximum frequency f_{max} to be used to reach RX by exploiting the reflection on the ionosphere.
2) Calculate the distance d between TX and RX, assuming the same conditions as in 1).
3) Keeping the same operational frequency f_{max}, what happens if the elevation angle is increased to $\theta_2 = 50°$?

Assume that the virtual reflection height is 1.2 of the height at which the wave is actually reflected.





Solution
1) The maximum frequency f_{max} to be used to guarantee total reflection is:

$$\cos\theta = \sqrt{1-\left(\frac{f_C}{f_{max}}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_m}}{f_{max}}\right)^2}$$

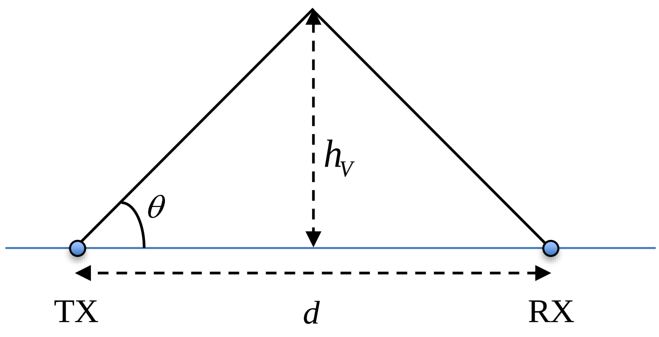
which yields (assuming $\theta = 40°$):

$$f_{max} = \sqrt{\frac{81N_m}{1-\left[\cos(\theta)\right]^2}} \approx 19.8 \text{ MHz}$$

For any frequency lower than f_{max}, given that elevation angle, the wave will be totally reflected.

2) Exploiting the concept of virtual reflection height d is given by:

$$d = \frac{2h_V}{\tan\theta} = 286 \text{ km}$$

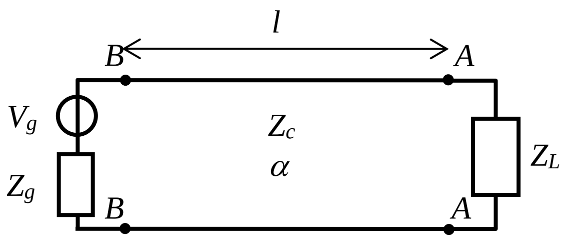


3) If the elevation angle increases beyond 40°, keeping the same f_{max}, the wave will cross the ionosphere.

Problem 3

A transmitter with voltage V_g = 10 V (sinusoidal regime) and internal impedance Z_g = 50 Ω is connected to a load Z_L = j10 Ω by a transmission line with characteristic impedance Z_C = 50 Ω and attenuation coefficient α = 20 dB/km . The line length is l = 55.2 m and the frequency is f = 300 MHz.
Calculate:

1) The power absorbed by the load.
2) The value of Z_L to maximize P_L, the power transferred to the load.
3) The value of P_L and the power absorbed by the line, for the conditions at point 2).



Solution

1) As the load is imaginary (it corresponds to an inductor), no power will be absorbed by Z_L.

2) The value of Z_L maximizing the power absorbed by the load is (total match):

$$Z_L = 50\ \Omega$$

In fact, in this case, the reflection coefficient at the load section will be zero, and so will be at the input section.

3) As a result, the power absorbed by the load will simply be given by:

$$P_L = P_{AV} e^{-2\alpha l}\ \text{W}$$

$$P_{AV} = \frac{|V_g|^2}{8\operatorname{Re}\{Z_g\}} = 0.25\ \text{W}$$

The attenuation coefficient must first be converted to Np/m:

$$\alpha = \frac{20}{8.686 \cdot 1000} = 0.0023\ \text{Np/m}$$

Therefore:

$$P_L = 0.1939\ \text{W}$$

Finally, the power absorbed by the line is:

$P_{line} = P_{AV} - P_L$ = 0.0561 W

Problem 4

Consider the downlink from a digital TV broadcast geostationary satellite to a ground user. The link, with elevation angle of $\theta = 30°$, is affected by rain and the carrier frequency is f = 20 GHz. Calculate the maximum bandwidth BW of the channel to guarantee a minimum SNR_{min} = 7 dB. Use the following data:

- the directivity of the ground antenna is D_R = 30 dB and its efficiency is η_R = 0.6
- the directivity of the satellite antenna is D_S = 5 dB and its efficiency is η_S = 0.7
- assume that both antennas are optimally pointed
- the power transmitted by the satellite is P_T = 1 kW
- the distance between the ground station and the satellite is H = 40000 km
- the receiver LNA equivalent noise temperature is T_{LNA} = 100 K
- the rain rate, assumed to be constant along the link, is R = 10 mm/h, the rain height is h_R = 3.5 km and the rain temperature is T_{rain} = 10 °C. For rain attenuation calculation use k_R = 0.0956 and α_R = 0.9933
- assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
- assume that there are no additional atmospheric constituents inducing attenuation besides rain

What is the maximum theoretical data rate (bit/s) achievable for this channel?

Solution

1) The wavelength is $\lambda = c/f$ = 0.01 m. The gains of the two antennas are:

$G_R = \eta_R D_R$ = 600 (converted to linear scale)

$G_S = \eta_S D_S$ = 2.2 (converted to linear scale)

The atmospheric attenuation is due to rain:

$A_R = k_R R^{\alpha_R} \dfrac{h_R}{\sin\theta}$ = 6.6 dB → A_R = 0.219

The received power is:

$$P_R = P_T G_S f_S \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R A_R = 2.59 \cdot 10^{-16} \text{ W}$$

where $f_S = f_R = 1$ (antennas optimally pointed).

The noise power depends on the total system equivalent noise temperature:

$T_{sys} \approx T_A + T_{LNA}$

The equivalent antenna noise (sky noise) is given by:

$T_A = T_{rain}(1-A_R) + T_C A_R$ = 221.7 K

Thus $T_{sys} \approx 321.7$ K

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{k T_{sys} B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). By inverting the last equation, the maximum bandwidth can be determined.

$$SNR = \frac{P_R}{kT_{sys}B} > SNR_{\min} \rightarrow B < \frac{P_R}{kT_{sys}SNR_{\min}} = 11.7 \text{ kHz}$$

The maximum theoretical data rate C for this channel is dictated by the Shannon theorem:

$$C = B\log_2(1+SNR) = 30.2 \text{ kb/s}$$

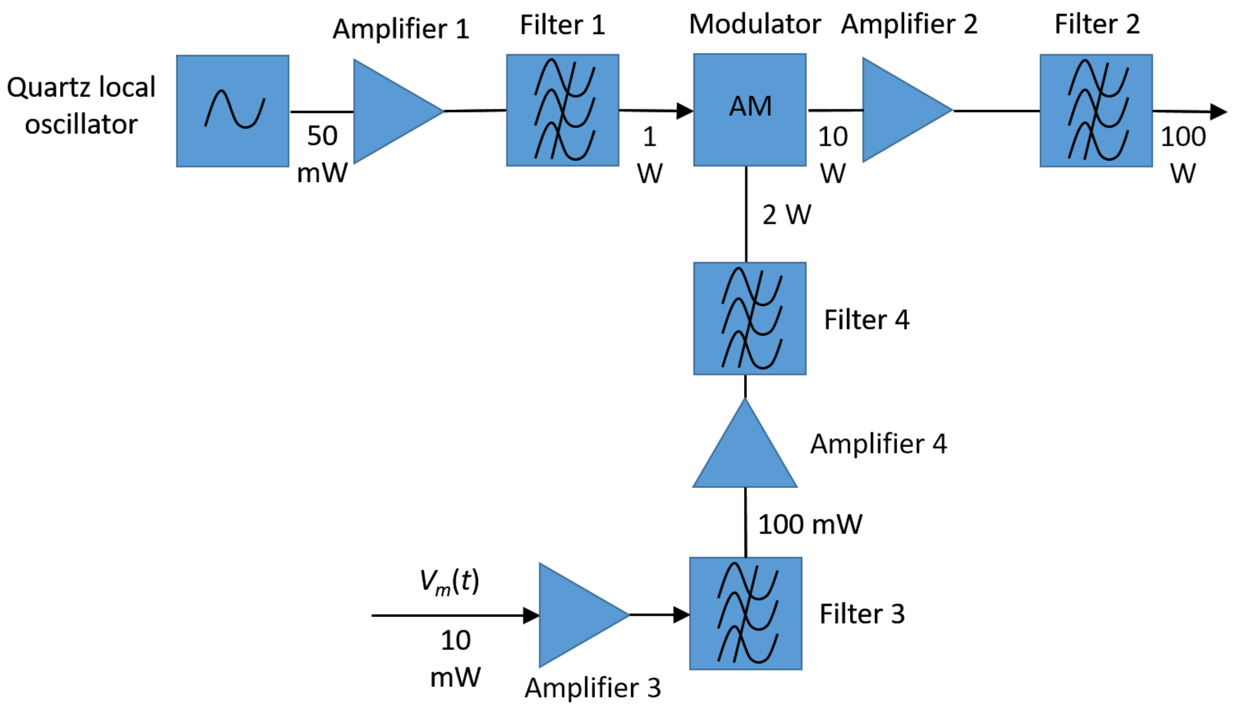
Problem 5

We need to transmit a 15 kHz musical signal by amplitude modulation over one of the 200 channels (the carrier frequency of our specific channel is 100 MHz) in the frequency range 95-105 MHz. The output power is 100 W. The modulation depth is 80%.

- Show the block diagram of the multi-channel transmitter
- Describe the role of each subsystem
- Design the equipment: bandwidth of the filters, gain of the different subsystems
- Propose the oscillator type: describe its behavior, and provide its design
- Show an example of the time series of the modulated signal.

Solution

The reference block diagram of a single-channel AM transmitter is reported below:



Here are some features of the various blocks/components:

- The local oscillator needs to be extremely stable (quartz oscillator) and provides the carrier frequency f_c.
- Amplifier 1 plus filter 1: this amplifier has a high gain and the filter needs to be as narrow as possible around the carrier frequency to reduce noise as much as possible.
- The modulating signal $V_m(t)$ is recorded by an analogic device (e.g. microphone).
- Amplifier 2 plus filter 2: this amplifier has a relatively high gain and the band-pass filter needs to be centred on the selected carrier frequency, with a bandwidth of at least two times the reference channel bandwidth of 15 kHz (e.g. 35 kHz); in fact the carrier frequencies of two adjacent channels are spaced by 10 MHz/200 = 50 kHz.
- Amplifier 3 plus filter 3: this amplifier has a low gain and the low-pass filter needs to have a bandwidth of at least 15 kHz (it should be as narrow as possible).
- Amplifier 4 plus filter 4: this amplifier has a high gain and the low-pass filter needs to have a bandwidth of at least 15 kHz (it should be as narrow as possible).

- The amplitude modulator is an amplifier with variable gain, which is driven by the amplified modulating signal.

The block diagram reports some reasonable values for the signal power, i.e. the amplifier gains.

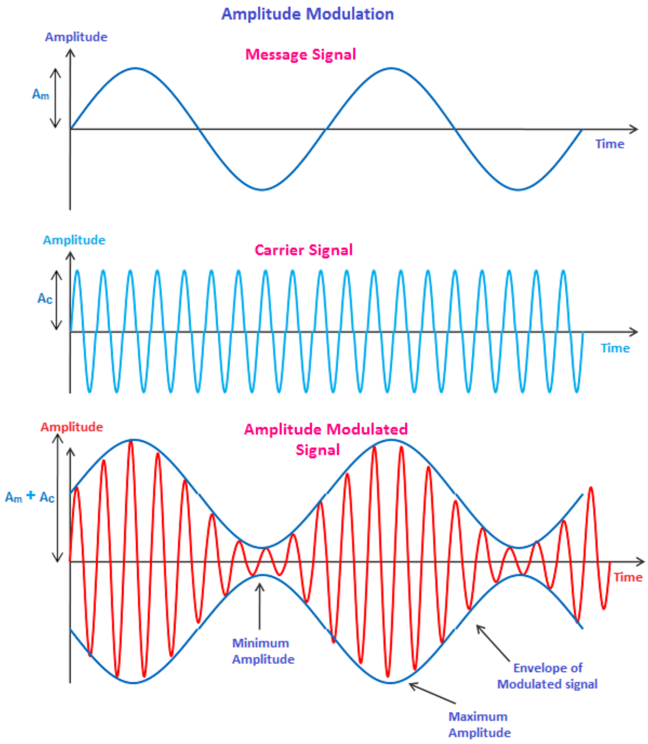
The output modulated signal is given by:

$$V(t) = (V_0 + k_a V_m(t))\cos\left(2\pi f_c t\right)$$

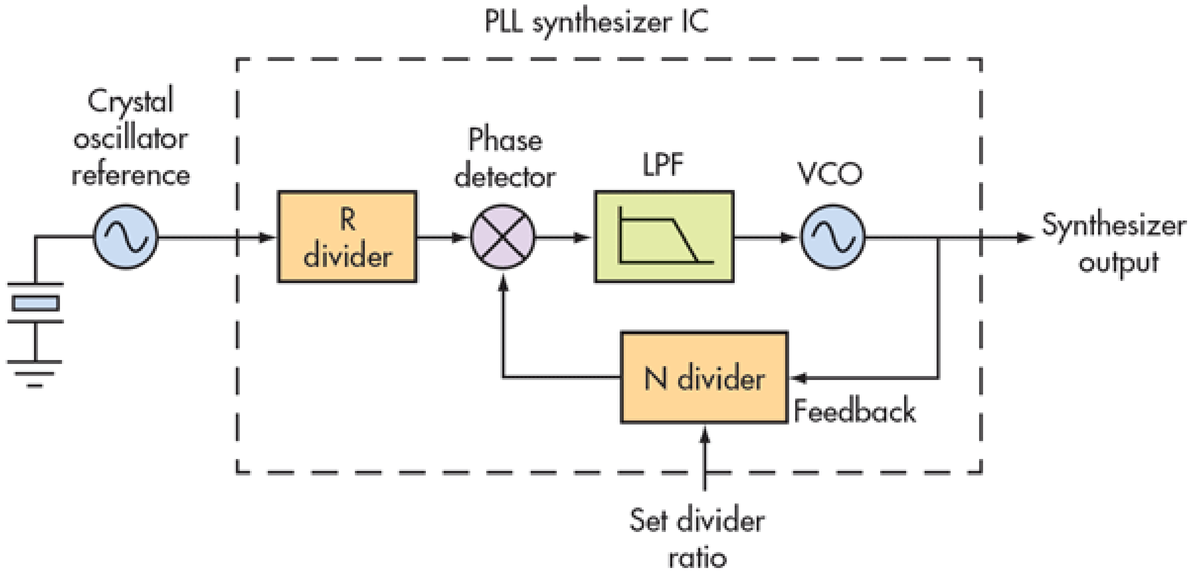
where V_0 is the amplitude of the input carrier frequency, $V_m(t)$ is the modulating signal, and f_c is the carrier frequency. Finally k_a is the coefficient that can be determined by knowing the modulation depth m_d:

$$m_d = \frac{k_a \max\{V_m(t)\}}{V_0}$$

An example of the modulated time series is given below:



In order to have a multichannel transmitter, the quartz oscillator can be replaced by a frequency synthesizer, which allows tuning the carrier frequency depending on the selected channel. This component consists of a reference very stable quartz oscillator and of a Voltage Controlled Oscillator (VCO), whose frequency is function of the input voltage. R and N are chosen so as to obtain the desired carrier frequency, so they will depend on the channel spacing (50 kHz).



In this case filter 1 needs to be changed accordingly: one possibility is to keep a very selective filter with tunable centre frequency; the other is to use a filter with a bandwidth larger enough to accommodate all the selectable carrier frequencies (95-105 MHz). The latter solution will introduce more noise though.