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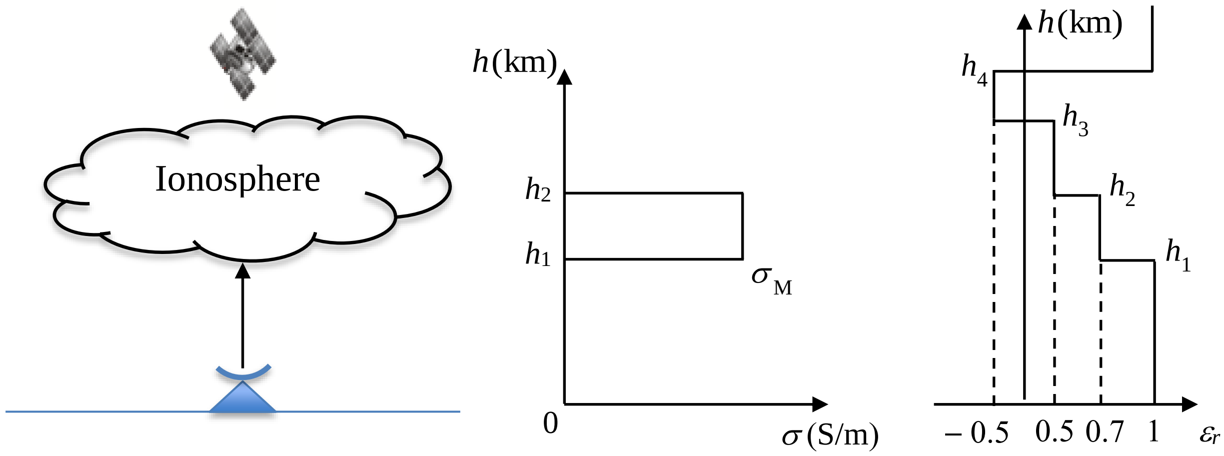
Problem 1

Making reference to the figure below, a ground station transmits an EM signal to a satellite (zenithal pointing). The figure also reports the vertical profile of the equivalent conductivity σ of the ionosphere ($\sigma_M = 1.03\times10^{-7}$ S/m, h_1 = 50 km, h_2 = 100 km), as well as the vertical profile of its equivalent relative permittivity ε_r (h_3 = 150 km, h_4 = 200 km). Both profiles are associated to the link frequency f = 9 MHz. For this scenario:

1) Determine if the wave reaches the satellite.
2) Calculate the peak electron content of the ionospheric profile.
3) Calculate the total path attenuation impairing the wave propagation.

Assume: if required, virtual reflection height $h_V = h_R$ (where h_R is the real reflection height); no tropospheric effects.



Solution

1) Looking at the vertical profile of the ionosphere, between h_3 and h_4, ε_r becomes negative (-0.5). This condition corresponds to having an evanescent wave, i.e. total reflection. As a result, at frequency f, the wave will not reach the satellite.

2) Recalling the expression of ε_r in the ionosphere (when there is no attenuation, which is the case for between h_3 and h_4, as $\sigma = 0$):

$$\varepsilon_r = 1 - \left(\frac{f_P}{f}\right)^2 = 1 - \left(\frac{9\sqrt{N}}{f}\right)^2$$

From the expression above, the higher the value of N, the more negative ε_r. The lowest value of ε_r is found between h_3 and h_4, i.e. where the highest value of N lies. Setting $N = N_{max}$, and inverting the equation above:

$$N_{\text{max}} = \frac{f^2(1-\varepsilon_r)}{81} = 1.5 \times 10^{12} \text{ e/m}^3$$

3) The wave travels from the ground up to h_3, where it is reflected back to ground. Given the profiles in the figure, the ionosphere attenuates only between h_1 and h_2, as $\sigma = 0$ elsewhere along the profile. The specific attenuation α is obtained from the propagation constant γ:

$$\gamma = \sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_r\varepsilon_0)} = 2.32 \times 10^{-5} + j0.158 \quad 1/\text{m}$$

where $\sigma = \sigma_M$ and $\varepsilon_r = 0.7$.

Thus:

$$\alpha = 2.32 \times 10^{-5} \text{ Np/m} \quad \rightarrow \quad \alpha_{dB} = 0.2014 \text{ dB/km}$$

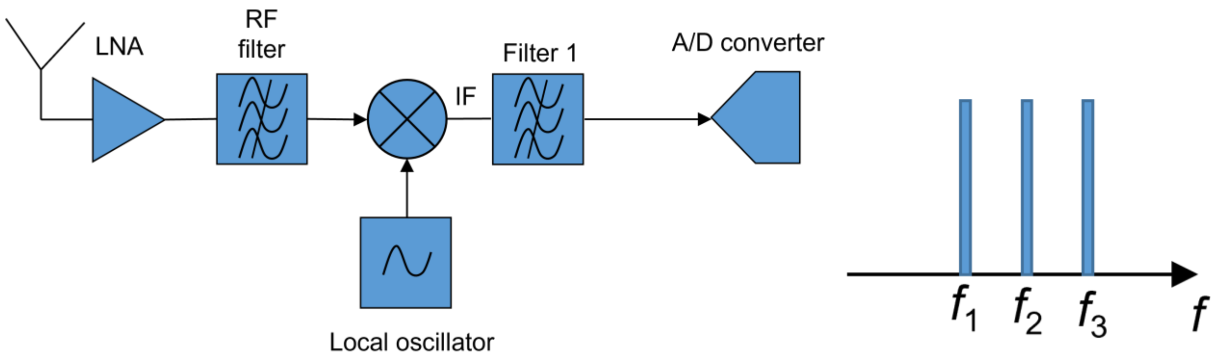
Therefore the path attenuation (one way) is:

$$A = \alpha_{dB}(h_2 - h_1) = 10 \text{ dB}$$

Problem 2

Consider the heterodyne receiver depicted below (left side), which aims at receiving the RF signal with carrier frequency f_2 = 30 GHz, to be digitalized at the end of the receiver chain. As indicated in the picture below (right side), the RF spectrum is occupied by multiple signals, all having the same bandwidth B = 100 MHz, with other carriers being f_1 = 28 GHz and f_3 = 32 GHz. The local oscillator frequency is f_{LO} = 29 GHz.

1) Assuming that the RF filter can be either a low-pass or a high-pass filter, indicate which one should be chosen and explain why. Finally, propose a suitable cutoff frequency for such a filter.
2) Calculate the optimum band of Filter 1 to maximize the SNR as input to the A/D converter.
3) Indicate the minimum sampling frequency of the A/D converter.
4) Calculate the exact SNR as input to the A/D converter knowing that:
   - the signal power as input to the LNA is P_1 = 1 pW;
   - the equivalent noise temperature as input to the LNA is T_{IN} = 250 K;
   - the equivalent noise temperatures of the LNA is T_{LNA} = 200 K and its gain is G_{LNA} = 30 dB;
   - that the other mixer and the RF filter are ideal, i.e. they do not introduce any loss/amplification/noise;
   - Filter 1 do not introduce any loss/amplification, but its equivalent noise temperatures is T_{F1} = 1000 K.



Solution

1) When down converting signals from RF to intermediate frequency (IF), image signals represent a problem. In this case, as the oscillator frequency is f_{LO} = 29 GHz, the image signal is the one with carrier f_1. As a result, the RF filter should be a high-pass filter, with cutoff frequency ideally around 29 GHz.

2) After down conversion, the carrier frequency f_2 is converted to $f_{IF} = f_2 - f_{LO}$ = 1 GHz. The optimum Filter 1 will have a lower cut-off frequency of $f_{min} = f_{IF}\text{-}B/2$ = 0.95 GHz and $f_{max} = f_{IF}+B/2$ = 1.05 GHz.

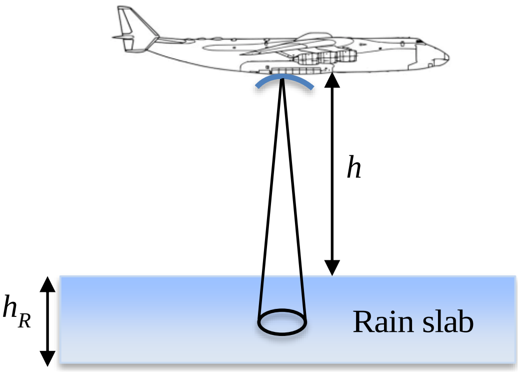
3) From the Nyquist theorem, the minimum sampling frequency of the A/D converter is $f_S = 2f_{max}$ = 2.1 MS/s.

4) The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_1 G_{LNA}}{kT_{sys}B} = \frac{P_1 G_{LNA}}{k(T_{IN} + T_{LNA} + T_{F1}/G_{LNA})B} = \frac{1\ \text{nW}}{6.22 \times 10^{-13}} = 1.61 \times 10^3 = 32.06\ \text{dB}$$

Problem 3

Making reference to the figure below, a pulsed radar onboard an airplane, operating with carrier frequency f = 10 GHz and pointed zenithally, is used to measure precipitation. The beam illuminates an area filled with rain at distance h = 8 km from the top of the rain slab. The height of the rain slab is h_R = 2 km. The rain drops density is N = 100 drops/m^3, they all have the same dimension and same extinction section, i.e. C = 3 mm^2. The power transmitted by the radar is P_T = 370 W and the radar antenna gain is G = 35 dB. Calculate if the radar can measure the precipitation at position h_R /2 from the ground, knowing that the sensitivity of the radar receiver is P_R = 1 pW and that, at any position in the rain slab, the total radar cross section due to rain is σ = 6 m^2.



Solution

First, let us calculate the power density reaching the point of interest in the rain volume:

$$S = \frac{P_T}{4\pi(h + h_R/2)^2} G A_R f = 8.516 \times 10^{-4}\ \mathrm{W/m^2}$$

where G = 3162, f = 1 (radar pointing to the volume).

A_R takes into account the attenuation induced by rain: in fact, as the wave penetrates into the rain slab, it gets partially reflected (see σ) and partially attenuated. The specific attenuation due to rain is calculated as:

$$\alpha = \frac{1}{2} NC = 1.5 \times 10^{-4}\ \mathrm{Np/m}$$

The path attenuation due to rain (from the beginning of the slab to the point of interest) is:

$$A_R = e^{-2\alpha \frac{h_R}{2}} = 0.74$$

The power reirradiated by the ensemble of rain drops is quantified by the total radar cross section due to rain σ (with gain = 1 according to the definition of radar cross section):

$$P_t = S\sigma = 0.0051\ \mathrm{W}$$

Thus, the power received by the radar is given by:

$$P_R = \frac{P_t}{4\pi(h + h_R/2)^2} A_R A_E = \frac{P_t}{4\pi(h + h_R/2)^2} A_R G \frac{\lambda^2}{4\pi} = 8.42 \times 10^{-13}\ \mathrm{W}$$

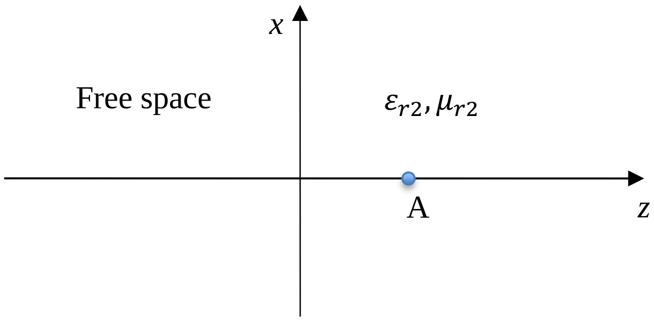
As P_R < 1 pW, the radar cannot measure the precipitation at the point of interest.

Problem 4

A uniform plane wave with horizontal polarization (frequency f = 300 MHz) propagates along z, from free space into a medium with the following electromagnetic features: ε_{r2} = 1-j, μ_{r2} = 1. The incident electric field at the origin of the axis is $\boldsymbol{\vec{E}_i(0) = E_0\vec{\mu}_y = 10\vec{\mu}_y}$ V/m.

For this scenario:

a) Calculate the expression of the magnetic field in the first medium (left side)
b) Calculate the power received by the isotropic antenna located at A(0,1,λ_2); to this aim assume that the antenna efficiency is η_A = 0.9 and that it is suitable to receive any polarization.



Solution

a) First, let us calculate the reflected wave, which implies first finding the reflection coefficient (no approximations are possible as the loss tangent is equal to 1):

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \approx 377\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{j\omega\varepsilon_2}} = 292.7 + j121.2\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.09 + j0.197$$

The total magnetic field in medium 1 is:

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r = -\frac{E_0}{\eta_0} e^{-j\beta_1 z}\vec{\mu}_x + \Gamma\frac{E_0}{\eta_0} e^{j\beta_1 z}\vec{\mu}_x \quad \text{A/m}$$

where $\beta_1 = 2\pi/\lambda_0 = 2\pi f/c$ = 6.28 rad/m

b) The wavelength in the second medium depends on the phase constant, which can be determined using the following expression (no approximations are possible as the loss tangent is equal to 1):

$$\gamma_2 = \sqrt{j\omega\mu_2 j\omega\varepsilon_2} = 2.86 + j6.91\ \text{1/m}$$

Therefore, the wavelength is:

$$\lambda_2 = \frac{2\pi}{\beta_2} = 0.91\ \text{m}$$

The power received at A can be calculated first by finding the transmitted electric field at z = 0:

$$\vec{E}(z=0) = \vec{E}_i(z=0) + \vec{E}_r(z=0) = E_0\vec{\mu}_x + E_0\Gamma\vec{\mu}_x = (9.1018 + j1.9737)\vec{\mu}_x \quad \text{V/m}$$

The power density reaching point A is:

$$S(A) = \frac{1}{2}\frac{\left|\vec{E}(z=0)\right|^2}{|\eta_2|^2}\cos(\measuredangle\, \eta_2)e^{-2\alpha_2\lambda_2} = 2.2\ \mu\mathrm{W/m^2}$$

The received power is:

$$P(A) = S(A)A_E = S(A)A_E = S(A)D\eta_A\frac{\lambda_0^2}{4\pi} = 0.157\ \mu\mathrm{W}$$

where:

D = 1 (directivity for an isotropic antenna)

λ_0 = free space wavelength