Telecommunication Systems
January 14th, 2020

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SURNAME AND NAME ____________________

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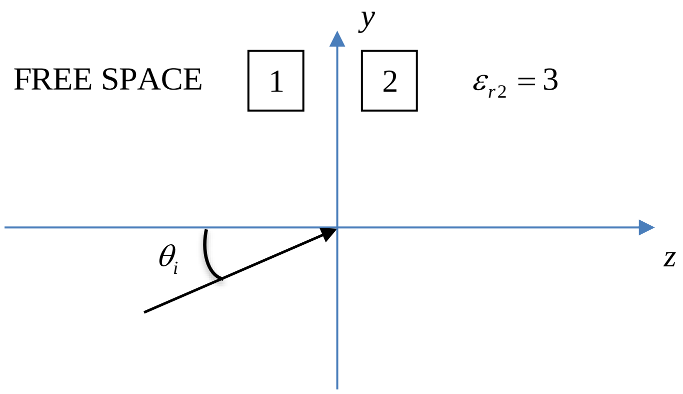
Problem 1

A plane sinusoidal EM wave propagates from free space into a medium with electric permittivity $\varepsilon_{r2} = 3$ ($\mu_{r2} = 1$), with incidence angle θ_i. The expression for the electric field in the first medium is:

$$\vec{E}_i(z,y) = \vec{E}_i^{TM} + \vec{E}_i^{TE} = \left[\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right) - j3\vec{\mu}_x\right] e^{-j\cos\theta 209.44 z} e^{-j\sin\theta 209.44 y} \text{ V/m}$$

For this wave:

1) Determine the frequency of the EM wave
2) Determine the polarization of the incident EM wave
3) Determine θ_i to obtain a linearly polarized reflected wave
4) OPTIONAL: write the equation of the reflected electric field for the incidence angle determined at point 3)



Solution:

1) The frequency of the incident EM wave can be derived from the phase constant $\beta = 209.44$ rad/m:

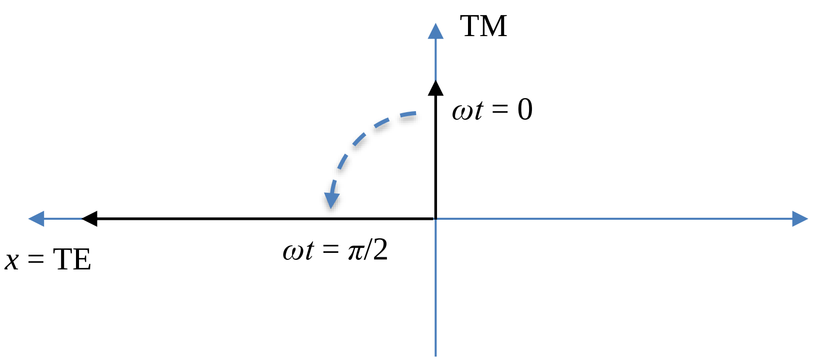
$$\beta = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}} \quad \Rightarrow \quad f = \frac{c\beta}{2\pi\sqrt{\varepsilon_{r1}}} = 10 \text{ GHz}$$

2) The polarization of the incident wave is LHEP (left-hand elliptical polarization) because the two TE and TM components have different amplitudes and a phase shift of $\pi/2$. In fact, setting y and z to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \text{Re}\left\{\left[\left(\cos\theta\,\vec{\mu}_y - \sin\theta\,\vec{\mu}_z\right) - j3\vec{\mu}_x\right]e^{j\omega t}\right\} = \cos(\omega t)\,\vec{\mu}_{TM} - 3\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_{TE} \text{ V/m}$$

Thus, making reference to the figure below that shows the reference system as seen from behind the wave, for $t = 0$ → $\vec{E}(0,0)\big|_{\omega t=0} = \vec{\mu}_{TM}$ V/m

Afterwards, for $\omega t = \pi/2$ → $\vec{E}(0,0)\big|_{\omega t=\pi/2} = 3\vec{\mu}_{TE}$ V/m



3) The incidence of the wave on the discontinuity will normally give birth to a reflected wave and a transmitted (refracted) wave, both for the TE and TM components. However, for the latter, the reflection is nullified when the incidence angle θ_i coincides with the Brewster angle θ_B, given by:

$$\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 60°$$

Thus, by imposing $\theta_i = \theta_B$, the TM component is completely transmitted into the second medium (no reflection), while the TE one is partially reflected and partially refracted. In this case, the reflected wave will be a TE linearly polarized wave.

4) First, we need to calculate the reflection coefficient for TE waves, which, in turn, requires calculating the refraction angle as:

$$\sqrt{\varepsilon_{r1}\mu_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}\mu_{r2}}\sin\theta_t \rightarrow \theta_t = 30°$$

$$\eta_1^{TE} = \frac{\eta_0}{\cos\theta_i\sqrt{\varepsilon_{r1}}} = 754 \ \Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_t\sqrt{\varepsilon_{r2}}} = 251.3 \ \Omega$$

$$\Gamma^{TE} = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0.5$$

The full expression for the reflected electric field is given by:

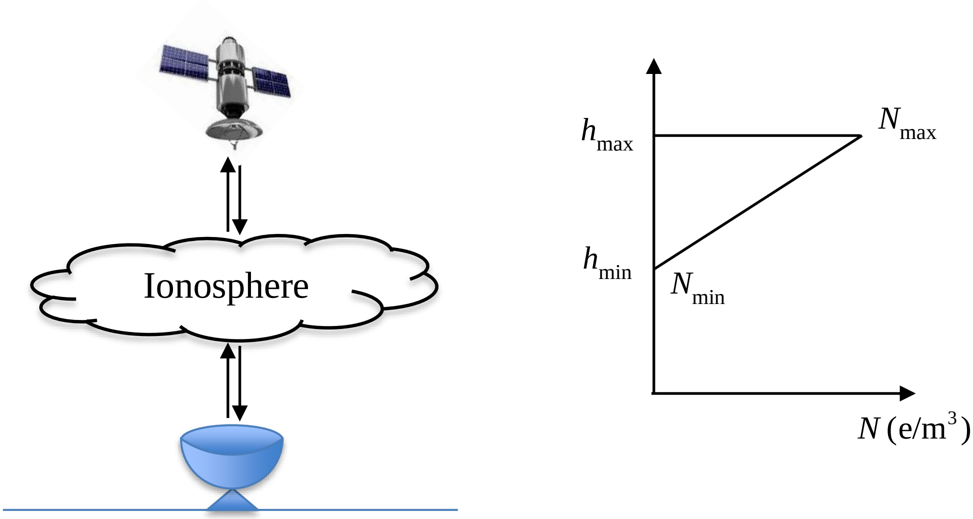
$$\begin{aligned}\vec{E}_r(z,y) &= \vec{E}_i^{TE}(0,0)\,\Gamma^{TE} e^{j\cos\theta_r 209.44z} e^{-j\sin\theta_r 209.44y} = \\ &= -j3\vec{\mu}_x(-0.5) e^{j\cos\theta_r 209.44z} e^{-j\sin\theta_r 209.44y} = \\ &= j1.5\vec{\mu}_x e^{j\cos\theta_r 209.44z} e^{-j\sin\theta_r 209.44y} \quad \text{V/m}\end{aligned}$$

where: $\theta_r = \theta_i$.

Problem 2

As reported in the figure below, there are two radars, both aiming to measure the altitude of the top layer of the ionosphere (h_{max}), for the electron content profile depicted on the right side (N_{max} = 5×10^{12} e/m^3 and N_{min} = 10^{10} e/m^3). The ground-based radar points up with 90° elevation angle and the space-borne one points down towards the center of the Earth. Determine:

- for both radars, the operational frequency to be used to properly measure h_{max};
- which radar will provide the best accuracy (justify the answer).



Solution

Ground-based radar: the operational frequency of the radar must be high enough to avoid that the wave is reflected by any electron content value between h_{min} and h_{max} and low enough to avoid that the wave crosses the ionosphere. Therefore, f is determined using the following equation, where $\theta = 90°$:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{max}}}{f}\right)^2} \quad \Rightarrow \quad f = \frac{9\sqrt{N_{max}}}{\sin\theta} \approx 20.12 \text{ MHz}$$

Space-borne radar: in this case, the first electron content value that the wave will interact with is N_{max}: like for the ground-based radar, for any frequency higher than f, the wave will cross the ionosphere; however, while a frequency lower than f for the ground-based radar will result in a reflection below h_{max}, this is not the case for the space-borne radar: any frequency lower than f will be anyway reflected by the layer at h_{max}.

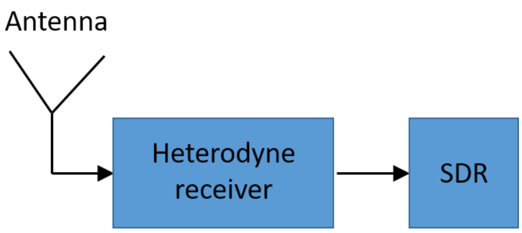
Which radar provides the best accuracy? The space-borne configuration is superior for the following reasons:

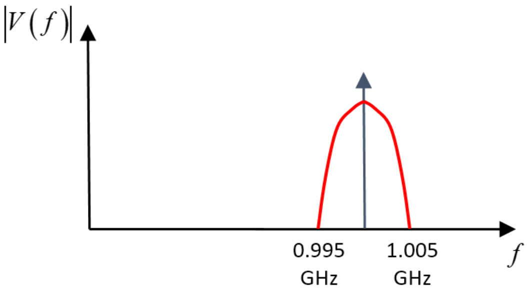
- based on the discussion above, the choice of the operational frequency is less critical for the space-borne radar than for the ground-based one;
- the pulse transmitted by the ground-based radar will travel through troposphere and through the ionosphere, in both cases accumulating extra delays (difficult to predict), which will translate into an uncertainty in measuring h_{max}. This is not the case for the space-borne radar, as the pulse will travel mostly in free space;
- finally, the ionosphere will also induce attenuation and depolarization, which, in turn, will further degrade the pulse emitted by the ground radar.

Problem 3

Consider the receiver depicted below (left side), consisting in a single-stage heterodyne receiver followed by a Software Defined Radio (SDR) card, which has the task to post-process the received signal digitally. The maximum sampling frequency of the SDR card is 50 MHz. The radio frequency signal to be received is shown below (right side): the carrier frequency is f_{RF} = 1 GHz and the signal bandwidth is 10 MHz. For this receiver:

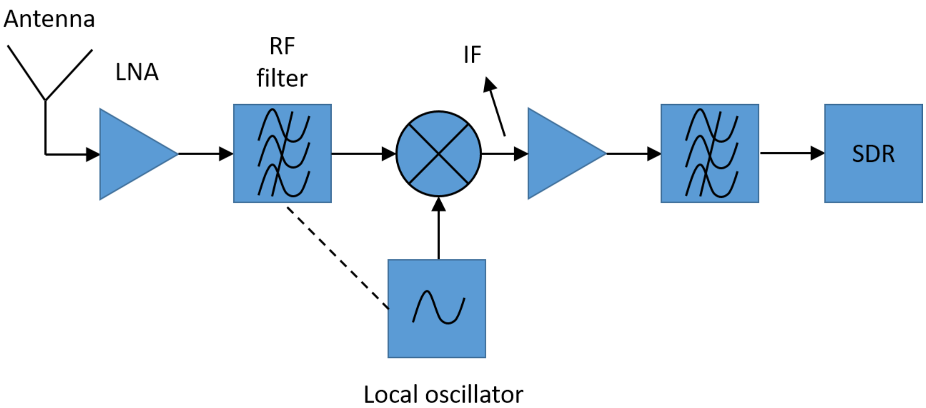
1. Draw the block scheme of the single-stage heterodyne receiver.
2. Determine the maximum carrier intermediate frequency to allow a proper digitalization of the received signal.
3. Determine the maximum frequency of the mixer local oscillator to achieve the condition at point 2.



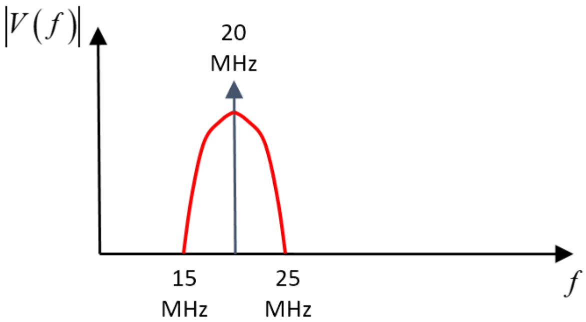


Solution

The block scheme of the single-stage heterodyne receiver is:



When an analog signal is digitalized, the sampling frequency f_s must be chosen according to the Nyquist-Shannon sampling theorem, which states that $f_s \geq 2f_{max}$, where f_{max} is the maximum frequency of the signal. The maximum f_s for the SDR is 50 MHz, therefore, when the signal is shifted to intermediate frequency, $f_{max} \leq f_s/2$ = 25 MHz; pushing the SDR to its limit, this means that after the down-conversion, we obtain:



Therefore the maximum carrier intermediate frequency is f_{IF} = 20 MHz.
After the mixer, the intermediate carrier frequency f_{IF} is given by:

$f_{IF} = f_{LO} - f_{RF}$

Therefore, the maximum local oscillator frequencies is:

$f_{LO} = f_{IF} + f_{RF}$ = 20 MHz + 1 GHz = 1.02 GHz

Problem 4

Consider a downlink from a spacecraft to a ground station. Determine whether it is more convenient to use f_1 = 20 GHz or f_2 = 30 GHz as carrier frequency for the link (i.e. which one will provide the highest *SNR* for the same target yearly availability of the link), considering that the statistics of the atmospheric attenuation *A* can be modelled using the following Complementary Cumulative Distribution Functions (probability expressed in percentage values, *A* expressed in dB) for the two frequencies:

$$P(A) = 100e^{-1.15A} \text{ at 30 GHz}$$

$$P(A) = 100e^{-2.3A} \text{ at 20 GHz}$$

Use the following data:

- the target yearly availability of the link is P_{AV} = 99.999%
- both antennas are parabolic with diameter of D = 2 m
- the efficiency of both antennas is η = 0.6 (for both frequencies)
- both antennas are optimally pointed
- the transmitted power is P_T = 5 W
- the distance between the ground station and the satellite is H = 36000 km
- the receiver LNA equivalent noise temperature is T_{LNA} = 120 K
- the mean radiating temperature of the troposphere is T_{mr} = 278 K
- assume that there are no additional losses in the transmitter and receiver chains, nor antenna pointing inaccuracies
- the system bandwidth is B = 50 MHz

Solution

In order to guarantee the target availability for the link (99.999% of the time), we need to set $P(A)$ = 100%-99.999% = 0.001%. As a result, we obtain $A_{30} \approx 10$ dB = 0.1 and $A_{20} \approx 5$ dB = 0.3162, at 30 GHz and 20 GHz, respectively. The SNR is calculated as:

$$SNR = \frac{P_T G_T \left(\dfrac{\lambda}{4\pi H}\right)^2 G_R A}{k T_{sys} B}$$

where f_T and f_R (radiation pattern) are assumed to be 1, as antennas are optimally pointed.

Several terms in such equation depend on the frequency, specifically:

<u>Gain</u>

Starting from the key formula $\dfrac{A_{eff}}{G} = \dfrac{\lambda^2}{4\pi}$, and using the available data, we obtain:

$$G = \frac{4\pi\eta}{c^2} A_{phy} f^2 = \frac{4\pi\eta}{c^2}\left(\frac{D}{2}\right)^2 \pi f^2 = \frac{\pi^2 \eta D^2}{c^2} f^2$$

Therefore G_{20} = 105275 = 50.2 dB and G_{30} = 236870 = 53.7 dB

<u>Wavelength</u>

λ_{20} = 0.015 m and λ_{30} = 0.01 m

<u>Equivalent antenna noise power</u>

$$T_A = T_{mr}\left(1 - A\right) + T_C A$$

Thus (T_C = 2.73 K):

$T_{A20} = 190.1$ K and $T_{A30} = 250.5$ K

Therefore, the total system equivalent noise temperature is:

$T_{sys} = T_A + T_{LNA}$

Thus:

$T_{sys20} = 310.9$ K and $T_{sys30} = 370.5$

Using these data in the *SNR* equation, we obtain:

$SNR_{20} = 89.8 = 19.53$ dB and $SNR_{30} = 53.6 = 17.3$ dB → 20 GHz is more convenient.