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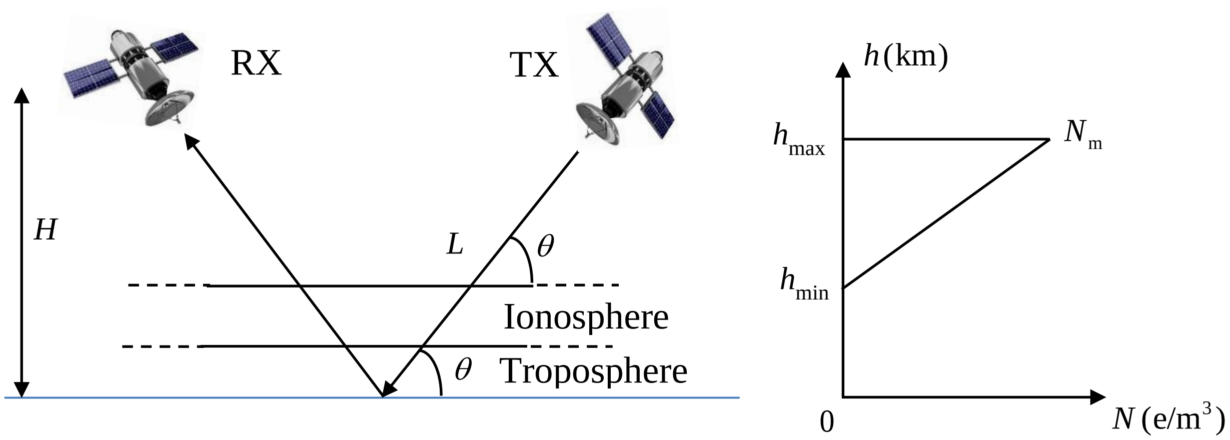
Problem 1

The figure below shows a bistatic radar system consisting of two LEO satellites flying along the same orbit (height above the ground H = 600 km). The radar extracts information on the ground by measuring the power received at RX by reflection. Making reference to the electron content profile (right side, $h_{\min}$ = 100 km, $h_{\max}$ = 400 km, $N_m = 4\times10^{12}$ e/m^3, N homogeneous horizontally) and to the simplified geometry (left side) reported below:

1) Determine the minimum value of the angle θ for the system to work properly, when the operational frequency is f_1 = 36 MHz.
2) Assuming the angle determined at point 1) and that the radar operational frequency changes to f_2 = 10 GHz, considering that the tropospheric attenuation along the path L is A = 3 dB, determine the power received at RX.

Assumptions: the gain of both antennas is G = 50 dB, the backscatter section of the ground is σ = 1000 m^2, the transmit power is P_T = 100 W.



Solution

1) For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{m}}}{f_1}\right)^2} \quad \Rightarrow \quad \theta_{min} = 30°$$

2) Fixing $\theta = \theta_{min}$, $L = H/\cos(90-\theta) = 1200$ km. Working at $f_2 = 10$ GHz, the ionospheric effects can be neglected, but not the atmospheric ones. The power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l = 2.77 \times 10^{-7}\ \mathrm{W}$$

where

$$A_l = 10^{-A/10} = 0.5$$

The power received by RX is:

$$P_R = \frac{S\sigma}{4\pi L^2} A_l A_E = 5.5 \times 10^{-17}\ \mathrm{W}$$

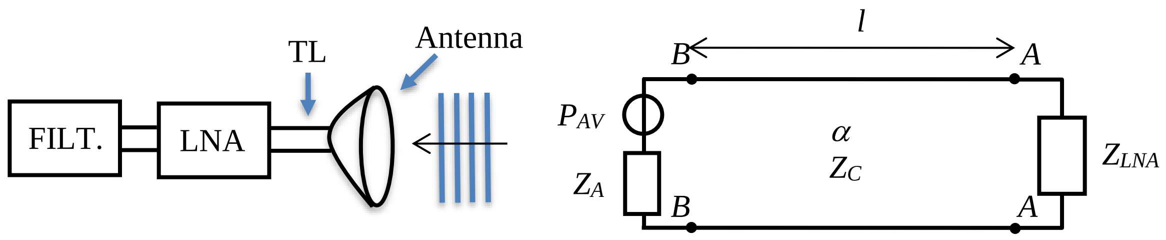
where:

$$A_E = \frac{\lambda^2}{4\pi} G = 7.16\ \mathrm{m}^2$$

Problem 2

The power received by an antenna is conveyed to a Low Noise Amplifier (LNA) via a lossy transmission line, with intrinsic impedance Z_C = 50 Ω and attenuation constant α_{dB} = 60 dB/km. The antenna acts as an equivalent generator with available power P_{AV} = 10 nW and internal impedance Z_A = 50 Ω; the impedance of the LNA is Z_{LNA} = 50 Ω. The frequency is f = 2 GHz. For this receiver:

1) Determine the maximum length l to guarantee that the power dissipated on the line is lower than 20% of the available power.
2) Using the l value determined at point 1), determine the minimum gain of the LNA G to guarantee an overall system equivalent noise temperature T_{sys} lower than 750 K (for the receiver chain, including a filter after the LNA, make reference to the left side of the figure below). To this aim, assume: antenna noise temperature T_A = 290 K, physical temperature of the transmission line T = 310 K, equivalent noise temperature of the LNA T_{LNA} = 350 K, equivalent noise temperature of the filter T_F = 600 K.



Solution

1) As there is total match, the power absorbed by the LNA is:

$P_L = P_{AV}e^{-2\alpha l}$

where $\alpha = 0.0069 = \alpha_{dB}/(8.686 \cdot 1000)$ Np/m.

The power crossing section BB is P_{AV}, so the power absorbed by the line is:

$P_{line} = P_{AV} - P_L$

Imposing that $P_{line} = 0.2\, P_{AV}$, replacing P_L with $P_{AV}e^{-2\alpha l}$, and solving for l:

$l < \frac{\ln(0.8)}{-2\alpha} = 16.15$ m

2) The total equivalent noise power of the system is:

$$T_{sys} = T_A + T_{TL} + T_{LNA} + \frac{T_F}{G}$$

The noise contribution introduced by the transmission line is:

$T_{TL} = T(1 - A) = 62\ K$

where:

$A = P_L/P_{AV} = 0.8$

Imposing T_{sys} = 750 K, solving for G:

$$G = \frac{T_F}{T_{sys} - (T_A + T_{TL} + T_{LNA})} = 12.5 = 10.97 \text{ dB}$$

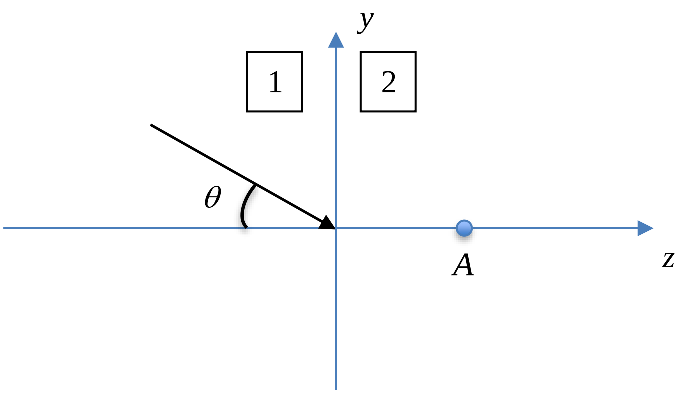
Therefore an LNA with a gain of 11 dB will allow having T_{sys} < 750 K.

Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 4 into free space (assume μ_r = 1 for both media). The incident electric field is:

$$\vec{E}_i(z,y) = -j\vec{\mu}_x e^{-j\beta\cos\theta z} e^{j\beta\sin\theta y}\ \text{V/m}$$

1) Determine the polarization of the incident field $\vec{E}_i$.
2) Determine the value of θ to maximize the power received in A(z = 1 m, y = 0 m, x = 10 m), where an isotropic antenna is located.
3) Calculate the power received by the antenna in A for the θ value determined at point 2).
4) Determine the value of θ to minimize the power received in A.



Solution

1) The wave has just one component, specifically the TE one, so the polarization is linear along -$\vec{\mu}_x$.

2) As the wave is a TE wave, there is no chance to have total transmission (this is possible only with the TM wave, when θ_i is the Brewster's angle). Therefore, the power density transmitted in the second medium will be maximized by minimizing the reflection coefficient, which occurs when θ_i = 0° (orthogonal incidence).

3) For θ_i = 0°, the reflection coefficients will be:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/\sqrt{\varepsilon_{r2}} - \eta_0/\sqrt{\varepsilon_{r1}}}{\eta_0/\sqrt{\varepsilon_{r2}} + \eta_0/\sqrt{\varepsilon_{r1}}} = 0.34$$

The power reaching A will be:

$$S_A = \frac{1}{2}\frac{|\vec{E}_i|^2}{\eta_0/\sqrt{\varepsilon_{r1}}}(1 - |\Gamma|^2) = 2.4\ \text{mW/m}^2$$

The power received by the antenna in A is:

$$P_R = S_A\, A_E = S_t \frac{\lambda^2}{4\pi} G = 0.21\ \mu\text{W}$$

where G = 1 (isotropic antenna).

4) The power received in A will be minimized (= 0 W) if θ_i is larger than the critical angle; in fact, total reflection (evanescent wave in the second medium) is possible as medium 1 is denser than medium 2 (electromagnetically). The critical angle is:

$$\theta_C = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

As a result, if $\theta_i \geq \theta_C$, no power will be received in A.

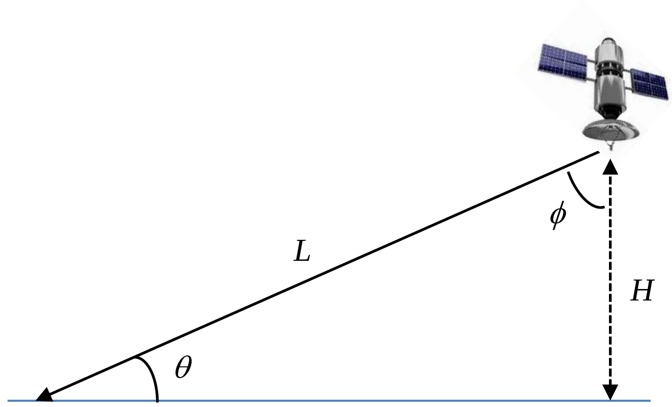
Problem 4

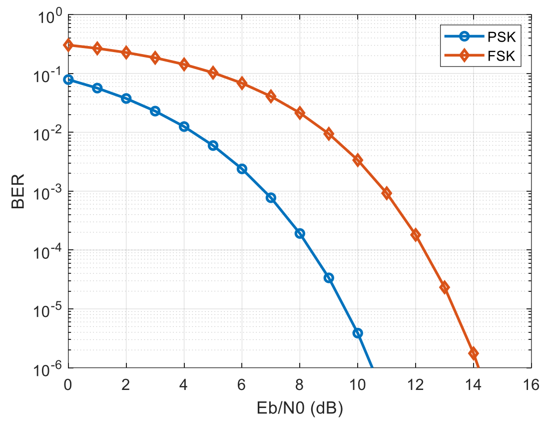
Consider a link from a LEO satellite to a ground station, operating at f = 30 GHz. For this system:

1) Determine if the link operates properly down to the target elevation angle θ = 20°, knowing that the minimum signal-to-noise ratio (SNR) at the ground station must be 5 dB and that the link needs to be available for 99.9% of the time. The CCDF of the zenithal tropospheric attenuation is given by:

$$P\left(A_T^{dB}\right) = 100 e^{-1.15 A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in \%})$$

2) Given the SNR determined at point 1), assuming to set the data rate equal to the system bandwidth and making reference to the figure below (right side), which modulation should be used to guarantee a BER lower than 10^{-4}?





Additional assumptions and data:

- use the simplified geometry depicted above (left side)
- the specific attenuation of the troposphere is homogeneous vertically and horizontally
- ground station tracking the satellite optimally
- power transmitted by the satellite P_T = 100 W
- LEO satellite pointing always to the centre of the Earth
- radiation patter of the LEO satellite antenna (circular symmetry): $f_T = \cos(\phi)$
- disregard the cosmic background temperature contribution
- mean radiating temperature T_{mr} = 290 K
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 30 dB
- altitude of the LEO satellite: H = 800 km
- bandwidth of the receiver: B = 1 MHz
- internal noise temperature of the receiver: T_R = 350 K

Solution

1) The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_T}{k[T_R + T_{mr}(1 - A_T)]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). As the link needs to be available for 99.9% of the time, the outage probability is P_{out} = 100%-99.9% = 0.1%. Using P_{out} in the CCDF expression, we obtain:

A_T^{dB} = 6 dB

Moreover, the attenuation scaled to the slant path is:

A_S^{dB} = 17.56 dB → $A_T = 0.0175$

Also, $f_T = \cos(90° - \theta) = 0.342$, while $f_R = 1$.

Finally, $L = H/\sin(\theta) = 2339$ km.

Solving the equation above → SNR = 9 dB > 5 dB → the system operates correctly down to the target elevation angle for 99.9% of the time.

2) Considering $R = B$ (R being the data rate), SNR = $Eb/N0$ = 9 dB will guarantee a BER < 10^{-4} using the PSK modulation.