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SURNAME AND NAME ______________________________

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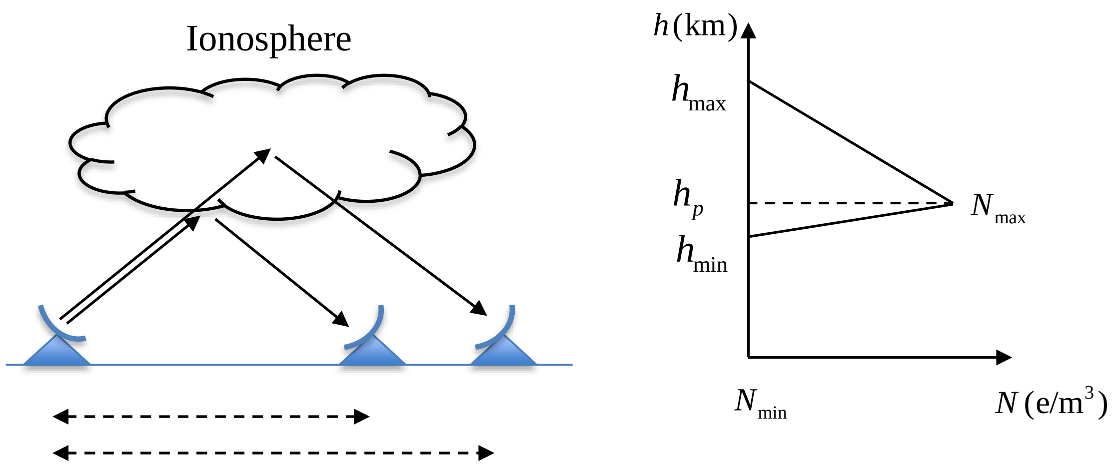
Problem 1

Making reference to the figure below, a bistatic ground-based pulsed radar system aims at measuring the profile of the ionospheric electron content (not the N values, but where the profile begins and ends), which is depicted in the figure below (N_{min} = 9×10^{10} e/m^3, N_{max} = 3×10^{12} e/m^3, h_{max} = 400 km, h_{min} = 100 km and h_p = 250 km, horizontally homogeneous). The transmitter elevation angle is θ_1 = 60°. The transmitter can work in an extended frequency range, and the receiver side consists of an array of antennas at different distance from the transmitter (see figure below). In this context:

1) Find the minimum frequency range to be used to achieve, as much as possible, the measurement of the electron content profile.
2) If the elevation angle decreases to θ_2 = 40°, assuming that also the frequency range needs to be changed accordingly to still achieve reflection, will the radar be more or less accurate compared to θ_1 = 60°? (MANDATORY: discuss qualitatively; OPTIONAL: perform calculations)



Solution

1) The operational frequency range of the radar can be obtained by exploiting the following expression:

$$\cos\theta_1 = \sqrt{1-\left(\frac{9\sqrt{N}}{f}\right)^2}$$

where f is the radar frequency. Therefore we can identify two frequencies associated to the minimum and maximum electron content:

$$f_{\min}^1 = \sqrt{\frac{81N_{\min}}{1-\left[\cos(\theta_1)\right]^2}} = 3.12 \text{ MHz}$$
$$f_{\max}^1 = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta_1)\right]^2}} = 18 \text{ MHz}$$

Using any frequency below $f_{\min}^1$ will guarantee total reflection at $h_{\min}$; increasing the frequency beyond $f_{\min}^1$ will allow progressively extending the reflection at altitudes higher than $h_{\min}$; for $f >$ $f_{\max}^1$, the wave will cross the ionosphere. Therefore the minimum frequency range is $f_{\min}^1 < f < f_{\max}^1$. As a result, only a portion of the profile will be measured.

2) If the elevation angle decreases to 40°, in order to still obtain reflection, the frequency range will need to increase. In fact:

$$f_{\min}^2 = \sqrt{\frac{81N_{\min}}{1-\left[\cos(\theta_2)\right]^2}} = 4.2 \text{ MHz}$$
$$f_{\max}^2 = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta_2)\right]^2}} = 24.3 \text{ MHz}$$

This will induce a lower ionospheric delay, which is given by the expression:

$$T_{iono} = \frac{1}{2c}\frac{81}{f^2}\text{TEC}$$

On the other hand, decreasing the elevation angle will increase the TEC along the path, i.e. the delay. Let us write T_{iono} for both elevation angles (taking as reference the minimum frequency):

$$T_{iono}^1 = \frac{1}{2c}\frac{81}{\left(f_{\min}^1\right)^2}\text{TEC}_1 = \frac{1}{2c}\frac{81}{\left(f_{\min}^1\right)^2}\frac{\text{TEC}}{\sin\theta_1} \qquad T_{iono}^2 = \frac{1}{2c}\frac{81}{\left(f_{\min}^2\right)^2}\text{TEC}_2 = \frac{1}{2c}\frac{81}{\left(f_{\min}^2\right)^2}\frac{\text{TEC}}{\sin\theta_2}$$

where TEC is the total electron content along the zenith. Therefore:

$$\Delta T_{iono} = T_{iono}^1 - T_{iono}^2 = \frac{81}{2c}\text{TEC}\left[\frac{1}{\sin\theta_1\left(f_{\min}^1\right)^2} - \frac{1}{\sin\theta_2\left(f_{\min}^2\right)^2}\right]$$

Even without calculating TEC (though we have sufficient data to do it), we can determine in which case we have a higher ionosphere delay by studying the factor within the square brackets:

$$\frac{1}{\sin\theta_1\left(f_{\min}^1\right)^2} - \frac{1}{\sin\theta_2\left(f_{\min}^2\right)^2} \approx 3.1\cdot 10^{-14} \text{ sec}^2$$

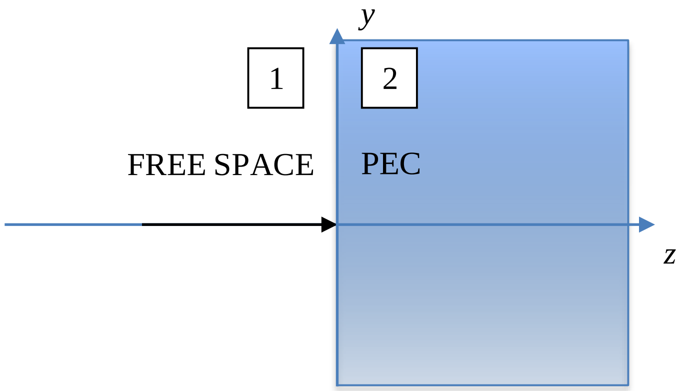
As this factor is positive, $T_{iono}^1 > T_{iono}^2$, i.e. the ionospheric delay will be higher at θ_1 = 60°. As a result, the radar will be more accurate using θ_2 = 40°.

Problem 2

A plane sinusoidal EM wave hits orthogonally the boundary between free space and a perfect electric conductor (PEC); the expression for the incident electric field is:

$$\vec{E}(z,y) = \left(\vec{\mu}_x + j\vec{\mu}_y\right)e^{-j\beta z} \ \text{V/m}$$

1) Determine the polarization of the incident wave.
2) Determine the polarization of the reflected wave.



Solution

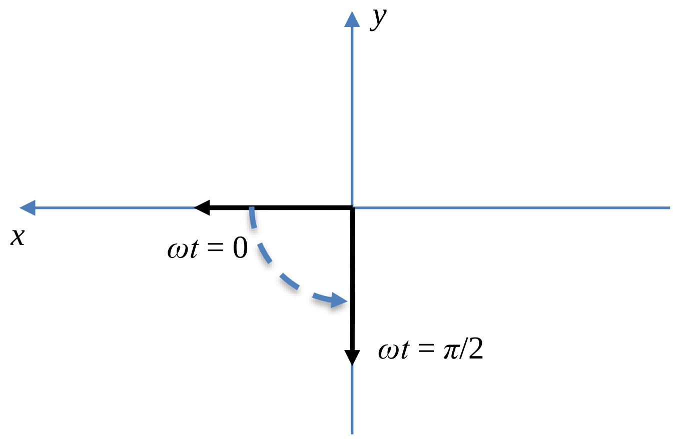
1) The polarization of the incident wave can be determined by setting z to 0, and by expressing the dependence on time:

$$\vec{E}(0,t) = \text{Re}\left\{\left(\vec{\mu}_x + j\vec{\mu}_y\right)e^{j\omega t}\right\} = \cos(\omega t)\,\vec{\mu}_x + \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_y \ \text{V/m}$$

Thus, for $t = 0$ → $\vec{E}(0)\Big|_{\omega t=0} = \vec{\mu}_x$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0)\Big|_{\omega t=\pi/2} = -\vec{\mu}_y$ V/m

Looking from behind the wave along its propagation direction, we can see the following:



The incident wave has left-hand circular polarization.

2) With a PEC, for both components of the wave, the reflection coefficient is equal to -1. Therefore the reflected wave is:

$$\vec{E}^r(z,y) = \left(-\vec{\mu}_x - j\vec{\mu}_y\right)e^{j\beta z} \text{ V/m}$$

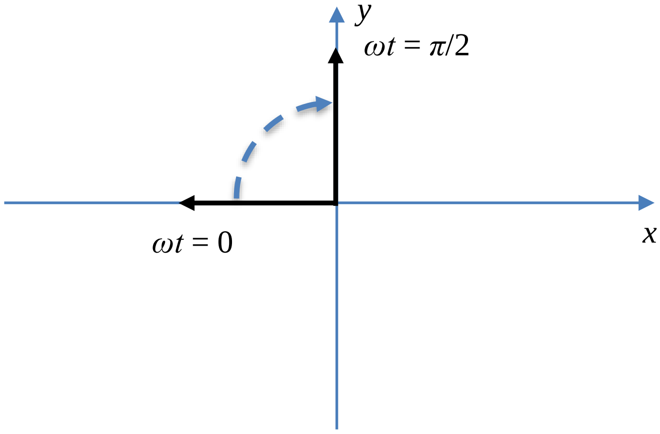
In this case:

$$\vec{E}(0,t) = \mathrm{Re}\left\{\left(-\vec{\mu}_x - j\vec{\mu}_y\right)e^{j\omega t}\right\} = -\cos\left(\omega t\right)\vec{\mu}_x - \cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_y \text{ V/m}$$

Thus, for $t = 0$ → $\vec{E}(0)\Big|_{\omega t=0} = -\vec{\mu}_x$ V/m

Thus, for $\omega t = \pi/2$ → $\vec{E}(0)\Big|_{\omega t=\pi/2} = \vec{\mu}_y$ V/m

Looking from behind the wave along its propagation direction, we can see the following:



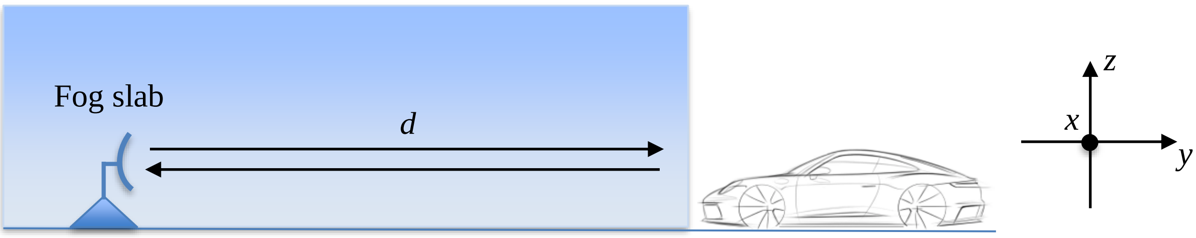
The reflected wave has right-hand circular polarization.

Problem 3

Making reference to the figure below, a ground-based pulsed radar, operating with carrier frequency of 45 GHz and pointed horizontally, is used to identify cars in low-visibility foggy conditions at a distance d = 18 km. The fog slab consists of spherical droplets, whose associated specific attenuation α = 0.08 dB/km is constant both horizontally and vertically. The polarization of the wave transmitted by the radar is vertical. In this context:

1) Determine the polarization of the wave in front of the car.
2) Calculate the minimum radar antenna diameter (assume a parabolic antenna) considering that the radar requires a minimum SNR_{min} = 7 dB to operate properly.

Consider the following data: radar transmit power P_T = 1 kW; car backscatter section σ = 4 m^2; neglect the attenuation due to gases; LNA equivalent noise temperature T_R = 300 K; mean radiating temperature of fog T_{mr} = 210 K; LNA very close to the radar antenna feed; radar bandwidth B = 1 GHz; antenna efficiency η = 0.7.



Solution

1) As the droplets are spherical, the wave will not be depolarized: in front of the car, the wave will still be vertically polarized.

2) First, let us calculate the power density reaching the car:

$$S_C = \frac{P_T}{4\pi d^2} G f A_F$$

where G is the antenna gain, f = 1 (radar pointing to the car) A_F is the fog attenuation in linear scale. This is first calculated in dB as:

$A_F^{dB} = \alpha d = 1.44 \text{ dB} \rightarrow A_F = 0.7178$

The power reirradiated by the car (with gain = 1 according to the definition of backscatter section), is:

$$P_A = S_C \sigma$$

The power density reaching the radar is:

$$S_R = \frac{P_A}{4\pi d^2} A_F \text{ W/m}^2$$

Finally, the power received by the radar is:

$$P_R = S_R A_E \text{ W}$$

Combining all the equations:

$$P_R = P_T G^2 \left(A_F\right)^2 \sigma \frac{\lambda^2}{\left(4\pi\right)^3 d^4}$$

The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{SYS}B}$$

where T_{SYS} is the system equivalent noise temperature given by:

$$T_{SYS} = T_R + T_A = T_R + T_{mr}(1 - A_F) = 359.3 \text{ K}$$

Therefore:

$$P_N = kT_{SYS}B = 4.96 \times 10^{-12} \text{ W}$$

By imposing SNR > SNR_{min} and combining the equations above:

$$SNR = \frac{P_R}{kT_{SYS}B} > SNR_{\min} \quad \rightarrow \quad G > \sqrt{\frac{SNR_{\min}(4\pi)^3 d^4 P_N}{P_T\left(A_F\right)^2 \sigma\lambda^2}} = 237880 = 53.76 \text{ dB}$$

As the effective area is:

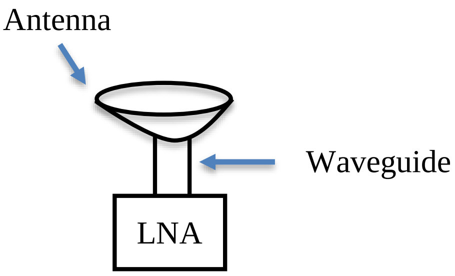
$$A_E = \frac{\lambda^2}{4\pi}G \quad \Rightarrow \quad D = \frac{\lambda}{\pi}\sqrt{\frac{G}{\eta}} \approx 1.24 \text{ m}$$

Problem 4

Consider the typical receiver sketched in the figure below. The antenna points zenithally to a satellite and the CCDF of the total tropospheric attenuation along the path, A_T, is given by the following expression (the mean radiating temperature of the atmosphere is T_{mr} =290 K):

$$P\left(A_T^{dB}\right) = 100e^{-1.15A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in \%})$$

The antenna is connected to a the low noise amplifier LNA (equivalent noise temperature T_{LNA} = 280 K) through a lossy waveguide with physical temperature T_{WG} = 20 °C, introducing an attenuation of A_{WG}^{dB} = 1 dB. The antenna, the waveguide and the LNA are all matched. The operational frequency is f = 30 GHz. Calculate the link availability P_{AV}, knowing that the maximum equivalent noise temperature for the receiving system to work properly is $\overline{T}_{sys}$ = 585 K. In the calculations, disregard the cosmic background noise.



Solution

The overall equivalent noise temperature for the receiving system is given by three contributions, those due to the antenna (T_{ANT}), the waveguide (T_{CON}) and the LNA (T_{LNA}):

$$T_{sys} = T_{ANT} + T_{CON} + T_{LNA}$$

The terms can be expanded as follows:

$$T_{sys} = T_{mr}(1 - A_T) + T_{WG}(1 - A_{WG}) + T_{LNA}$$

The tropospheric attenuation is not deterministic but we know its CCDF. The link will experience outage when T_{sys} will be higher than $\overline{T}_{sys}$ = 585 K, .i.e.:

$$T_{sys} > \overline{T}_{sys} \rightarrow T_{mr}(1 - A_T) + T_{WG}(1 - A_{WG}) + T_{LNA} > \overline{T}_{sys} \rightarrow A_T < 1 - \frac{\overline{T}_{sys} - T_{WG}(1 - A_{WG}) - T_{LNA}}{T_{mr}} = 0.1562$$

This result is in linear scale; in dB:

$$A_T^{dB} = 8.0637 \text{ dB}$$

Using this value in the CCDF equation, we obtain the link outage probability:

P_{OUT} = 0.0094%

Therefore the link availability is:

$P_{AV} = 100 - P_{OUT}$ = 99.9906%