Telecommunication Systems
July 18th, 2019

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SURNAME AND NAME ______________________________

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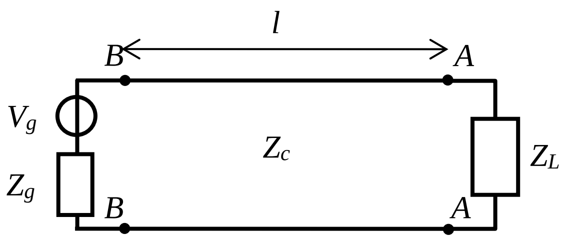
Problem 1

A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a transmission line with characteristic impedance Z_C = 75 Ω, the frequency is f = 600 MHz and the length of the line is l = 4.5 m.

a) Determine the value of the load Z_L to maximize the absorbed power.
b) Calculate the power absorbed by the load using the value of Z_L determined at point a)
c) OPTIONAL: calculate the power absorbed by the internal impedance Z_g = 50 Ω using the value of Z_L determined at point a)



Solution

a) The wavelength is:

$$\lambda = \lambda_0 = 0.5 \text{ m}$$

As a consequence, the length of the line normalized to the wave length is:

$$l / \lambda = 9$$

In other terms, the line is a multiple of λ. In this case:

$$Z_{BB} = Z_L$$

Therefore, in order to maximize the power absorbed by the load → $Z_L = Z_g$ = 50 Ω. In fact:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \frac{Z_L - Z_g}{Z_L + Z_g} = 0$$

b) Therefore, the power absorbed by the load is:

$$P_L = P_{AV} = \frac{\left|V_g\right|^2}{8\,\mathrm{Re}\left[Z_g\right]} = 0.25 \text{ W}$$

c) At section BB, the voltage on Z_g is given by:

$$V_{Zg} = V_g \frac{Z_g}{Z_g + Z_L} = \frac{V_g}{2} \text{ V}$$

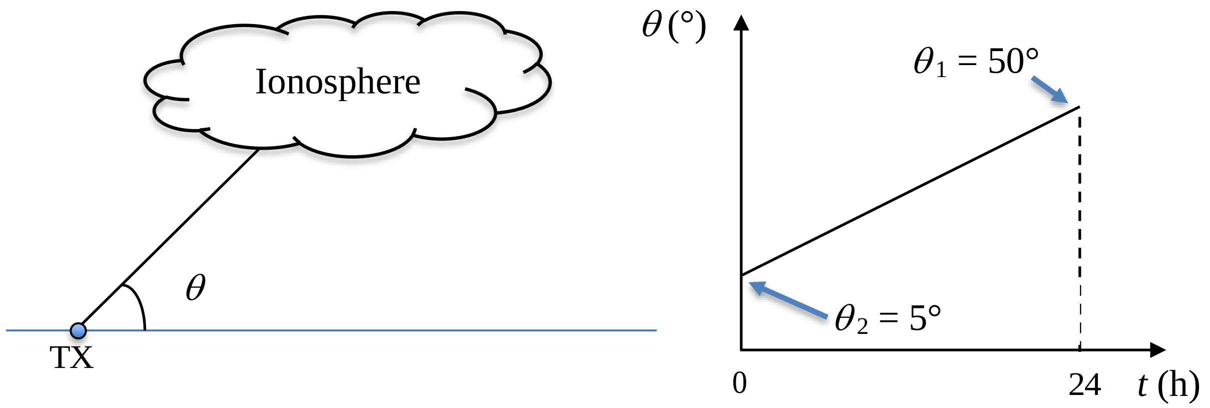
Therefore, the power absorbed by Z_g is:

$$P_g = \frac{1}{2}\left|V_{Zg}\right|^2 \mathrm{Re}\left[\frac{1}{Z_g}\right] = 0.25 \text{ W}$$

As expected in the case of perfect matching, the power absorbed by Z_g and by Z_L is the same.

Problem 2

Making reference to the figure below, a ground station points to a spacecraft with variable elevation angle θ, according to the graph reported below (right side). The link operational frequency is f = 30 MHz and maximum electron content along the ionospheric profile is constant and equal to N_{max} = 1.8×10^{12} e/m^3. Calculate the percentage of time during the day for which the link to the spacecraft can be established.



Solution

To determine if the ionosphere can be crossed, we need to invert the following equation to determine the critical angle θ_C, given f and N_{max}:

$$\cos\theta_C = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2} \quad \Rightarrow \quad \theta_C = \cos^{-1}\left(\sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2}\right) = 23.7°$$

If $\theta > \theta_C$, the wave crosses the ionosphere, otherwise it is completely reflected.

The trend of θ in the figure is given by (t expressed in hours):

$$\theta = \frac{\theta_1-\theta_2}{24}t+\theta_2$$

By imposing that $\theta > \theta_C$:

$$\frac{\theta_1-\theta_2}{24}t+\theta_2 > \theta_C \quad \Rightarrow \quad t > \frac{24\left(\theta_C-\theta_2\right)}{\theta_1-\theta_2} \approx 10 \text{ h}$$

The time percentage of the day (24-10 = 14 h) is therefore P = 58.3 %.

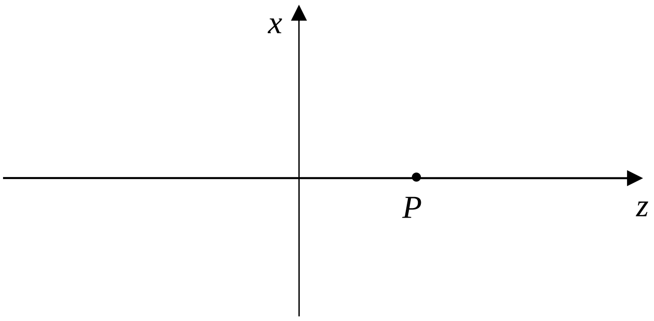
Problem 3

A uniform plane wave propagates in a perfect dielectric with ε_{r1} = 9 and μ_{r1} = 1, and hits the surface of a medium characterized by conductivity σ = 0.5 S/m, ε_{r2} = 1 and μ_{r2} = 1. The incident electric field is

$$\vec{E}_i = e^{-j0.00628z}\ \vec{\mu}_x \text{ V/m}$$

Calculate:

a) The wavelength of the wave in the second medium
b) The electric field reaching point P(0,0,3 m)



Solution

a) First, it is necessary to calculate the frequency. Based on β_1 = 0.00628 rad/m:

$$f = \frac{\beta_1 c}{2\pi\sqrt{\varepsilon_{r1}}} = 100 \text{ kHz}$$

The wavelength in the second medium depends on the propagation constant. The loss tangent for this medium is:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_0} = 9\times10^4$$

Thus the good conductor approximations can be used; therefore:

$$\gamma_2 = \alpha_2 + j\beta_2 = \sqrt{\frac{\omega\mu_2\sigma}{2}} + j\sqrt{\frac{\omega\mu_2\sigma}{2}} = 0.44(1+j) \text{ 1/m}$$

The wavelength in the second medium is:

$$\lambda_2 = \frac{2\pi}{\beta_2} = 14.1 \text{ m}$$

b) The electric field reaching point P depends on the reflection coefficient, which, in turn, depends on the intrinsic impedance of the medium. Using the same kind of approximation:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}} = 125.7 \quad \Omega$$

$$\eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma}} = 8.9\times10^{-1} \quad \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.9859 + j0.0139$$

The electric field in position P is:

$$\vec{E}_t(P) = (1+\Gamma)e^{-\alpha_2 z_P}e^{-j\beta_2 z_P}\ \vec{\mu}_x = (4.5\text{-}j2.8)\vec{\mu}_x \text{ mV/m}$$

Problem 4

Consider the downlink from a GEO satellite to a ground station at elevation angle of $\theta = 40°$. The link operational frequency is f = 15 GHz. Calculate the required ground station pointing accuracy P_{ACC} to guarantee a received power P_R higher than 5×10^{-10} W.
To this aim, use the following data:

- the gain of both antennas (parabolic type) is G = 50 dB and their efficiency is $\eta = 0.6$
- antennas are optimally pointed
- pointing loss of the satellite antenna P_{SAT}^{LOSS} = 0.1 dB
- the power transmitted by the satellite is P_T = 100 W
- the distance between the ground station and the satellite is H = 38000 km
- no satellite on-board losses and no waveguide losses in the receiver
- atmospheric attenuation A = 3 dB

If rain starts, the requirement on P_{ACC} will be more stringent or more relaxed?

Solution

The wavelength is $\lambda = c/f = 0.02$ m. The gain of the two antennas is:

$$G_T = G_R = 50\text{ dB} = 100000\ \text{ dB}$$

The atmospheric attenuation is:

A = 3 dB ≈ 0.5

The received power is:

$$P_R = P_T G_T f_T P_{SAT}^{LOSS} \left(\frac{\lambda}{4\pi H}\right)^2 G_R f_R P_G^{LOSS} A$$

where $f_T = 1$ and f_R = 1 (antennas are optimally pointed), while P_{SAT}^{LOSS} and P_G^{LOSS} are the pointing losses of the satellite and ground antennas, respectively. The ground station pointing loss includes the pointing accuracy of the ground station (P_{ACC} expressed in degrees and P_G^{LOSS} in dB):

$$P_G^{LOSS} = 12\left(\frac{P_{ACC}}{70\lambda / D}\right)^2$$

where D is the diameter of the antennas, which can be calculated from the antenna efficiency and the gain as:

$$A_{eff} = \eta A = \frac{\lambda^2}{4\pi}G \quad \Rightarrow \quad \eta\left(\frac{D}{2}\right)^2 \pi = \frac{\lambda^2}{4\pi}G \quad \Rightarrow \quad D = \frac{\lambda}{\pi}\sqrt{\frac{G}{\eta}} = 2.6\ \text{m}$$

Rearranging the link budget equation to isolate P_G^{LOSS} (P_{SAT}^{LOSS} must be converted in linear scale):

$$P_G^{LOSS} = \frac{P_R}{P_T G_T P_{SAT}^{LOSS}\left(\frac{\lambda}{4\pi H}\right)^2 G_R A} = 0.5824 = 2.35\text{ dB}$$

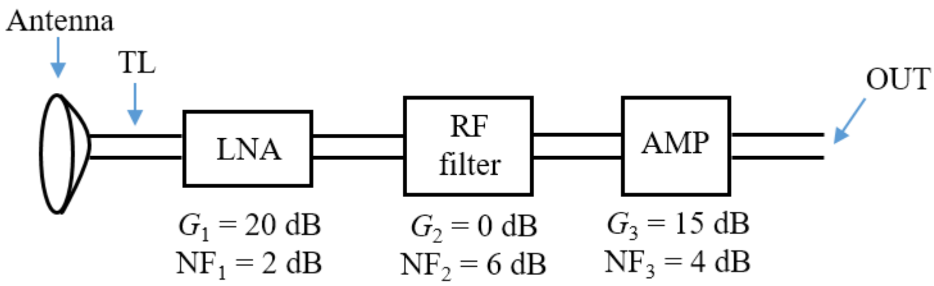
Finally, by inverting P_G^{LOSS}:

$$P_{ACC} < 70\frac{\lambda}{D}\sqrt{\frac{P_G^{LOSS}}{12}} = 0.24°$$

The presence of rain will increase the atmospheric attenuation, thus reducing the received power: the requirement on P_{ACC} will be more stringent (higher pointing precision).

Problem 5

Consider the block diagram below, relative to the front end of a radio receiver. The antenna noise temperature is T_A = 160 K. The transmission line has a physical temperature T_{WG} = 30 °C, its length is l = 3 m and is characterized by a specific attenuation of α = 0.3 dB/m. The receiver chain consists in the cascade of the low noise amplifier, a passband filter and a second amplifier. The gain G and the noise figure NF associated to each component are reported below. Calculate the equivalent noise temperature at the end of the receiver chain (position labelled as OUT).



Solution

The equivalent noise temperature at the end of the receiver chain is given by:

$$T_{OUT} = T_A + T_W + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2}$$

where T_W is the noise introduced by the waveguide, while T_i is the equivalent noise temperature of the i-th element in the receiver chain. Each of the latter can be calculated from the noise figure as:

$$T_i = 290\left(10^{\frac{NF_i}{10}} - 1\right) \rightarrow T_1 = 169.62 \text{ K},\ T_2 = 864.51 \text{ K and } T_3 = 438.45 \text{ K}$$

The attenuation introduced by the waveguide is:

$$A_{WG}^{dB} = \alpha l = 0.9 \text{ dB} \rightarrow A_{WG} = 0.8128$$

The equivalent noise temperature introduced by the waveguide is therefore:

$$T_W = T_{WG}(1 - A_{WG}) = 56.74 \text{ K}$$

As a result (G_1 = 100, G_2 = 1 and G_3 = 31.6, in linear scale):

T_{OUT} = 399.39 K