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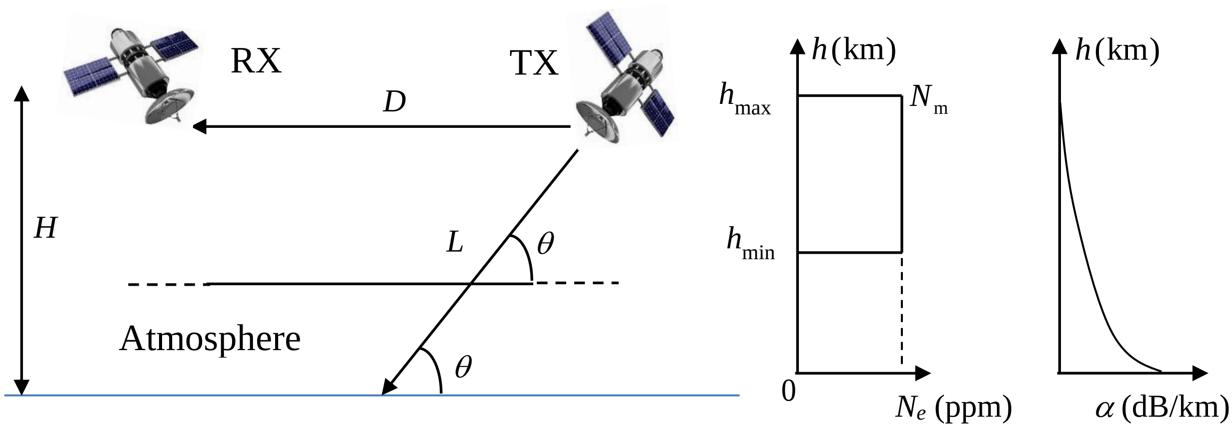
Problem 1

The figure below shows a bistatic radar system (carrier frequency f_1 = 1 GHz), consisting of two LEO satellites (same orbit, satellite height above the ground H = 500 km, elevation angle θ = 45°). The radar extracts information on the atmosphere by measuring the difference between the power received along the direct path and the downlink to the ground station. Making reference to the right, where the electron content (N_e) profile ($h_{\min}$ = 100 km, $h_{\max}$ = 400 km, N_m = 5×10^{12} e/m^3, N homogeneous horizontally) and the tropospheric specific attenuation profile ($\alpha = 0.01e^{-h/50}$, where h is in km and α, horizontally homogeneous, is in dB/km) are shown, and to the left side, where the simplified geometry is reported:

1) Calculate the difference of the power received along the two paths.
2) Will the difference between the power received along the two paths decrease or increase (compared to point 1) if the frequency changes to f_2 = 25 MHz?

Assume: transmit power P_T = 100 W; gain of all antennas (circular parabolic reflectors) G = 45 dB; antenna radiation pattern $f = [\cos(\phi)]^2$, where ϕ is the angle between the specific direction and the antenna axis; perfect pointing for the downlink, on both ends.



Solution

1) The power received along the direct path (free space) is simply given by:

$P_{R1} = P_T G_T f_T (\lambda/4\pi D)^2 G_R f_R$

where:

$$D = \frac{2H}{\text{tg}(\theta)}$$

$$G_T = G_R = G$$

$$f_T = f_R = \cos{(\theta)}$$

Therefore → $P_{R1} = 14.2\ \mu\text{W}$

Regarding the second path, the power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l$$

where $L = H/\sin(\theta)$ and A_l is the tropospheric attenuation (the ionospheric one can be neglected given the carrier frequency in the GHz range). The zenithal tropospheric attenuation can be calculated as:

$$A_Z = \int_0^H \alpha dh = \alpha_0 \int_H^{H_S} e^{-\frac{h}{h_0}} dh = -\alpha_0 h_0 \left[e^{-\frac{h}{h_0}} \right]_0^H$$

As $H >> h_0 = 50$ km:

$$A_Z = -\alpha_0 h_0 \left[e^{-\frac{h}{h_0}} \right]_0^\infty = \alpha_0 h_0 = 0.5\ \text{dB}$$

The slant path attenuation in linear scale is:

$$A_l = 10^{-(A_Z/\sin{(\theta)})/10} = \alpha_0 h_0 = 0.85\ \text{dB}$$

The power received on the ground is:

$$P_{R2} = S A_{RX} = \frac{P_T}{4\pi L^2} G A_l \frac{\lambda^2}{4\pi} G = 96.9\ \mu\text{W}$$

The differential power is:

$$\Delta P_R = P_{R2} - P_{R1} = 82.6\ \mu\text{W}$$

2) Using the second frequency (range of tens of MHz), it is worth checking if the wave can actually cross the ionosphere. For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as by:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\text{m}}}{f_2} \right)^2} \quad \Rightarrow \quad \theta_{min} = 53.6°$$

As $\theta < \theta_{min}$, the wave will be totally reflected and it will not be received by RX.

Problem 2

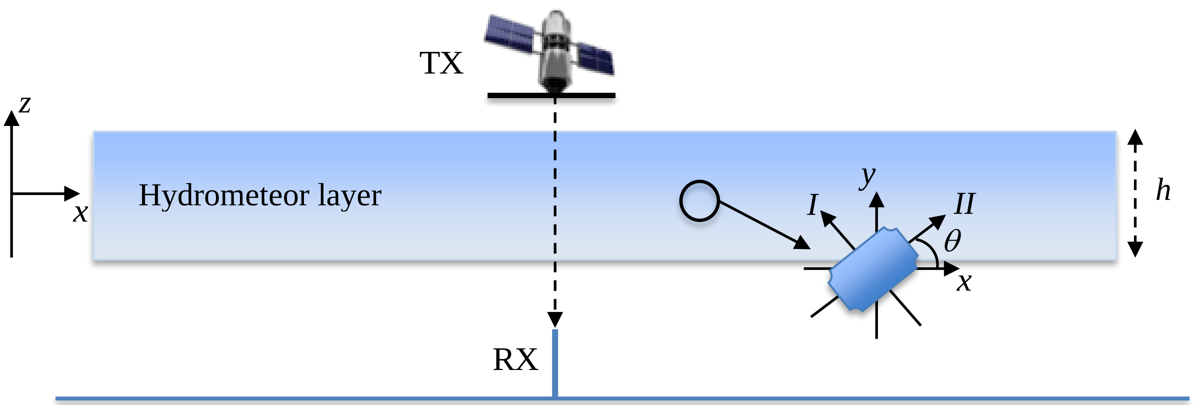
A plane EM wave ($|\vec{E}_{TX}| = 10$ V/m) at 30 GHz propagates across a layer of melting hydrometeors (layer thickness h = 1 km, θ = 45° tilt angle), which is characterized by the following propagation constants (see sketch below):

$\gamma_I = 0.6413 + j628676$ 1/km

$\gamma_{II} = 0.6413 + j628674.43$ 1/km

Both the transmitter (TX) and the receiver (RX) employ linear antennas; the TX antenna is horizontal linear; as for the RX antenna, see point 2). For this link:

1) Determine the wave polarization in front of the receiver RX (no need to specify the tilt angle for a linear polarization, nor the rotation direction for a circular/elliptical polarization).
2) Based on point 1, determine the best direction of the RX (linear) antenna to maximize the received power.



Solution

1) As is clear from the propagation constants, the hydrometeor slab induces the same attenuation on both I and II components of the wave; on the other hand, the differential phase shift is:

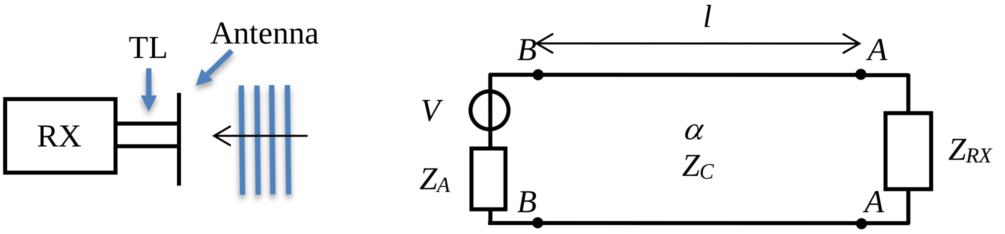
$$\Delta\beta = \beta_{II} - \beta_I = -1.57 = -\pi/2 \text{ rad/km}$$

The TX antenna emits a vertical polarization, so the amplitude of components I and II are equal and given by $|\vec{E}_{TX}|\cos(45°) = 7.07$ V/m. Therefore, in front of RX, the two components I and II will have: 1) same amplitude (attenuated by the slab); 2) a differential phase shift of $\pi/2$ rad/km → the polarization will be circular.

2) Given the circular polarization determined at point 1, a circular polarized antenna is the best choice.

Problem 3

A plane wave (linear vertical polarization), propagating in free space and whose absolute value of the electric field is $|\vec{E}| = 1\ \mathrm{V/m}$, is received by a linear vertical antenna (same direction as the wave polarization, gain G = 6 dB). The power available at the generator section, assumed to be equal to the power received by the antenna, is conveyed into the receiver RX via a lossy coaxial cable (attenuation constant α = 30 dB/km), with intrinsic impedance Z_C = 50 Ω. The antenna acts as an equivalent generator with internal impedance Z_A = 100 Ω. The RX, which acts as a load, is matched to the transmission line. The frequency is f = 600 MHz. The line length is l = 5.2 m. Determine the power absorbed by RX, P_{RX}.



Solution

The wavelength is $\lambda = c/f$ = 0.5 m. The available power is calculated from the power density of the incident wave and the antenna effective area. The former is:

$$S = \frac{1}{2}\frac{|E|^2}{\eta_0} = 1.3\ \mathrm{mW/m^2}$$

The latter is:

$$A_{RX} = \frac{\lambda^2}{4\pi}G = 0.0792\ \mathrm{m^2}$$

Therefore, the available power is:

$$P_{AV} = SA_{RX} = 1.05 \times 10^{-4}\ \mathrm{W}$$

Having a look at the impedances, there is match at the load section, but not at the generator one. In this case, there will be only one reflection in the circuit, specifically at section BB. Therefore, the power absorbed by the load is:

$$P_L = P_{AV}\left(1 - |\Gamma_g|\right)e^{-2\alpha l}$$

The reflection coefficient at section AA is given by:

$$\Gamma_L = \frac{Z_{RX} - Z_C}{Z_{RX} + Z_C} = 0$$

Therefore, at section BB:

$$\Gamma_{BB} = \Gamma_L e^{-2\alpha l} e^{-2j\beta l} = 0 \rightarrow Z_{BB} = Z_C$$

Finally:

$$\Gamma_g = \frac{Z_{BB} - Z_A}{Z_{BB} + Z_A} = \frac{Z_C - Z_A}{Z_C + Z_A} = -0.333$$

The attenuation coefficient is converted in Np/m as:

$$\alpha_l = \frac{\alpha}{8.686 \times 1000} = 0.0035\ \mathrm{Np/m}$$

Therefore:

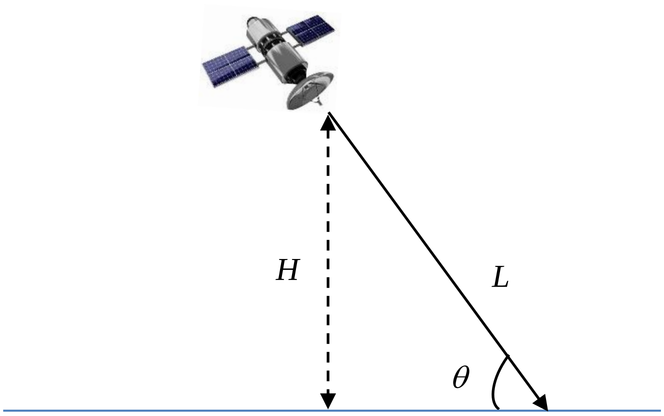
$$P_L = P_{AV}\left(1 - |\Gamma_g|\right)e^{-2\alpha l} = 9 \times 10^{-5}\ \mathrm{W}$$

Problem 4

Consider a satellite link, implementing adaptive coding and modulation, which adapts the data rate depending on the link signal to noise ratio (SNR). The link elevation angle is θ = 45° and the link operates at f = 20 GHz. The Complementary Cumulative Distribution Function (CCDF) of the zenithal tropospheric attenuation is given by:

$$P(A_{dB}^Z) = 100e^{-0.545\,A_{dB}^Z} \quad (A_{dB}^Z \text{ in dB and } P \text{ in } \%)$$

Determine the yearly time for which 10 Mbit/s can be guaranteed to the user.



15 dB < SNR ≤ 20 dB	D = 20 Mbit/s
10 dB < SNR ≤ 15 dB	D = 10 Mbit/s
SNR ≤ 10 dB	D = 1 Mbit/s

Additional assumptions and data:

- both antennas pointed optimally
- disregard the cosmic background radiation
- power transmitted by each satellite P_T = 100 W
- mean radiating temperature T_{mr} = 290 K
- gain of both antennas G = 35 dB
- satellite altitude H = 600 km
- bandwidth of the receiver: B = 100 MHz
- internal noise temperature of the receiver: T_R = 300 K

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A}{k[T_R + T_{mr}(1-A)]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = f_R = 1$, $L = H/\sin(\theta)$, $G_R = G_T = G$ (same antenna features). The target data rate is obtained for 10 dB < SNR ≤ 15 dB: to calculate the yearly time for which 10 Mbit/s can be guaranteed to the user, the threshold SNR_{min} = 10 dB is used. Therefore, the above equation can be solved for A (slant path attenuation in linear scale):

$$A = \frac{SNR_{min} kB(T_R + T_{mr})}{P_T G^2 (\lambda/4\pi L)^2 + SNR_{min} kBT_{mr}} = 0.0041$$

The slant path attenuation in dB is:

$$A_{dB} = -10\log_{10}(A) = 23.9 \text{ dB}$$

The zenithal path attenuation in dB is:

$A_{dB}^{Z} = A_{dB}\sin(\theta) = 16.9$ dB

Using such a value in the CCDF of the tropospheric attenuation:

P = 0.01%, which corresponds to approximately 0.89 hours in a year. Therefore, 10 Mbit/s can be guaranteed for 99.99% of the yearly time, i.e. always but 0.89 hours (53 minutes) in a year.