Telecommunication Systems – Prof. L. Luini,
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Problem 1

Making reference to the figure below, a ground station transmits an EM signal to a satellite with elevation angle $\theta = 60°$. The figure also reports the vertical profile of the relative permittivity ε_r in the ionosphere ($h_1 = 100$ km, $h_2 = 150$ km and $h_3 = 200$ km), associated to the link frequency $f = 12$ MHz. For this scenario:

1) Draw the vertical profile of the total electron content N.
2) Determine if the wave reaches the satellite.
3) Determine the change in the elevation angle necessary to modify the condition at point 2 (from crossing to reflection; from reflection to crossing).
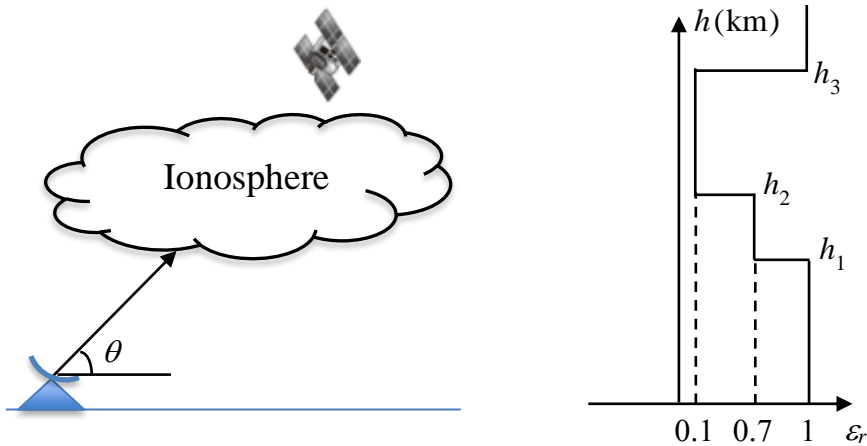
Solution

1) Inverting the usual relationship and using the values in the profile:

$$\sqrt{\varepsilon_r} = \sqrt{1 - \left(\frac{f_P}{f}\right)^2} = \sqrt{1 - \frac{81N}{f^2}}$$

we obtain → $N_1 = 5.3 \times 10^{11}$ e/m³ (value between h_1 and h_2) and $N_2 = 1.6 \times 10^{12}$ e/m³ (value between h_2 and h_3).

2) The condition to obtain total reflection is

$$\cos(\theta) = \sqrt{\varepsilon_r}$$

For the given elevation angle, $\cos(\theta) = 0.5$. As this value lies between $\sqrt{\varepsilon_{r1}} = 0.3162$ and $\sqrt{\varepsilon_{r2}} = 0.8367$, the wave is totally reflected at h_2.

3) For the wave to cross the ionosphere, it is necessary to increase the elevation angle:
$$\cos(\theta') = \sqrt{0.1} = 0.3162 \rightarrow \theta' = 71.57°$$
For any elevation angle higher that θ', the wave will cross the ionosphere.

Problem 2

A signal, whose baseband spectrum is indicated in the first figure below, is transmitted with circular polarization from the ground to a satellite, with carrier frequency $f_C = 1$ GHz.

1) For that carrier frequency, which atmospheric effect(s) need to be taken into account?

Before transmission, the signal is digitalized.

2) Indicate a suitable sampling frequency to be used.

The receiving system is a heterodyne receiver characterized by a system equivalent noise temperature $T_{sys} = 1000$ K. The power received by the antenna is $P_R = 1$ fW.

3) Considering the second figure below, which modulation(s) can be used to guarantee an error free communication (BER $< 10^{-6}$)?

Assume that the signal bandwidth is the same also at radio frequency, regardless of the modulation type.
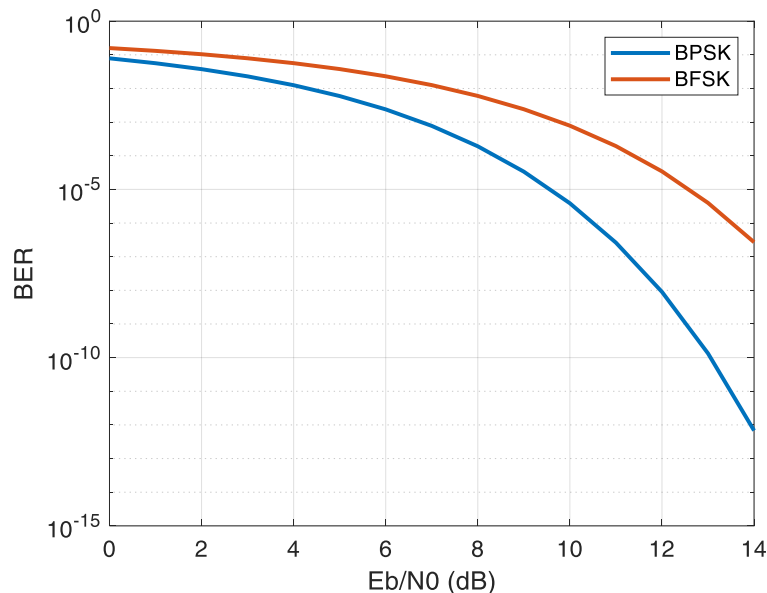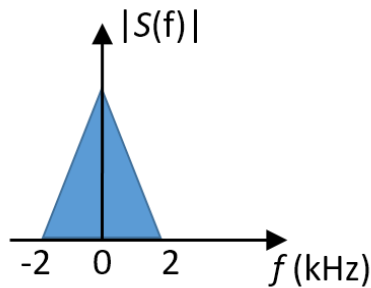
Solution

1) At that carrier frequency, the ionosphere becomes almost transparent, and the troposphere has quite limited effects as well. The only effect to be taken into account is delay, induced both by the electron content and by the tropospheric gasses. Given the circular polarization, also depolarization can be neglected.

2) The maximum frequency component of the baseband signal if $f_M = 2$ kHz: the minimum sampling frequency will be $f_S = 2f_M = 4$ kb/s.

3) Given the specifications, the SNR will be:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B} = 12.58 \text{ dB}$$

Using the curves in the graph, only the BPSK can guarantee an error free communication.

Problem 3

Making reference to the figure below, a satellite, equipped with a linear antenna oriented along x, transmits a signal to a ground station, whose receiving antenna is linear and oriented along z. The distance between the satellite and the ground station is $d = 600$ km and the link operates at $f = 20$ GHz. The signal crosses the layer consisting of anisotropic particles ($h = 400$ m, $h_S = 598$ km), all oriented as indicated in the figure below. The propagation constants associated to the two principal axes of the particles are:

$\gamma_I = 3.4 \times 10^{-4} + j1623$ 1/m
$\gamma_{II} = 2.4 \times 10^{-4} + j1082$ 1/m

The absolute value of the electric field emitted by the satellite antenna is $E_0 = 100$ V/m.

For this link, calculate the power received at RX.

Additional data: disregard additional attenuation due to the atmosphere; the effective area of the receiver antenna $A_E = 0.004$ m². Assume plane wave propagation. Assume free space propagation outside the melting layer.
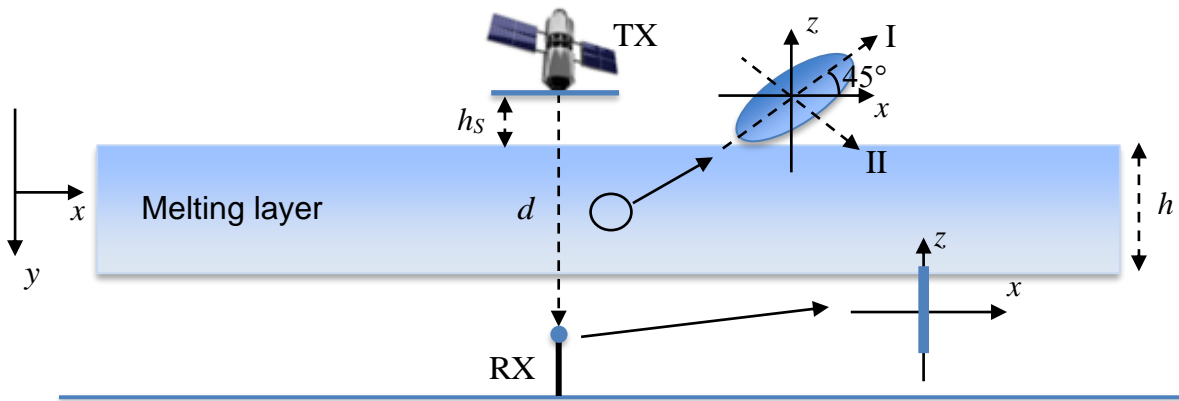
Solution

The power received by the RX antenna will depend on the z electric field component at RX. The propagation along the two principal axes of the particle will be different, according to the associated propagation constants. First, the x component of the electric field is projected along directions I and II:

$E_I(y = 0) = E_0(0) \cos(45°) = E_0(0)/\sqrt{2}$
$E_{II}(y = 0) = E_0(0) \sin(45°) = E_0(0)/\sqrt{2}$

Assuming plane wave propagation, these two components will propagate in the same way from the satellite to the upper limit of the melting layer and from its lower limit to RX, while they will propagate differently in the melting layer. Thus, the two components reaching RX will be:

$E_I(y = d) = E_0(0)/\sqrt{2} \; e^{-j\beta_0 h_S} e^{-\gamma_I h} e^{-j\beta_0 (d - h_S - h)}$
$E_{II}(y = d) = E_0(0)/\sqrt{2} \; e^{-j\beta_0 h_S} e^{-\gamma_{II} h} e^{-j\beta_0 (d - h_S - h)}$

Finally, the field along z is obtained as:

$E_z(y = d) = E_I(y = d)\sin(45°) - E_{II}(y = d)\cos(45°) = -64 + j9.5$ V/m → $|E_z(y = d)| = 26.4$ V/m

Note that, for the calculation of the absolute value of E_z, in turn needed to calculate the power density reaching the RX antenna, the propagation terms in "free space" involving β_0 can be neglected (no additional depolarization is introduced).

The power density reaching RX is therefore:

$$S_{RX} = \frac{1}{2} \frac{|E_z(y = d)|^2}{\eta_0} \approx 0.9249 \text{ W}$$

Finally, the received power is:

$$P_{RX} = S_{RX} A_E = 3.7 \text{ mW}$$

Problem 4

A submarine transmits electromagnetic signals towards the sea surface to keep track of its depth. Assuming that the submarine antenna emits plane waves with electric field $|\vec{E}_{out}| = 5 \text{ V/m}$ at frequency $f = 1$ kHz, and that its depth is $d = 40$ m, calculate:

1) The wavelength underwater.
2) The propagation velocity underwater.
3) The power density reaching back the submarine after the reflection on the sea surface.

AIR (EM parameters as in free space)

z

$\varepsilon_{r1} = 81$
$\mu_{r1} = 1$
$\sigma_1 = 10$ S/m

$d = 40$ m
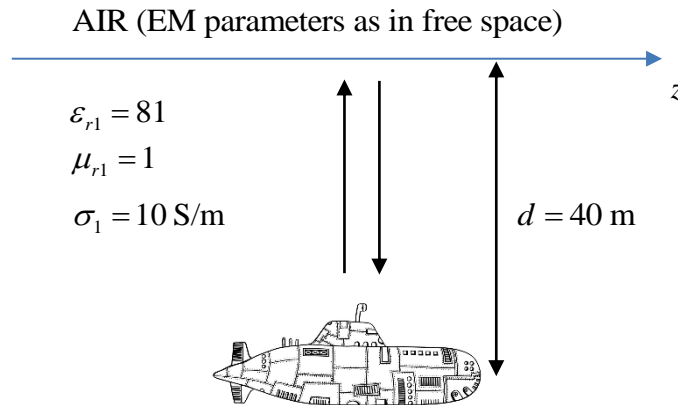
Solution

1) First we need to characterize the electromagnetically the first medium (sea water). In this case, the loss tangent is $\dfrac{\sigma}{\omega\varepsilon} = 2.2 \times 10^6 \gg 1$. Therefore the second medium can be well approximated as a good conductor. Accordingly, the attenuation and propagation constants are:

$$\alpha_1 = \beta_1 = \sqrt{\pi f \mu_1 \sigma_1} = 0.1987 \ 1/\text{m}$$

As for the intrinsic impedance, we obtain:

$$\eta_1 = \sqrt{\frac{\pi f \mu_1}{\sigma_1}}(1 + j) = 0.0199(1 + j) \ \Omega$$

The wavelength is:

$$\lambda_1 = \frac{2\pi}{\beta_1} = 31.62 \ \text{m}$$

2) The propagation velocity is:

$$v_1 = \frac{\omega}{\beta_1} = 3.162 \times 10^4 \ \text{m/s}$$

3) For the second medium (air/free space), $\eta_2 = 377 \ \Omega$. The reflection coefficient is therefore:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx 1$$

The power density emitted by the submarine antenna is:

$$S_{out} = \frac{1}{2}\frac{|\vec{E}_{out}|^2}{|\eta_1|}\cos(\angle \eta_1) = 314.6 \ \text{W/m}^2$$

The power density reaching the submarine after reflection is: 4.9 pW/m^2