Telecommunication Systems – Prof. L. Luini,
June 24th, 2021

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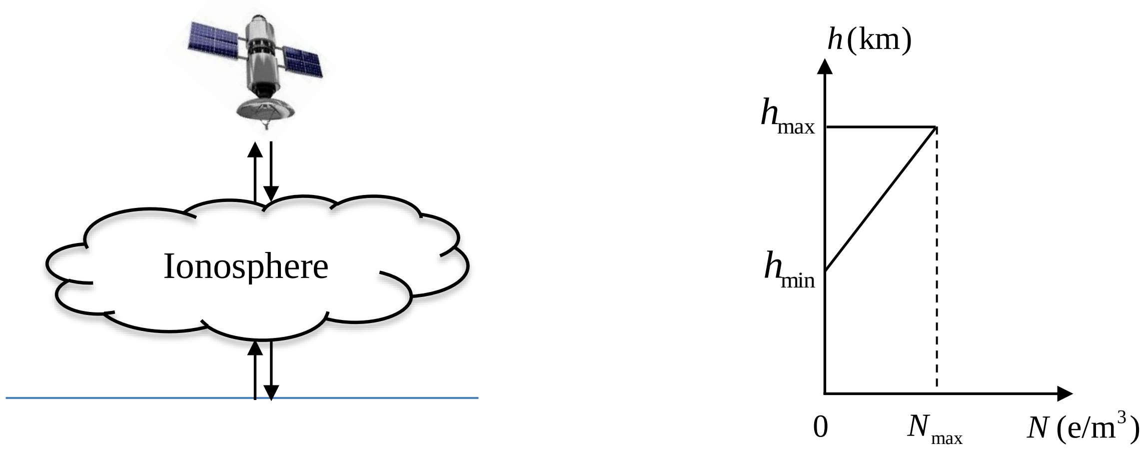
Problem 1

Making reference to the figure below, a satellite, whose altitude is $d = 500$ km, is used as a radar altimeter. To this aim, the satellite transmits an electromagnetic pulse (zenithal pointing) and the altitude of the ground is measured by knowing the time required for the pulse to reach back the satellite after the reflection on the Earth's surface. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where $h_{max} = 470$ km, $h_{min} = 70$ km and $N_{max} = 9\times10^{12}$ e/m^3. For this system:

1) Determine the altimeter accuracy when the radar operates at $f = 20$ MHz.
2) Determine the minimum transmit power P_T necessary to guarantee that the radar operates correctly, when $f = 1$ GHz, knowing that the radar receiver sensitivity is $P_R = 5\times10^{-15}$ W.

Assumptions: the radar antenna is a reflector with diameter $D = 10$ m, the antenna efficiency is $\eta = 0.9$, the backscatter section of the ground is $\sigma = 900$ m^2, disregard the ionospheric attenuation.



Solution

1) For the radar to work properly, the pulse needs to cross the ionosphere, i.e., for zenithal pointing, the operational frequency needs to be higher than the critical frequency f_C, whose value is:

$$f_C \approx 9\sqrt{N_{\max}} = 27 \text{ MHz}$$

As the operational frequency f = 20 MHz < f_C, the pulse will be totally reflected by the ionosphere, therefore the radar will be totally inaccurate.

2) Using f = 1 GHz > f_C, the pulse will cross the ionosphere. To calculate the transmit power necessary for the radar to properly operate, let us first calculate the power density reaching the ground.

$$S = \frac{P_T}{4\pi d^2} f_T G_T$$

where:

$$G_T = \frac{4\pi}{\lambda^2}\eta\left(\frac{D}{2}\right)^2 \pi \approx 9870 \text{ and } f_T = 1$$

The power reaching back the radar is:

$$P_R = \frac{S\sigma}{4\pi d^2} f_R A_E$$

where:

$$A_E = \eta\left(\frac{D}{2}\right)^2 \pi \approx 70.7 \text{ m}^2 \text{ and } f_R = 1$$

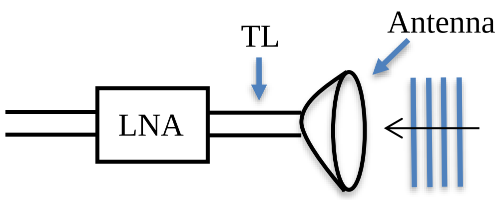
Thus:

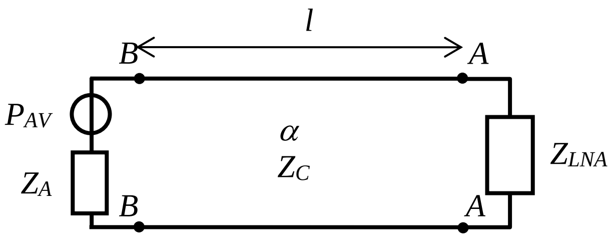
$$\frac{P_R}{P_T} = \frac{G_T \sigma A_E}{\left(4\pi d^2\right)^2} = 6.36\times10^{-17} \rightarrow P_T \geq 78.6 \text{ W}$$

Problem 2

The power received by an antenna is conveyed to a Low Noise Amplifier (LNA) via a lossy transmission line, with intrinsic impedance Z_C and attenuation constant α_{dB} = 30 dB/km. The antenna acts as an equivalent generator with available power P_{AV} = 1 nW and internal impedance Z_A = 50 Ω; the impedance of the LNA is Z_{LNA} = 50 Ω. The line length is l = 12 m and the frequency is f = 600 MHz. For this receiver:

1) Determine the value of Z_C to maximize the power absorbed by the LNA.
2) Using the Z_C value determined at point 1), calculate the power absorbed by the line.
3) Using the Z_C value determined at point 1), calculate the total equivalent noise power of the system. To this aim, assume: equivalent noise temperature of the antenna T_A = 200 K, physical temperature of the transmission line T = 290 K, equivalent noise temperature of the LNA T_{LNA} = 350 K, system bandwidth B = 100 kHz.





Solution

1) Total match can be achieved if Z_C = 50 Ω. In this case, the power absorbed by the LNA is:

$$P_L = P_{AV} e^{-2\alpha l} = 9.2\times 10^{-10}\ \text{W}$$

where α = 0.0035 = α_{dB}/(8.686·1000) Np/m.

2) The power crossing section BB is P_{AV}, so the power absorbed by the line is:

$$P_{line} = P_{AV} - P_L = 7.95\times 10^{-11}\ \text{W}$$

3) The total equivalent noise power of the system is:

$$T_{sys} = T_A + T_{TL} + T_{LNA}$$

The noise contribution introduced by the transmission line is:

$$T_{TL} = T(1-A) = 23.1\ \text{K}$$

where:

$A = P_L/P_{AV}$ = 0.9205 = 0.36 dB = $a_{dB}/1000\times l$

Therefore T_{sys} = 573.1 K.

Finally the system noise power is:

$$P = kT_{sys}B = 7.9\times 10^{-16}\ \text{W}$$

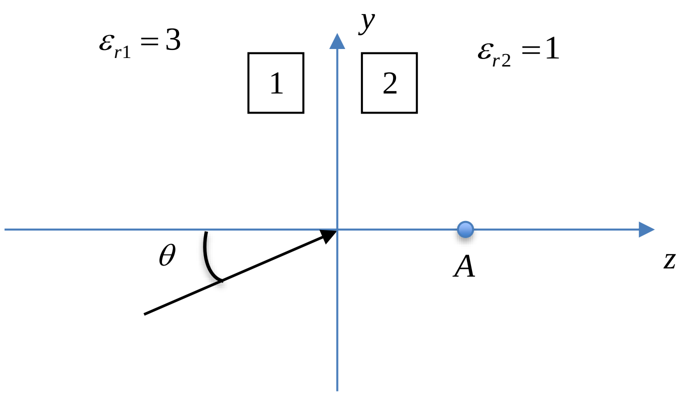
Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 3 into free space (assume μ_r = 1 for both media). There are two possible incident electric fields, whose expressions are:

$$\vec{E}_1(z,y) = -jE_0\vec{\mu}_x e^{-j\beta\cos\theta z}e^{-j\beta\sin\theta y} \text{ V/m}$$

$$\vec{E}_2(z,y) = E_0\left(\sin\theta\,\vec{\mu}_z - \cos\theta\,\vec{\mu}_y\right)e^{-j\beta\cos\theta z}e^{-j\beta\sin\theta y} \text{ V/m}$$

1) Determine which electric field and which θ value should be used to maximize the power received by the antenna located in A(x = 1 m, y = 0 m, z = 1 m)
2) Under the conditions at point 1), calculate the value of E_0 to guarantee that the antenna in A, whose gain is 10 dB, receives 8 μW.
3) What is the polarization of the incident wave resulting from the concurrent transmission of both $\vec{E}_1$ and $\vec{E}_2$, i.e. $\vec{E}_i = \vec{E}_1 + \vec{E}_2$?



Solution

1) $\vec{E}_1$ is the electric field of a TE wave, while $\vec{E}_2$ represents a TM wave. Both waves carry the same power (the absolute value of the electric field is E_0 V/m in both cases), so the choice depends just on the reflection coefficients. The latter can be zero only for the TM case, specifically when θ corresponds to the Brewster angle:

$$\theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}}\right) = 30°$$

Therefore if $\theta = \theta_B$, and we pick $\vec{E}_2$, the wave is totally transmitted, and the power received at A is maximized.

2) Checking Snell's law:

$$\sin\theta_i\sqrt{\varepsilon_{r1}} = \sin\theta_t\sqrt{\varepsilon_{r2}} \quad \Rightarrow \quad \theta_t = \sin^{-1}\left(\sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 60°$$

As the reflection coefficient $\Gamma = 0 \rightarrow$ the transmission coefficient $T = 1$. Therefore the power density reaching A will be:

$$S_t^z = S_i^z \rightarrow S_t \cos\theta_t = S_i \cos\theta \rightarrow S_t = \frac{1}{2}\frac{\left|\vec{E}_2\right|^2}{\eta_0 / \sqrt{\varepsilon_{r1}}}\frac{\cos\theta}{\cos\theta_t} = \frac{1}{2}\frac{\left|E_0\right|^2}{\eta_0 / \sqrt{\varepsilon_{r1}}}\frac{\cos\theta}{\cos\theta_t}$$

The power received in A is:

$$P_R = S_t A_E = \frac{1}{2}\frac{\left|E_0\right|^2}{\eta_0 / \sqrt{\varepsilon_{r1}}}\frac{\cos\theta}{\cos\theta_t}\frac{\lambda^2}{4\pi}G$$

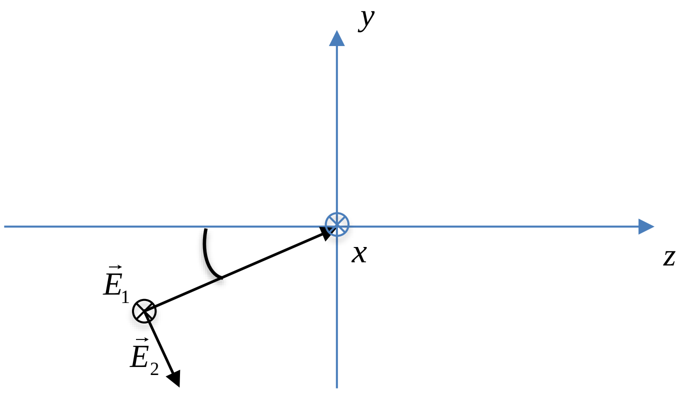
Setting P_R = 8 μW, and inverting the equation → E_0 = 1.5 V/m.

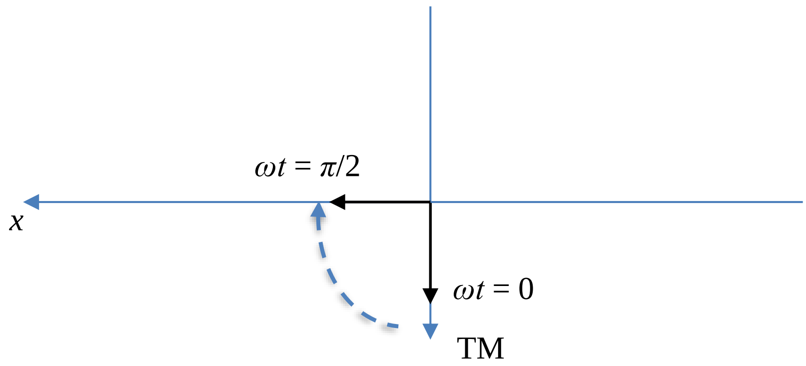
3) Considering the full expressions of the two electric fields in time, it turns out that the wave has a right end circular polarization (RHCP). In fact, both components (TE and TM) have the same amplitude and have a phase shift of ±π/2. Setting y and z to 0, and expressing the dependence on time, we can determine the electric field rotation direction:

$$\vec{E}(0,0,t) = \mathrm{Re}\left\{\left[E_0\left(\sin\theta\,\vec{\mu}_z - \cos\theta\,\vec{\mu}_y\right) - jE_0\vec{\mu}_x\right]e^{j\omega t}\right\} = E_0\cos(\omega t)\,\vec{\mu}_{TM} + E_0\cos\left(\omega t - \frac{\pi}{2}\right)\vec{\mu}_x \text{ V/m}$$

Thus, making reference to the figure below that shows the reference system as seen from behind the wave, for $\omega t = 0$ → $\vec{E}(0,0)\big|_{\omega t=0} = E_0\vec{\mu}_{TM}$ V/m

Afterwards, for $\omega t = \pi/2$ → $\vec{E}(0,0)\big|_{\omega t=\pi/2} = E_0\vec{\mu}_x$ V/m





Problem 4

Consider a link from a LEO satellite to a ground station with elevation angle $\theta = 90°$, operating at f = 30 GHz. The satellite is used to measure the rain rate from space. The signal-to-noise ratio (SNR) measured at the ground station before the rain event is SNR_1 = 23 dB; during the rain event, the SNR drops to SNR_2 = 9.15 dB. Assuming that the rain rate is uniform horizontally and vertically, that the rain height is h = 4 km and that the specific attenuation due to rain is given by $\alpha = 0.23R^{0.93}$ dB/km.

1) Calculate P_T, the power transmitted by the LEO satellite assuming that, in non-rainy conditions, no additional attenuation is induced by the atmosphere.
2) Assuming the value of P_T calculated at point 1), derive the value of the rain rate affecting the link during the precipitation event.
3) Keeping the same system geometry described above, and assuming that rain drops are all aligned horizontally (with minor axis parallel to the zenithal direction), what is the best linear polarization to be used for this radar system?

Additional assumptions and data:

- antennas optimally pointed
- disregard the cosmic background temperature contribution
- mean radiating temperature (rainy case) T_{mr} = 290 K
- gain of the antennas (on board the satellite and on the ground): $G_T = G_R$ = 20 dB
- altitude of the LEO satellite: H = 800 km
- bandwidth of the receiver: B = 1 MHz
- internal noise temperature of the receiver: T_R = 359 K

Solution

1) In case of no rain, the signal-to-noise ratio (SNR) is given by:

$$SNR_1 = \frac{P_R}{P_N} = \frac{P_T G \left(\lambda/4\pi H\right)^2 G}{kT_R B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). Note that there is no attenuation at all and therefore also the antenna noise temperature is 0 K (according to the assumptions).

By inverting the equation above → P_T = 100 W.

2) In case of rain, the SNR becomes

$$SNR_2 = \frac{P_T G \left(\lambda/4\pi H\right)^2 GA}{k\left[T_R + T_{mr}(1-A)\right]B}$$

where A is the rain attenuation (in linear scale), which decreases the received power and increases the receiver noise. Solving for A:

$$A = \frac{SNR_2 kB\left(T_R + T_{mr}\right)}{SNR_2 kT_{mr}B + P_T G^2 \left(\lambda/4\pi H\right)^2} = 0.0722$$

In dB:

A_{dB} = 11.417 dB

As: $A_{dB} = 0.23 R^{0.93} h$ → R = 15 mm/h.

3) Given the assumptions, the rain drop section seen by the transmitter and the receiver will be circular, so any linear polarization will be subject to the same rain attenuation (no differences among polarizations).