Telecommunication Systems – Prof. L. Luini,
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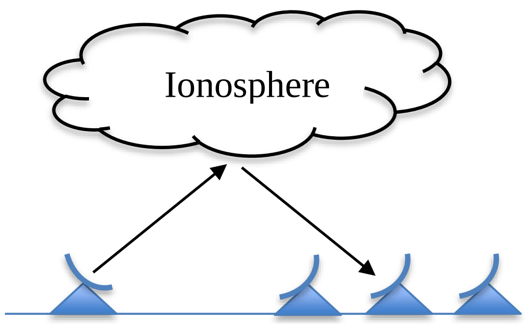
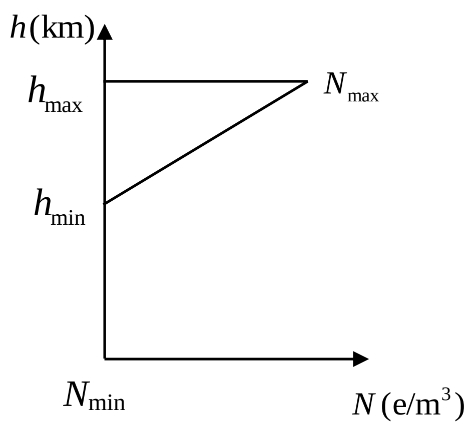
Problem 1

Making reference to the figure below, a bistatic ground-based radar system aims at measuring the extent of the ionosphere, as well as the minimum and maximum value of the electron content. The transmitter elevation angle is $\theta = 40°$. The electron content profile is shown in the figure below (right side), where h_{max} = 400 km, h_{min} = 100 km. The radar operates by gradually switching the carrier frequency from 5 MHz to 50 MHz, and the receiver consists of an array of terminals at increasing distance from the transmitter.

1) Determine the value of N_{min} knowing that the radar starts to receive the reflected signal at f_1 = 10 MHz and determine the distance d_1 between the transmitter and the terminal receiving the signal.
2) Determine the value of N_{max} knowing that the radar no longer receives the reflected signal for frequencies higher than f_2 = 30 MHz and determine the distance d_2 between the transmitter and the terminal receiving the signal.
3) Calculate the propagation time of the EM pulse from the transmitter to the receiver for the conditions at point 1)
4) Calculate the propagation time of the EM pulse from the transmitter to the receiver for the conditions at point 2)

Assume: virtual reflection height $h_V = 1.1h_R$ (where h_R is the real reflection height), no tropospheric effects.





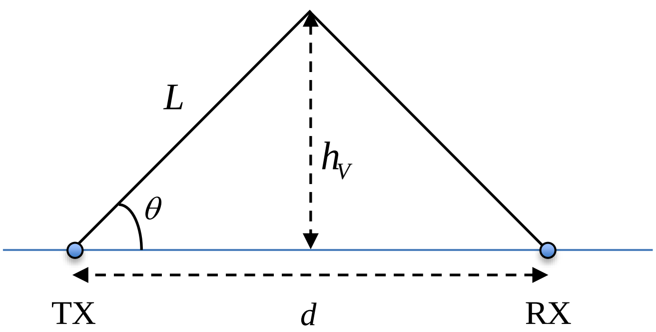
Solution

1) The radar starts to receive some signal when the first reflection at the base of the ionosphere occurs, due to $N_{\min}$. It is obtained as:

$$N_{\min} = \frac{f_1^2(1 - [\cos(\theta)]^2)}{81} \approx 5.1 \times 10^{11}\ \mathrm{e/m^3}$$

Exploiting the concept of virtual reflection height, and making reference to the figure below, d_1 is given by:

$$d_1 = \frac{2h_{V1}}{\mathrm{tg}(\theta)} = \frac{2.2\ h_{\min}}{\mathrm{tg}(\theta)} = 262.2\ \mathrm{km}$$



2) The radar no longer receives the reflected signal when the wave crosses the ionosphere, which occurs at 30 MHz. Using the same equations above, but with f_2:

$$N_{\max} = \frac{f_2^2(1 - [\cos(\theta)]^2)}{81} \approx 4.6 \times 10^{12}\ \mathrm{e/m^3}$$

and

$$d_2 = \frac{2h_{V2}}{\mathrm{tg}(\theta)} = \frac{2.2\ h_{\max}}{\mathrm{tg}(\theta)} = 1048.7\ \mathrm{km}$$

3) Assuming to neglect the effect of the troposphere, the propagation time in this case is simply:

$$t_1 = \frac{2L_1}{c} = \frac{2(h_{V1}/\sin(\theta))}{c} = 0.0011\ \mathrm{s}$$

4) As the signal crosses partially the ionosphere, it will be delayed by it. In this case, the propagation time is:

$$t_2 = \frac{2L_2}{c} + \frac{40.3}{cf_2^2} 2 \frac{TEC}{\sin(\theta)}$$

The zenithal TEC is calculated as:

$$TEC = \frac{(h_{\max} - h_{\min})(N_{\max} - N_{\min})}{2} = 6.12 \times 10^{17}\ \mathrm{e/m^2}$$

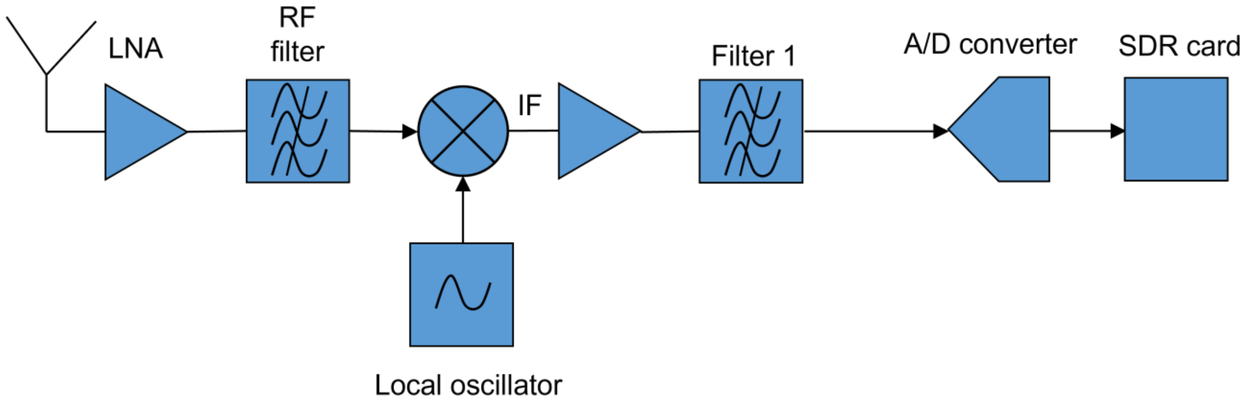
Therefore:

$$t_2 = 0.0048\ \mathrm{s}$$

Problem 2

Consider the heterodyne receiver depicted below, which aims at receiving an RF signal, whose carrier frequency is f_{RF} = 20 GHz, to be converted from analog to digital at the end of the receiver chain. The signal bandwidth is B = 20 MHz. The A/D converter sampling frequency if f_S = 80 MHz. The LNA amplifier gain is G = 30 dB and its noise figure is NF = 2 dB. The voltage at the input of the LNA is V_1 = 2 μV. Equivalent noise temperature at the input of the LNA is T_1 = 350 K.

1) Determine the minimum and maximum local oscillator frequency f_{LO} for the signal to be properly sampled by the A/D converter.
2) Calculate the ideal filter bandwidth of filter 1 (specify minimum and maximum frequencies of the pass band).
3) Determine the minimum gain of the amplifier after the mixer to obtain a minimum voltage at the input of the A/D converter equal to V_2 = 20 nV (assume a loss of 1 dB for each filter, and no loss nor gain for the active mixer).
4) Calculate the noise power in input to the A/D converter.



Solution

1) The A/D converter has a fixed sampling frequency of 80 MHz. The sampling theorem states that:

$f_S > 2f_{\max}$

where $f_{\max}$ is the maximum frequency of the whole signal to be sampled. The local oscillator frequency f_{LO} needs to be chosen such that the signal is shifted down at intermediate frequency at the same time having a maximum frequency component equal to $f_{\max}$. Given the sampling frequency, from the equation above → $f_{\max}$ = 40 MHz. Considering that the bandwidth of the signal is 20 MHz, the IF frequency must be lower than or equal to f_{IF} = 30 MHz.

When the RF signal is mixed with the local oscillator, we have:

$f_{\mathrm{IF}} = f_{RF} - f_{\mathrm{LO}}$

Therefore:

$f_{\mathrm{LO}} = f_{RF} - f_{\mathrm{IF}}$ = 19.97 GHz

The above equation is valid for $f_{\mathrm{LO}} < f_{\mathrm{RF}}$. If $f_{\mathrm{RF}} > f_{\mathrm{LO}}$:

$f_{\mathrm{IF}} = f_{LO} - f_{\mathrm{RF}}$

Therefore:

$f_{\mathrm{LO}} = f_{IF} + f_{\mathrm{RF}}$ = 20.03 GHz

To sum up → 19.97 GHz < f_{LO} < 20.03 GHz.

2) Given the calculations above, the ideal filter band spans from 20 MHz to 40 MHz.

3) The total gain required to amplify the signal along the chain is:

$G_T = 20\log_{10}\left(\frac{V_1}{V_2}\right) = 40$ dB

The total gain is given by:

$G_T = G_{LNA} - G_{F-RF} - G_{MIX} + G_A - G_{F-IF}$

The loss of both filters is 1 dB, the LNA gain is 30 dB, while the loss due to the mixer is set to 0 dB (see the data). Therefore:

G_A = 12 dB

4) Given the high gain of the LNA, the contributions to the noise power due to the elements after the LNA can be neglected. Therefore, the noise power in input to the A/D converter is given by:

$P_N = k(T_1 + T_{LNA})B = k\left[T_1 + \left(10^{\frac{NF}{10}} - 1\right)290\right]B = 14.3$ pW

Problem 3

Consider a link to a MEO satellite of the O3b constellation, which transmits a telemetry beacon signal centred about f =19.8 GHz.

1) Which wave polarization is likely used for such a beacon signal?

A ground station is equipped with a steerable vertically polarized antenna, which features a closed-loop tracking system to point at the MEO satellite in an optimal way. Such a system is based on the received power and it requires a minimum SNR = 15 dB for an accurate pointing to the satellite.

2) Determine the minimum elevation angle guaranteeing the proper tracking of the satellite in clear sky conditions. To this aim, assume:

- that the LNA noise temperature is T_R = 200 K;
- to neglect the cosmic background temperature;
- that the mean radiating temperature of is T_{mr} = 20 °C;
- to use a parabolic antenna with Cassegrain configuration with gain G_R = 50 dB;
- that the equivalent antenna noise temperature measured at 20° (elevation peak of the satellite) is T_A = 170 K.
- that the MEO satellite features an isoflux system, i.e. the power density reaching the ground is S = 1.25 pW/m^2, regardless of the link elevation angle.
- that the system bandwidth is B = 1 MHz.
- the atmosphere is horizontally homogeneous.

Solution

1) The best polarization to be used for non-geosynchronous satellites is the circular one (LHCP or RHCP), mainly for geometrical reasons.

2) First, we need to derive the reference tropospheric attenuation at 20°, as follows:

$$T_A = T_{mr}\left(1 - A_{20°}\right) \;\rightarrow\; A_{20°} = 1 - \frac{T_A}{T_{mr}} = 0.4201$$

As:

$$A_{20°} = 10^{-\frac{A_{90°}^{dB}}{10\sin(20)}} \;\rightarrow\; A_{90°}^{dB} = 1.288 \text{ dB}$$

The system noise temperature is (for the Cassegrain configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}\left(1 - A_\theta\right)$$

where:

$$A_\theta = 10^{-\frac{A_\theta^{dB}}{10\sin(\theta)}}$$

The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B}$$

The received power can be conveniently expressed as follows:

$$P_R = \frac{P_T}{k\pi L^2} G_T A_E A_\theta\, 0.5 = S A_E A_\theta$$

The effective area of the receiving antenna is given by:

$$A_E = \frac{\lambda^2}{4\pi} G_R = 1.8268$$

while the 0.5 takes into account the fact that the antenna is linearly polarized, while the signal is circularly polarized.

Inserting P_R into the SNR equation and solving for A_θ:

$$A_\theta = \frac{k\ SNR\ B(T_R + T_{mr})}{S\ A_E + k\ SNR\ B\ T_{mr}} = 0.2063 \ \rightarrow \ A_\theta^{dB} = -10\log_{10}(A_\theta) = 6.8553$$

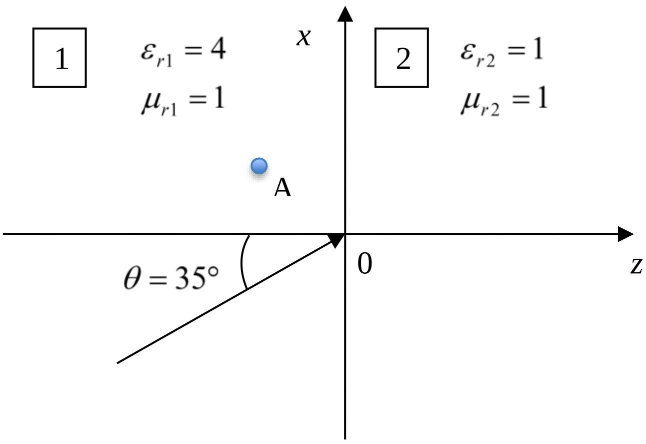
As:

$$A_\theta^{dB} = \frac{A_{90°}^{dB}}{\sin(\theta)} \ \rightarrow \ \theta = 10.83°$$

Problem 4

Consider the plane sinusoidal wave depicted below hitting the interface between two lossless media, whose EM characteristics are reported in the figure. The incidence angle is $\theta = 35°$ as shown below. The wave frequency is f = 1 GHz. The incident electric field in medium 1 at the origin of the system is:

$$\vec{E}_i(0,0,0) = j\vec{\mu}_y \text{ V/m}$$



For this scenario, calculate:

a) The reflection coefficient at the interface.
b) The power density flowing in the *z* direction in the second medium.
c) The absolute value of the total electric field in point A(x = 1, y = 2, z = -1).

Solution:

a) The wave is TE. To calculate the reflection coefficient, first, let us calculate the refraction angle, which is obtained from the Snell's law:

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \Rightarrow \quad \sin\theta_2 = 1.1472 \quad \Rightarrow \quad$ evanescent wave

The wave is evanescent, which implies that the absolute value of the reflection coefficient is 1, i.e. all the power density is reflected. For the detailed calculation of the reflection coefficient, we start from the intrinsic impedances:

$$\eta_{TE}^1 = \frac{\eta_0}{\sqrt{4}} \frac{1}{\cos\theta_1} = 230.1\ \Omega$$

$$\eta_{TE}^2 = \frac{\eta_0}{\cos\theta_2} = \frac{\eta_0}{\sqrt{1-(\sin\theta_2)^2}} = \frac{\eta_0}{\pm j0.5622} \Rightarrow \eta_{TE}^2 = j670.6\,\Omega$$

Note that this mathematical solution is picked to achieve a physical solution for the wave equation in the second medium. Finally, the reflection coefficient is:

$$\Gamma = \frac{\eta_{TE}^2 - \eta_{TE}^1}{\eta_{TE}^2 + \eta_{TE}^1} = 0.789 + j0.614$$

b) As the wave is evanescent wave, no power density will travel in the *z* direction in the second medium.

c) The total electric field in the first medium is the combination of the incident wave and of the reflected wave, i.e.:

$$\vec{E}(x,y,z) = \vec{E}_i(x,y,z) + \vec{E}_r(x,y,z) = j\vec{\mu}_y e^{-j\frac{2\pi}{\lambda}(\cos(\theta)z+\sin(\theta)x)} + \Gamma\left(j\vec{\mu}_y\right)e^{-j\frac{2\pi}{\lambda}(-\cos(\theta)z+\sin(\theta)x)} \quad \text{V/m}$$

with $\lambda = c/\left(\sqrt{4}f\right) = 0.15$ m.

The absolute value of the electric field in A can be obtained by setting x to 1 and z to -1 in the equation above and by taking the absolute value. This leads to:

$\vec{E}_t(A) = (1.6626 - j0.2231)\vec{\mu}_y$ V/m → $\left|\vec{E}_t(A)\right| = 1.6775$ V/m