Telecommunication Systems – Prof. L. Luini,
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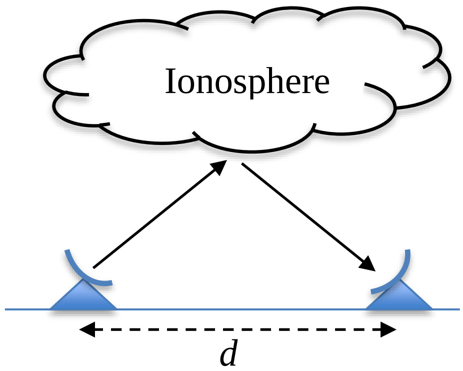
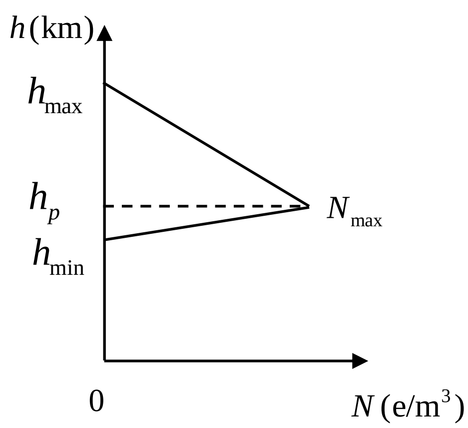
Problem 1

Making reference to the figure below, a bistatic ground-based radar system aims at measuring the position of the ionosphere peak electron content, whose value is $N_{max} = 2\times10^{12}$ e/m^3. The transmitter elevation angle is $\theta = 30°$. The electron content profile is shown in the figure below (right side), where $h_{max} = 400$ km, $h_{min} = 100$ km and $h_p = 200$ km.

1) Calculate the radar operational frequency f_R to measure the position of N_{max}.
2) Calculate the distance d between the transmitter and the receiver.
3) Fixing f_R, what happens if the elevation angle increases? (discuss qualitatively)
4) Fixing f_R and θ, what happens if the peak electron content decreases to $N''_{max} = 0.9\ N_{max}$? (discuss qualitatively)

Assume: antenna gain $G = 60$ dB (both for the transmitter and the receiver), flat Earth, virtual reflection height $h_V = 1.1h_R$ (where h_R is the real reflection height).





Solution

The operational frequency of the radar f_R can be obtained by using the following expression:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f_R}\right)^2}$$

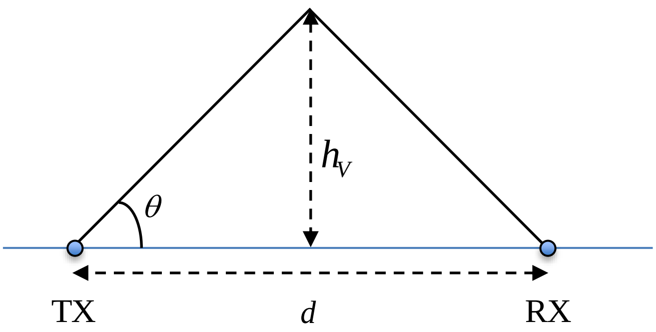
Solving for the frequency f_R, we obtain:

$$f_R = \sqrt{\frac{81N_{\max}}{1-\left[\cos\left(\theta\right)\right]^2}} = 25.46 \text{ MHz}$$

Using this frequency, the wave will be reflected exactly at $N_{\max}$.

2) Exploiting the concept of virtual reflection height d is given by:

$$d = \frac{2h_V}{\tan\theta} = 762.1 \text{ km}$$



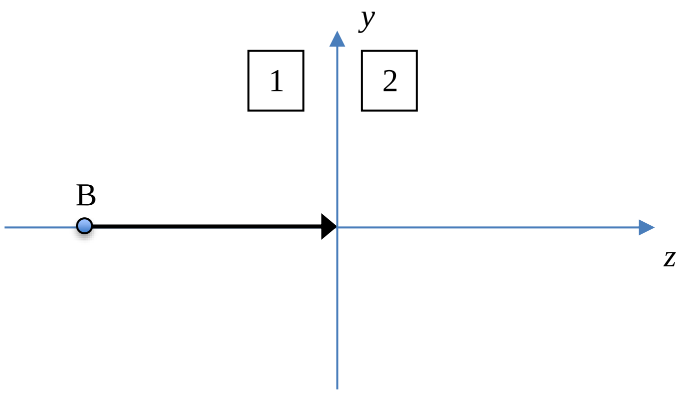
3) If the elevation angle increases, the wave will cross the ionosphere.

4) Also in this case, the wave will cross the ionosphere.

Problem 2

A plane sinusoidal wave (f = 12 GHz) propagates orthogonally from a medium characterized by ε_{r1} = 2, σ_1 = 0.0134 S/m, μ_{r1} = 1 into a medium characterized by ε_{r2} = 4-j3, μ_{r2} = 1. The incident electric field in B(x = 0 m, y = 0 m, z = -1 m) is $\vec{E}_i(B) = 5\vec{\mu}_y$ V/m. In this context:

1) Calculate the wavelength in medium 1.
2) Calculate the full expression of the magnetic field in medium 1.



Solution

1) The plane wave is a TEM wave with vertical polarization. To answer the first question, the propagation constant in the first medium is required. To this aim, it is worth checking the loss tangent:

$$\tan\delta = \frac{\sigma_1}{\omega\varepsilon_{r1}} = 0.01$$

As medium 1 is a good dielectric, the attenuation and phase constants can be simply calculated as:

$$\alpha_1 \approx \frac{\sigma_1}{2}\sqrt{\frac{\mu_1}{\varepsilon_1}} = 1.7848 \text{ Np/m}$$

$$\beta_1 \approx \omega\sqrt{\varepsilon_1\mu_1} = 355.67 \text{ rad/m}$$

Therefore the wavelength in medium 1 is:

$$\lambda_1 = {2\pi}/{\beta_1} = 0.0177 \text{ m}$$

2) E_0 can be calculated as:

$\vec{E}_i(B) = E_0 e^{-\gamma_1 z_B}\vec{\mu}_y = 5\vec{\mu}_y \quad \Rightarrow \quad E_0 = \dfrac{5}{e^{-\gamma_1 z_B}} = \text{-}0.6562 + \text{j}0.523$ V/m, which yields $|E_0| = 0.8391$ V/m. As expected, due to the lossy properties of medium 1, the amplitude of the electric field decreases moving from B to the interface.

To calculate the full electric field in the first medium, we need the reflection coefficient. The intrinsic impedances of the two media (no approximations are possible for the second medium as the loss tangent is Im(ε_{r2})/Re(ε_{r2}) = 0.75) are:

$$\eta_1 \approx \sqrt{\frac{\mu_1}{\varepsilon_1}} = 266.4\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_0\mu_{r2}}{j\omega\varepsilon_0\varepsilon_{r2}}} = 159.8 + j53.3\ \Omega$$

The reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.231 + j0.154$$

Therefore, the full expression of the electric field in medium 1 is:

$$\vec{E}_1(z) = \vec{E}_i(z) + \vec{E}_r(z) = E_0 e^{-\gamma_1 z}\ \vec{\mu}_y + \Gamma E_0 e^{\gamma_1 z}\ \vec{\mu}_y$$

Note the change in the sign at the exponent of the second term (reflected wave).

3) The full expression of the magnetic field in medium 1 is:

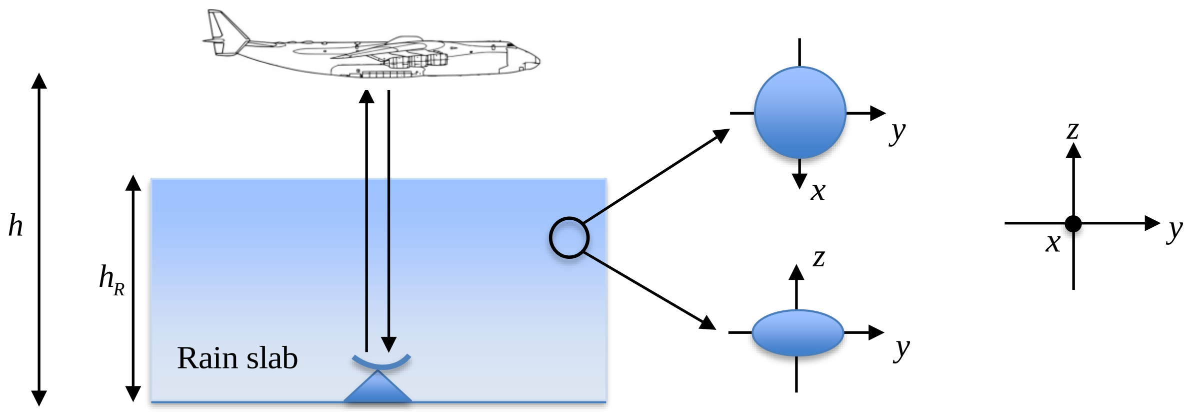
$$\vec{H}_1(z) = \vec{H}_i(z) + \vec{H}_r(z) = -\frac{E_0}{\eta_1} e^{-\gamma_1 z}\ \vec{\mu}_x + \Gamma \frac{E_0}{\eta_1} e^{\gamma_1 z}\ \vec{\mu}_x \text{ V/m}$$

Problem 3

Making reference to the figure below, a ground-based pulsed radar, operating with carrier frequency of 20 GHz and pointed zenithally, is used to identify aircrafts flying at altitude h = 10 km. The radar operates in all weather conditions, including rain, as depicted blow, where a rain slab of constant (both horizontally and vertically) rain rate R = 25 mm/h is considered. The rain drops are oblate spheroids, all equi-oriented with major axis parallel to the ground and the rain height is h_R = 3.5 km. The wave transmitted by the radar is RHCP. In this context:

1) Determine the polarization of the wave in front of the aircraft.
2) Calculate the radar bandwidth to achieve a minimum SNR_{min} = 10 dB at the radar.

Consider the following data: radar transmit power P_T = 1 kW; radar antenna gain G = 50 dB; aircraft backscatter section σ = 10 m^2, coefficients for the specific attenuation due to rain: a = 0.0928 and b = 1.0381; attenuation due to clouds and gases A_{CG} = 2 dB; neglect the cosmic background noise; LNA equivalent noise temperature T_R = 350 K; mean radiating temperature T_{mr} = 300 K; LNA very close to the radar antenna feed.



Solution

1) Any linear polarization transmitted by the radar will be subject to the same effects induced by the rain drops: in fact, for the geometry indicated in the figure, the wave will see a circular section for the drops (on the xy plane). As a result, both linear (phase shifted) components of the RHCP wave will be subject to the same specific attenuation and delay → the wave will not be depolarized.

2) First, let us calculate the power density reaching the aircraft:

$$S_A = \frac{P_T}{4\pi h^2} G f A_{ATM}$$

where G = 10000, f = 1 (radar pointing to the aircraft) A_{ATM} is the atmospheric attenuation in linear scale, which includes the effects of rain, cloud and gases. This is first calculated in dB as:

$$A_{ATM}^{dB} = A_R + A_{CG} = aR^b h_R + A_{CG} = 11.18 \text{ dB} \rightarrow A_{ATM} = 0.0762$$

Therefore:

$$S_A = 0.0061 \text{ W/m}^2$$

The power reirradiated by the aircraft (with gain = 1 according to the definition of backscatter section), is:

$$P_A = S_A \sigma = 0.061 \text{ W}$$

The power density reaching the radar is:

$$S_R = \frac{P_A}{4\pi h^2} A_{ATM} = 3.68 \cdot 10^{-12} \text{ W/m}^2$$

Finally, the power received by the radar is:

$$P_R = S_R A_E = 6.59 \cdot 10^{-12} \text{ W}$$

where A_E is the equivalent area, expressed as:

$$A_E = G \frac{\lambda^2}{4\pi} = 1.7905 \, \text{m}^2$$

The SNR is given by:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{SYS} B}$$

where T_{SYS} is the system equivalent noise temperature given by:

$$T_{SYS} = T_R + T_A = T_R + T_{mr}(1 - A_{ATM}) = 627 \text{ K}$$

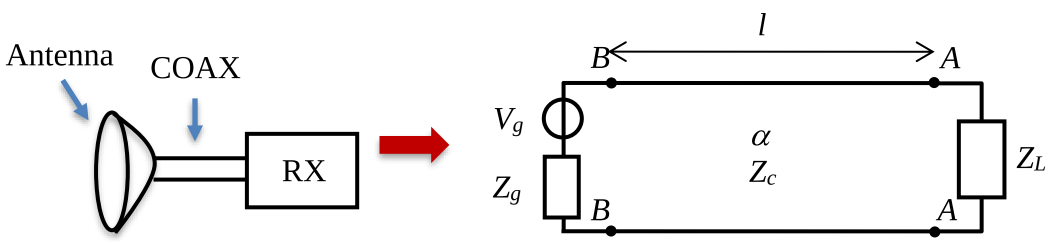
By imposing SNR > SNR_{min}:

$$\frac{P_R}{kT_{SYS} B} > SNR_{\min} \quad \rightarrow \quad B < \frac{P_R}{kT_{SYS} SNR_{\min}} = 76.1 \text{ MHz}$$

Problem 4

Consider the typical receiver sketched in the figure below. The antenna is modeled as a voltage generator V_g = 50 V (with internal impedance Z_g = 50 Ω), which is connected to the receiver RX through a lossy coaxial cable (COAX) with characteristic impedance Z_C = 50 Ω and attenuation constant α = 30 dB/km. The input impedance of the receiver is Z_L = 50 Ω, and the frequency is f = 3 GHz.

1) Determine the maximum length l to guarantee a minimum absolute value of the voltage on Z_L, $|V_L|$ = 24 V.
2) Calculate the equivalent noise temperature of the coaxial cable, whose physical temperature is T = 20 °C.



Solution

1) Given the value of Z_L, Z_C and Z_g, maximum power transfer is achieved. As a result, the power absorbed by the load is simply:

$$P_L = P_{AV} e^{-2\alpha l}$$

where:

$$P_{AV} = \frac{|V_g|^2}{8\,\mathrm{Re}\left[Z_g\right]} = 6.25 \text{ W}$$

Also, $\alpha = 3.5\times10^{-3}$ Np/m

The absorbed power P_L can also be expressed as:

$$P_L = \frac{1}{2}|V_L|^2 \,\mathrm{Re}\left(\frac{1}{Z_L}\right)$$

Therefore:

$$\frac{1}{2}|V_L|^2 \,\mathrm{Re}\left(\frac{1}{Z_L}\right) = P_{AV} e^{-2\alpha l}$$

By setting V_L = 10 V, solving for l:

$$l = \frac{\ln\left[\frac{1}{2}|V_L|^2 \,\mathrm{Re}\left(\frac{1}{Z_L}\right) \Big/ P_{AV}\right]}{-2\alpha} = 11.82 \text{ m}$$

2) The power absorbed by the load is:

$$P_L = P_{AV} e^{-2\alpha l} = 5.76 \text{ W}$$

As a result, the attenuation introduced by the coaxial cable is:

$$A = \frac{P_L}{P_{AV}} = 0.92$$

Therefore, the equivalent noise temperature of the coaxial cable is:

$$T_E = T\left(1 - A\right) = 23 \text{ K}$$