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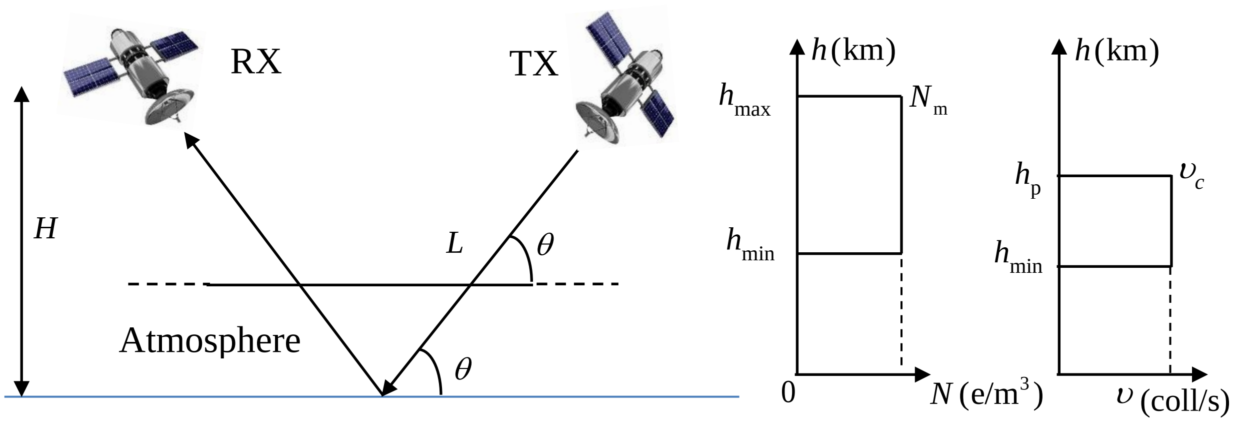
Problem 1

The figure below shows a bistatic radar system consisting of two LEO satellites (same orbit, satellite height above the ground H = 500 km). The radar extracts information on the ground measuring the power received at RX by reflection. Making reference to the right, where the electron content profile ($h_{\min}$ = 50 km, $h_{\max}$ = 400 km, $N_m = 4\times10^{12}$ e/m^3, N homogeneous horizontally) and the collisions frequency profile (h_p = 80 km, $\upsilon_c = 10^4$ coll/s, υ homogeneous horizontally) are shown, and to the left side, where the simplified geometry is reported:

1) Determine the minimum value of θ for the system to avoid reflection from the atmosphere, when the operational frequency is f = 21 MHz.
2) Considering to work with θ calculated at point 1, determine which type of target is on the ground, according to the associated backscatter section, σ: if $\sigma < 100$ m^2 → rural area; if $\sigma \geq 100$ m^2 → urban area.

Additional data: the gain of TX antenna is G = 45 dB, the transmit power is P_T = 100 W, the power density reaching RX is $S_{RX} = 10^{-18}$ W/m^2.



Solution

1) For the wave to avoid total reflection due to the atmosphere (ionosphere at 21 MHz), the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_{\mathrm{m}}}}{f_1}\right)^2} \quad \Rightarrow \quad \theta_{min} = 59°$$

2) To calculate correctly the link budget, first it is necessary to consider the attenuation induced by the ionosphere. The equivalent conductivity of the ionosphere is:

$$\sigma = \frac{N_{\max} e^2 \nu_C}{m(\nu_C^2 + \omega^2)} = 6.5 \cdot 10^{-8} \text{ S/m}$$

where $m = 9 \cdot 10^{-31}$ kg is the mass of the electron and $e = -1.6 \cdot 10^{-19}$ C is its charge.

The plasma angular frequency (squared) is:

$$\omega_P^2 = \frac{N_{\max} e^2}{m\varepsilon_0} = 1.285 \cdot 10^{16} \text{ rad}^2/\text{s}^2$$

from which we can calculate the equivalent relative permittivity of the ionosphere:

$$\varepsilon_r = 1 - \frac{\omega_P^2}{\nu_C^2 + \omega^2} = 0.26$$

The propagation constant thus is:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 2.4 \cdot 10^{-5} + j0.2252 \text{ 1/m}$$

The total path attenuation is obtained by considering that the conductivity is not zero only between $h_{\min}$ and h_p, and that the zenith attenuation can be scaled to the slant path using the cosecant law. Therefore:

$$\alpha_{dB} = \alpha \cdot 8.686 \cdot 1000 = 0.21 \text{ dB/km}$$

$$A_{IONO} = \alpha_{dB}(h_p - h_{\min})/\sin(\theta) \approx 7.3 \text{ dB} \rightarrow A_l = 0.186$$

Working at f = 21 MHz, the tropospheric effects can be neglected, but not the ionospheric ones. The power density reaching the ground is:

$$S = \frac{P_T}{4\pi L^2} G A_l = 1.37 \cdot 10^{-7} \text{ W/m}^2$$

where $L = H/\sin(\theta) = 583.3$ km. The power density reaching RX is:

$$S_{\mathrm{RX}} = \frac{S\sigma}{4\pi L^2} A_l$$

Combining both equations:

$$S_{\mathrm{RX}} = \frac{P_T}{4\pi L^2} G A_l \frac{\sigma}{4\pi L^2} A_l = P_T G \sigma \left(\frac{A_l}{4\pi L^2}\right)^2$$

Inverting the equation to solve for σ:

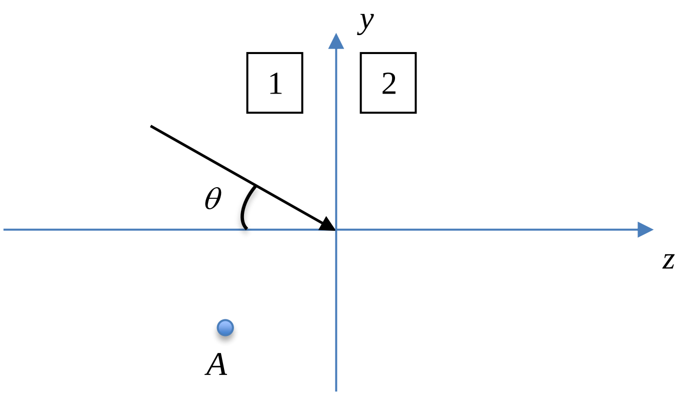
$$\sigma = \frac{S_{\mathrm{RX}}}{G P_T} \left(\frac{4\pi L^2}{A_l}\right)^2 = 167.7 \text{ m}^2 \rightarrow \text{urban area}$$

Problem 2

A plane sinusoidal EM wave (f = 9 GHz) propagates from a medium with electric permittivity ε_{r1} = 3 into free space (assume μ_r = 1 for both media) with incidence angle θ = 30°. The expression of the incident electric field is (E_0 = 44.84 V/m):

$$\vec{E}_i(z,y) = E_0\left(\frac{\sqrt{3}}{2}\vec{\mu}_y + \frac{1}{2}\vec{\mu}_z + \frac{j}{2}\vec{\mu}_x\right)e^{-j\beta\frac{\sqrt{3}}{2}z}e^{j\frac{\beta}{2}y}\ \text{V/m}$$

1) Determine the polarization of the incident field $\vec{E}_i$ (it is sufficient to state whether the polarization is elliptical, circular or linear; additional details, such as rotation direction or tilt angle, are not required).
2) Determine the polarization of the reflected field $\vec{E}_r$.
3) Calculate the power density S_T of the electric field in A(x = -1 m, y = -1 m, z = -1 m) by consider only the contribution of the reflected field.



Solution

1) The wave has two components: a TE one, along x, and a TM one, given by the combinations of the two fields along y and z. The absolute value of the components is E_0 V/m (TM) and $E_0/2$ V/m (TE), and the differential phase shift is $\pi/2$

2) Considering that the wave also has a TM component, it is worth checking the Brewster angle:

$$\theta_B = \text{tg}^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right) = 30°$$

As $\theta = \theta_B$ → the TM component is totally transmitted into the second medium. As a result, the polarization of the reflected wave will be linear (along x).

3) To determine E_0, it is first necessary to calculate the reflection coefficient for the TE component. To this aim, the transmission angle is:

$$\theta_2 = \sin^{-1}\left(\sin(\theta)\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\right) = 60°$$

The TE reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) - \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})}{\eta_0/(\cos(\theta_2)\sqrt{\varepsilon_{r2}}) + \eta_0/(\cos(\theta)\sqrt{\varepsilon_{r1}})} = 0.5$$

The reflected field is therefore given by:

$$\vec{E}_r(z, y) = j\frac{E_0}{2}\Gamma\vec{\mu}_x e^{j\beta\frac{\sqrt{3}}{2}z} e^{j\frac{\beta}{2}y}\ \mathrm{V/m}$$

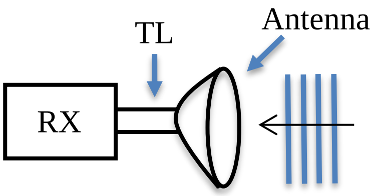
The power density reaching A will be:

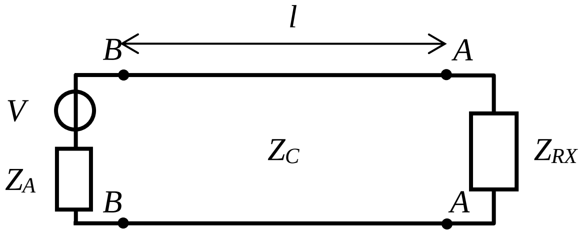
$$S_T = \frac{1}{2}\frac{\left|\vec{E}_r(A)\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}} = \frac{1}{2}\frac{\left|\frac{E_0}{2}\Gamma\right|^2}{\eta_0/\sqrt{\varepsilon_{r1}}} = 0.5\ \mathrm{W/m^2}$$

Problem 3

The power received by an antenna is conveyed into the receiver RX via a lossless coaxial cable, with intrinsic impedance Z_C = 50 Ω. The antenna acts as an equivalent generator with voltage V = 10^{-3} V and internal impedance Z_A = 50 Ω; the RX, which acts as a load, has impedance Z_{RX} = 150 Ω. The frequency is f = 600 MHz. The line length is l = 5.2 m.

1. Determine the power absorbed by RX, P_{RX}.
2. Using the same coaxial cable, propose changes to the circuit to achieve maximum power transfer from the antenna to RX: how much would P_{RX} be in that case?





Solution

1) The wavelength is $\lambda = c/f$ = 0.5 m. The reflection coefficient at section AA is given by:

$$\Gamma_L = \frac{Z_{RX} - Z_C}{Z_{RX} + Z_C} = 0.5$$

The solution is simplified by the partial match at section BB, so the power absorbed by the load can be simply calculated as (only one reflection at the load section):

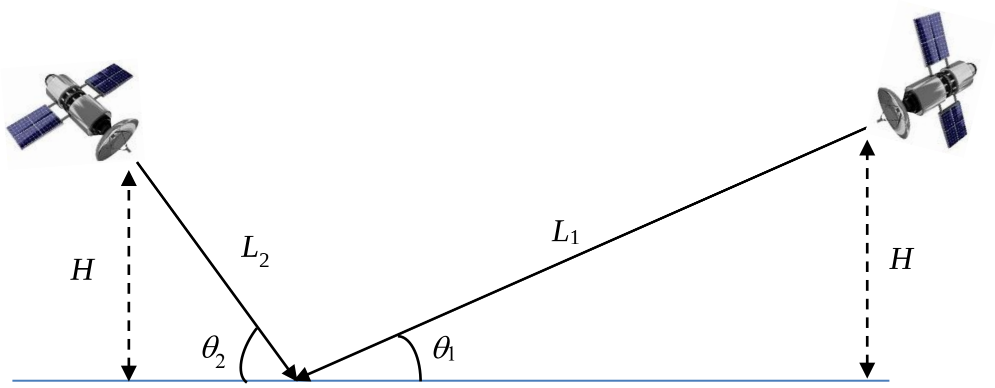
$$P_{RX} = P_{AV}(1 - |\Gamma_L|^2) = \frac{|V|^2}{8Z_A}(1 - |\Gamma_L|^2) = 1.875 \text{ nW}$$

2) Maximum power transfer is achieved by perfect matching, i.e. $Z_A = Z_{RX} = Z_C = 50\ \Omega$. In this case, no reflections occur and the power transferred to RX is the whole available power. In this case:

$$P_{AV} = \frac{|V|^2}{8Z_C} = 2.5 \text{ nW}$$

Problem 4

Consider a ground station, implementing orbital diversity (i.e. which always selects the satellite with the best SNR): it can be potentially served by two satellites. Satellite 1 (elevation θ_1 = 20°) operates at f_1 = 20 GHz , while satellite 2 (elevation θ_2 = 40°) operates at f_2 = 30 GHz. The ground station is affected by a zenithal rain attenuation at 20 GHz, $A_{R1}^{Z} = 6$ dB (rain rate is constant, both horizontally and vertically). Determine if the ground station connects to satellite 1 or to satellite 2.



Additional assumptions and data:

- use the simplified geometry depicted above (flat Earth)
- typical frequency scaling for rain attenuation from 20 GHz to 30 GHz
- ground station tracking the satellites optimally
- both satellites can maintain a perfect pointing to the ground station
- power transmitted by each satellite P_T = 100 W
- mean radiating temperature T_{mr} = 290 K
- LEO satellite antenna and ground antenna: parabolic reflectors with diameter D = 0.5 m and efficiency η = 0.6
- altitude of the LEO satellites: H = 800 km
- bandwidth of the receiver: B = 100 MHz
- internal noise temperature of the receiver: T_R = 350 K

Solution

The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_R}{k[T_R + T_{mr}(1 - A_R) + T_C A_R]B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K), $f_T = f_R = 1$, $L = H/\sin(\theta)$, $G_R = G_T$ (same antenna features). Depending on the link, several quantities will change. As for rain attenuation A_R:

Zenith: $A_{R1}^{Z} = 6$ dB → Slant: $A_{R1}^{S} = \frac{A_{R1}^{Z}}{\sin(\theta_1)} = 17.5$ dB → Linear scale: $A_{R1}^{L} = 0.0178$

Zenith: $A_{R2}^{Z} = A_{R1}^{Z}\left(\frac{f_2}{f_1}\right)^{1.72} = 12$ dB → Slant: $A_{R2}^{S} = \frac{A_{R2}^{Z}}{\sin(\theta_2)} = 18.7$ dB → Linear scale $A_{R2}^{L} = 0.0135$

As for the antenna gains, the effective areas of all antennas, at both frequencies, is:

$$A_E = \eta\pi \frac{D^2}{4} = 0.1178 \text{ m}^2$$

The gain of the both the satellite and ground antennas at 20 GHz is:

$$G_1 = \frac{4\pi}{\lambda_1^2} A_E = 6588.7 = 38.2 \text{ dB}$$

The gain of the both the satellite and ground antennas at 30 GHz is:

$$G_2 = \frac{4\pi}{\lambda_2^2} A_E = 14825 = 41.7 \text{ dB}$$

λ_1 and λ_2 are associated to f_1 and f_2, respectively.

Therefore:

$$SNR_1 = \frac{P_T G_1 (\lambda_1/4\pi L_1)^2 G_1 A_{R1}^L}{k[T_R + T_{mr}(1 - A_{R1}^L) + T_C A_{R1}^L]B} = 22.72 \text{ dB}$$

$$SNR_2 = \frac{P_T G_2 (\lambda_2/4\pi L_2)^2 G_2 A_{R2}^L}{k[T_R + T_{mr}(1 - A_{R2}^L) + T_C A_{R2}^L]B} = 31.1\text{dB}$$

It is more convenient to establish the connection with satellite 2.