**Telecommunication Systems – Prof. L. Luini,**
**June 27th, 2023**

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SURNAME AND NAME ______________________________

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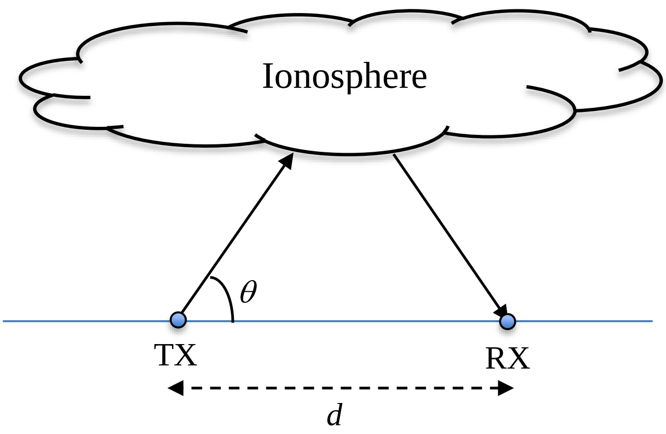
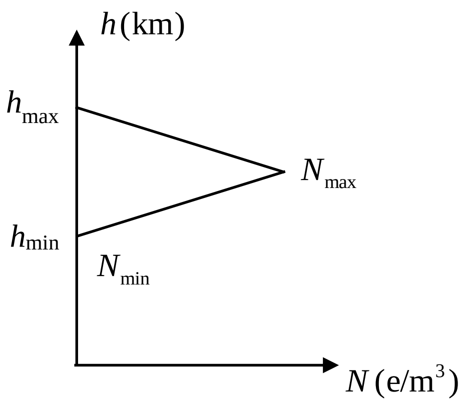
## Problem 1

Making reference to the figure below, we want the transmitter TX to reach the user RX at distance $d$ by exploiting the ionosphere (elevation angle $\theta$ = 60°). The ionosphere is modelled with the symmetric electron density profile (daytime) sketched in the figure (right side), where $N_{\max} = 6\times10^{12}$ e/m$^3$, $N_{\min} = 4\times10^{10}$ e/m$^3$, $h_{\min}$ = 100 km and $h_{\max}$ = 400 km.

1) Determine the maximum distance $d$ achievable for the TX → RX link.
2) Determine the operational frequency $f$ to achieve the conditions at point 1).
3) Indicate a reasonable margin on $f$ found at point 2) to guarantee the TX → RX link notwithstanding ionospheric variations.
4) Indicate the best polarization to be used for the TX → RX link.

Assume: the virtual reflection height $h_V$ is 1.2 times $h_R$, the height at which the wave is actually reflected; the Earth is flat; no tropospheric effects to be considered.

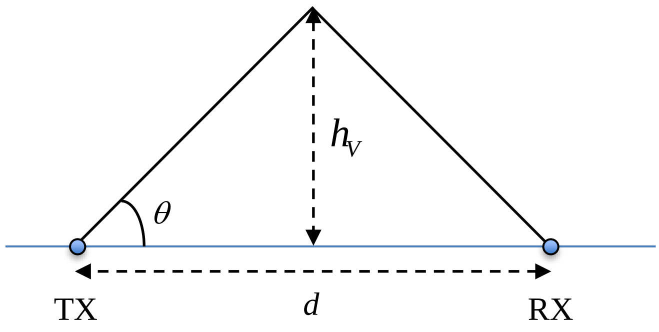



**Solution**

1) The distance $d$ is maximized if the reflection occurs as high as possible in the ionosphere, i.e. at the height $h_p$ = 250 km, correspondent to $N_{max}$. Considering the figure below, the distance can be found by inverting the following expression:

$h_V = 1.2\, h_p = {}^{d}/_{2} \tan\theta \rightarrow\ d = \frac{2.4\, h_p}{\tan\theta} = 346.4$ km




2) The link operational frequency *f* can be determined by inverting the following equation:

$$\cos\theta = \sqrt{1 - \left(\frac{f_C}{f_m}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N_{\max}}}{f_m}\right)^2}$$

Solving for the frequency $f_m$, we obtain:

$$f_m = \sqrt{\frac{81 N_{\max}}{1 - [\cos(\theta)]^2}} = 25.5 \text{ MHz}$$

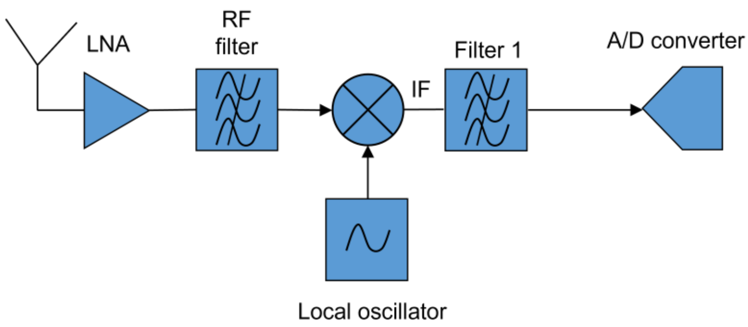
3) During the night, the peak values of the electron content will decrease: it is a good rule of thumb to use reduce by 10% the peak frequency to avoid that the wave crosses the ionosphere at nighttime. Therefore → $f' = 0.9f = 22.95$ MHz.

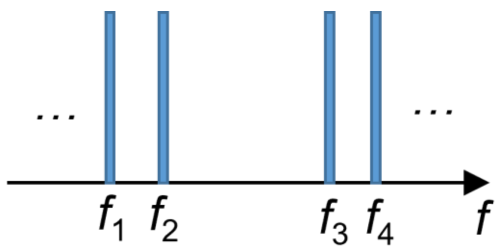
4) Depolarization in the ionosphere affects linearly polarized waves, but not circularly polarized ones. Therefore, the best polarization is LHCP or RHCP.

**Problem 2**

Consider the heterodyne receiver depicted below (left side), which aims at receiving the RF signal with carrier frequency $f_2$ = 20 GHz, to be digitalized at the end of the receiver chain by using an A/D converter with a maximum sampling frequency $f_S$ = 4.2 GS/s. As indicated in the picture below (right side), the RF spectrum is occupied by multiple signals, all having the same bandwidth $B$ = 200 MHz, with the closest carriers being $f_1$ = 19 GHz and $f_3$ = 25 GHz (more signals are present as indicated in the figure).

1) Design the RF filter and local oscillator considering the A/D converter features and the aim of reducing as much as possible the RF filter complexity (i.e. selectivity): specifically, provide a suitable $f_{LO}$, indicate the type of filter to be used for the RF filter and its cutoff(s) frequency(ies).
2) Indicate the ideal bandwidth of Filter 1.
3) Considering that the equivalent noise temperature at the input of the LNA is $T_{IN}$ = 300 K and that the gain of the LNA is $G_{LNA}$ = 30 dB, determine the maximum LNA equivalent noise temperature to achieve a noise power lower than $P_N$ = 1.5 pW in front of the A/D converter.




**Solution**

1) When down-converting signals from RF to intermediate frequency (IF), image signals represent a problem. The same IF can be obtained using a local oscillator $f_{LO}$ higher or lower than the target carrier frequency $f_2$. If $f_{LO} < f_2$, the image signals to be rejected using the RF filter are those lower than $f_2$; if $f_{LO} > f_2$, the image signals to be rejected using the RF filter are those higher than $f_2$. As a consequence, the RF filter for the case $f_{LO} < f_2$ will need to be much more selective than the one for the $f_{LO} > f_2$ case. Therefore, to meet the requirements expressed at point 1, the $f_{LO} > f_2$ case is to be selected. To reduce the RF filter complexity (low-pass filter) as much as possible, the ideal cutoff frequency is, for example, 22.5 GHz, exactly in the middle of the $f_3$- $f_2$ interval (other choices are possible). Regarding the value for the local oscillator frequency, if $f_{LO}$ = 2.5 GHz, then $f_{IF} = f_{LO} – f_2$ = 2.5 GHz, with a maximum frequency of the IF signal equal to $f_{max}$ = 2.6 GHz. However, according to the Nyquist theorem, in this case, the minimum sampling frequency of the A/D converter should be $f_S = 2f_{max}$ = 5.2 GS/s. This exceeds the available specifications for the A/D converter, i.e. $f_S$ = 4.2 GS/s. Using this value as an additional constraint, a possible final optimum design is: $f_{LO}$ = 22 GHz, $f_{IF} = f_{LO} – f_2$ = 2 GHz, $f_{max}$ = 2.1 GHz, $f_S = 2f_{max}$ = 4.2 GS/s.

2) Given the design at point 1, the optimum Filter 1 (bandpass filter) will have a lower cut-off frequency of $f_{min} = f_{IF}$-$B$/2 = 1.9 GHz and $f_{max} = f_{IF}$+$B$/2 = 2.1 GHz.

3) The LNA gain value is high enough to consider negligible the additional contributions to noise introduced by the other circuital elements. Therefore the noise power is:

$P_N = k(T_{IN} + T_{LNA})B$
Imposing $P_N < 1$ pW, we obtain → $T_{LNA} < 243.5$ K.

**Problem 3**

We need to design a link to a deep-space probe orbiting Mars and operating at Ka-band, specifically at $f$ = 26 GHz. The ground station is equipped with a steerable antenna to track the probe.

1) Calculate the reflector antenna diameter of the ground station (Gregorian configuration with efficiency $\eta$ = 0.5) necessary to guarantee that the probe can be correctly tracked down to an elevation angle $\theta$ = 30° for 99.9% of the time in a year, i.e. that the minimum SNR is 5 dB. To this aim, assume:
   - that the atmosphere is stratified;
   - that the ground station LNA noise temperature is $T_R$ = 50 K;
   - to neglect the cosmic background temperature;
   - that the mean radiating temperature of the atmosphere is $T_{mr}$ = 10 °C;
   - that the probe makes use of a parabolic antenna with gain $G_T$ = 45 dB;
   - the transmit power is $P_T$ = 110 W;
   - the probe antenna always points at the ground station;
   - the distance between the probe satellite and the ground station is $L$ = 225000000 km;
   - the receiver bandwidth is $B$ = 1 kHz;
   - that the CCDF of the zenithal atmospheric attenuation $A_T$ is modelled by:

$$P\left(A_Z^{dB}\right) = 100e^{-0.69A_Z^{dB}} \quad (A_Z \text{ in dB and } P \text{ in } \%)$$

2) Calculate the maximum data rate achievable for this channel with the conditions at point 1), considering a negligible bit error rate.

**Solution**

1) The zenithal attenuation $A_Z^{dB}$ is determined using the CCDF model. 99.9% availability corresponds to $P$ = 0.1% exceedance. Inverting the CCDF formula:

$$A_Z^{dB} = -\frac{1}{0.69}\ln\left(\frac{0.1}{100}\right) \approx 10 \text{ dB}$$

Scaling the zenithal attenuation to the target elevation angle:

$$A_S^{dB} = \frac{A_Z^{dB}}{\sin(\theta)} \approx 20 \text{ dB}$$

which, in linear scale, corresponds to:

$$A_L = 10^{\frac{A_S^{dB}}{10}} \approx 0.01$$

The system noise temperature is (for the Gregorian configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}(1 - A_L) = 330.3 \text{ K}$$

The SNR is given by:

$$SNR = \frac{P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_L}{k T_{sys} B}$$

where $f_R$ = 1 and $f_T$ = 1.

Inverting the expression above to solve for $G_R$ (by setting SNR = $SNR_{min}$ = 5 dB):

$G_R \approx$ 74 dB

Recalling that:

$$\frac{\eta A_g}{G_R} = \frac{\lambda^2}{4\pi}$$

where $A_g$ is the geometrical area of the antenna:

$$A_g = \left(\frac{D_R}{2}\right)^2 \pi$$

the antenna diameter $D_R$ is obtained:

$D_R \approx 26$ m

This is indeed the dimension of Ka-band deep-space antennas installed at NASA Deep Space Network (DSN) sites (Goldstone, Madrid, Canberra).

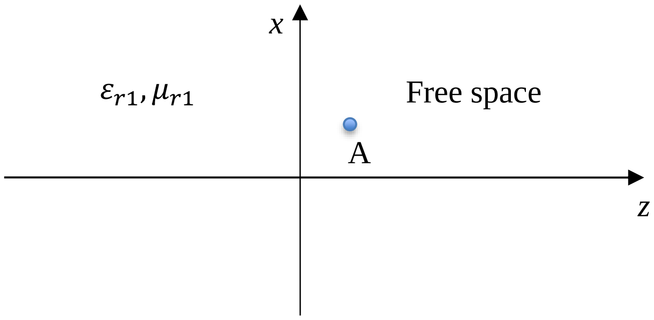
2) The reply is given by the Shannon limit:
$C = B\log_2(1 + SNR_{min}) \approx 2$ kb/s

**Problem 4**

A uniform plane wave (frequency $f$ = 300 MHz) propagates along $z$ into free space from a medium with the following electromagnetic features: $\varepsilon_{r1}$ = 3, $\mu_{r1}$ = 1 and $\sigma_1$ = 0.05 S/m. The incident electric field at the origin of the axis is $\vec{\boldsymbol{E}}_{\boldsymbol{i}}(\boldsymbol{z} = \mathbf{0}\ \mathbf{m}) = \boldsymbol{jE_0}\vec{\boldsymbol{\mu}}_{\boldsymbol{x}}$ V/m.

For this scenario:

1) What is the wave polarization?
2) Calculate the expression of the incident magnetic field in the first medium (left side) in the time domain.
3) Calculate the wavelength in medium 1.
4) Calculate $E_0$ knowing that power density power at point A(1,1,$\lambda_2$) is $S(A)$ = 3 $\mathrm{mW/m^2}$.




**Solution**

1) The propagation in medium 1 is regulated by the propagation constant. As no approximations are possible (the loss tangent is roughly 1):

$\gamma_1 = \sqrt{j\omega\mu_1(\sigma_{1+}j\omega\varepsilon_1)} = 4.95 + j11.96$ 1/m

Therefore, the expression of the electric field in medium 1 is:

$\vec{\boldsymbol{E}}_{\boldsymbol{i}} = \boldsymbol{jE_0 e^{-\gamma_1 z}}\vec{\boldsymbol{\mu}}_{\boldsymbol{x}} = \boldsymbol{jE_0 e^{-4.95z} e^{-j11.96z}}\vec{\boldsymbol{\mu}}_{\boldsymbol{x}}$ V/m.

To find the magnetic field, we first need to calculate the intrinsic impedance of the medium:

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{(\sigma_{1+}j\omega\varepsilon_1)}} = 169.1 + j70\ \Omega$$

In the phasor domain, the incident magnetic field is:

$$\vec{\boldsymbol{H}}_{\boldsymbol{i}} = \frac{\boldsymbol{E_0}}{\boldsymbol{\eta_1}}\boldsymbol{e^{-4.95z}e^{-j11.96z}}\vec{\boldsymbol{\mu}}_{\boldsymbol{y}} = \frac{\boldsymbol{E_0}}{|\boldsymbol{\eta_1}|}\boldsymbol{e^{-4.95z}e^{-j11.96z}e^{-j\measuredangle(\eta_1)}}\vec{\boldsymbol{\mu}}_{\boldsymbol{y}}$$

where $|\eta_1| = 183\ \Omega$ and $\measuredangle\ (\eta_1) = 0.3924$ rad.

Therefore, the expression of the incident magnetic field in the time domain is (considering also $j$, which translates into $\pi/2$ in the cosine argument):

$$\vec{\boldsymbol{H}}_{\boldsymbol{i}}(\boldsymbol{t}) = \frac{\boldsymbol{E_0}}{|\boldsymbol{\eta_1}|}\boldsymbol{e^{-4.95z}}\mathbf{cos}\ (\boldsymbol{2\pi ft} - \mathbf{11.96}\boldsymbol{z} - \mathbf{1.18})\vec{\boldsymbol{\mu}}_{\boldsymbol{y}}$$

2) $\lambda_1 = \frac{2\pi}{\beta_1} = 0.525$ m

3) First, it is necessary to calculate the reflection coefficient, given by:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.359\ - j\ 0.174$$

where

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \approx 377\ \Omega$$

The transmitted electric field, at $z$ = 0 m, is:

$$\vec{E}_t(z = 0\text{ m}) = jE_0 T\vec{\mu}_x = jE_0(1 + \Gamma)\vec{\mu}_x = E_0(0.174 + j1.359)\vec{\mu}_x\ \text{V/m.}$$

The power density reaching point $A$ is (as there are no losses in the second medium, the power density does not change along $z$):

$$S(A) = \frac{1}{2}\frac{\left|\vec{E}_t(z = 0)\right|^2}{\eta_2} = \frac{1}{2}\frac{(E_0)^2|T|^2}{\eta_2} = 0.0025\ (E_0)^2 = 3\ \text{mW/m}^2$$

Therefore:

$$E_0 = 1.1\ \text{V/m}$$