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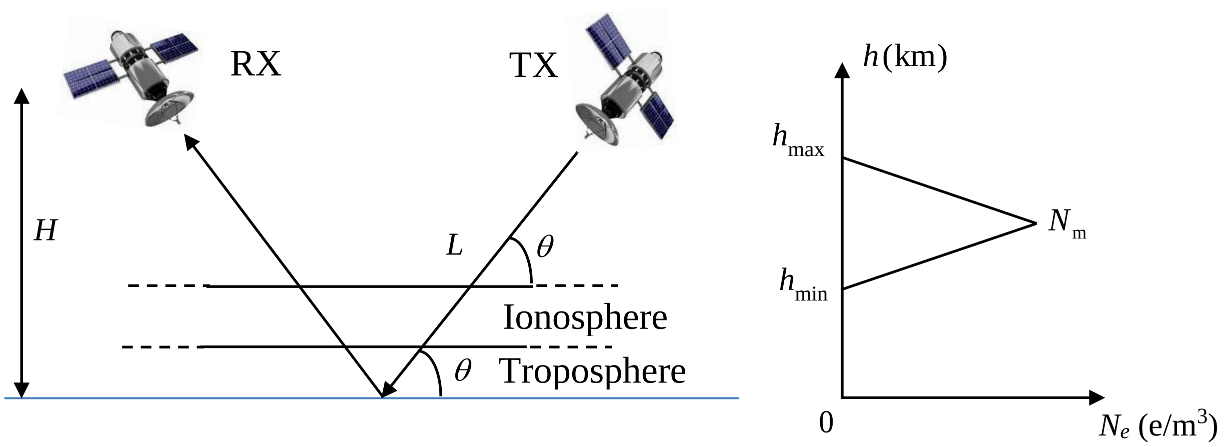
Problem 1

The figure below shows a bistatic radar system (operational frequency f = 51 MHz) consisting of two LEO satellites flying along the same orbit (height above the ground H = 700 km). The radar estimates the ground altitude by measuring the signal propagation time between TX at RX via reflection. To model the ionosphere, consider the symmetric electron content profile sketched below ($h_{\min}$ = 50 km, $h_{\max}$ = 450 km, N_m = 1×10^{10} e/m^3, N_e homogeneous horizontally); to model the troposphere, consider the following vertical profile for the refractivity N: $N = N_0 e^{-h/h_0}$ (N in ppm, h in km, N_0 = 800 ppm, h_0 = 25 km, N homogeneous horizontally). Taking into account the simplified geometry (left side) below:

1) Determine the minimum value of the angle θ for the system to work properly?
2) Calculate the error in estimating the ground altitude (in m) induced on the radar by the troposphere and to the ionosphere.
3) Assuming no ITU-R constraints, what would be the optimum frequency range for the radar operation (justify your answer)?



Solution

1) For the wave to avoid total reflection due to the ionosphere, the angle θ needs to be higher than θ_{min}, determined as:

$$\cos(\theta_{min}) = \sqrt{1 - \left(\frac{9\sqrt{N_\mathrm{m}}}{f}\right)^2} \quad \Rightarrow \quad \theta_{min} \approx 1°$$

2) The error due to the atmosphere is associated to the additional signal delay introduced by the ionosphere and the troposphere. The former can be calculated as:

$$ds_I = \frac{40.3}{f^2}\mathrm{TEC} \approx 30.1\ \mathrm{m}$$

where:

$$\mathrm{TEC} = \frac{(h_{max} - h_{min})N_m}{2} = 2 \times 10^{15}\,\frac{\mathrm{e}}{\mathrm{m}^2}$$

The latter can be calculated as:

$$ds_T = 10^{-6}\left[\int_0^H N dh\right]$$

For an exponential profile, considering that $H >> h_0$:

$$ds_T = h_0 N_0 10^{-6} = 20\ \mathrm{m}$$

The total error in calculating the ground altitude is:

$$ds = ds_T + ds_I = 50.1\ \mathrm{m}$$

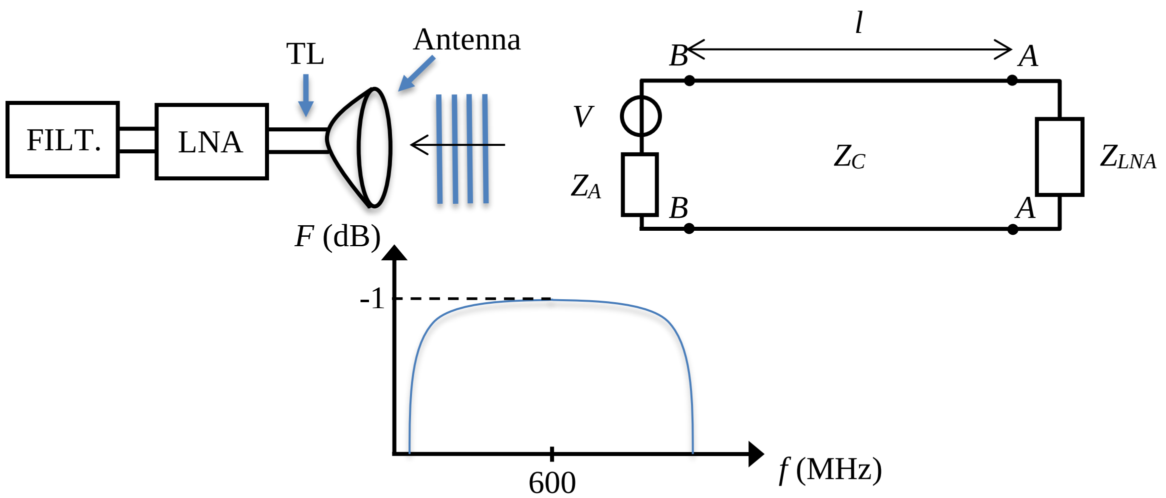
This value is already the zenithal one, i.e. expressing the error on measuring the altitude: theoretically, the delay should be calculated along the slant path and then projected to the zenithal direction, but the approach above inherently assumed that both N_e and N are horizontally homogeneous.

3) As the delay due to the ionosphere is frequency dependent, increasing the frequency will decrease the error. However, beyond 10 GHz, precipitation can cause a strong attenuation, which would impair the link to the point that the SNR at RX becomes too low. Therefore the best frequency range is the X band. For example, selecting f = 9 GHz, the error would reduce to 20 m (basically only due to the troposphere), with no significant impact on the wave amplitude.

Problem 2

The power received by an antenna is conveyed into a Low Noise Amplifier (LNA), whose gain is G = 20 dB, via a lossless transmission line, with intrinsic impedance Z_C = 50 Ω. The antenna acts as an equivalent generator with voltage $V = 5{\times}10^{-5}$ V and internal impedance Z_A = 50 Ω; the LNA has impedance Z_{LNA} = 80 Ω. The frequency is f = 600 MHz. The line length is l = 10.125 m. For this receiver:

1) Determine the signal power at the output of the filter: make reference to the receiver chain and to the filter transfer function F in the figure below.
2) Determine the noise power of the system after the filter. To this aim assume: that the antenna noise temperature is T_A = 280 K, the physical temperature of the transmission line T = 300 K, the noise figure of the LNA is NF_{LNA} = 3 dB, the equivalent noise temperature of the filter T_F = 800 K, the system bandwidth is B = 10 MHz.



Solution

1) To determine the signal power after the filter, we need to consider: the power absorbed by the LNA, the gain of the LNA, the insertion loss of the filter. To this aim, given that l is a multiple of $\lambda/4$ (being the wavelength λ = 0.5 m), the impedance at section BB is simply given by:

$$Z_{BB} = \frac{Z_C^2}{Z_{LNA}} = 31.25\ \Omega$$

The reflection coefficient at section BB is therefore:

$$\Gamma = \frac{Z_{BB} - Z_A}{Z_{BB} + Z_A} = -0.23$$

The power absorbed by the LNA is:

$$P_{LNA} = P_{AV}(1 - |\Gamma|^2) = \frac{|V|^2}{8Z_A}(1 - |\Gamma|^2) = 5.9\ \text{pW}$$

After the LNA, the power is:

$$P_{FILT} = P_{LNA}G_{lin} = 5.9 \times 10^{-10}\ \text{W}$$

After the filter, the power is:

$$P_{OUT} = P_{FILT}F_{lin} = 4.7 \times 10^{-10}\ \text{W}$$

2) The total equivalent noise power of the system is:

$$T_{sys} = T_A + T_{TL} + T_{LNA} + \frac{T_F}{G} = 576.6\ \text{K}$$

where T_{TL}, the noise contribution introduced by the transmission line, is zero, as it is assumed to be lossless, and:

$T_{LNA} = \left(10^{NF_{LNA}/10} - 1\right)290 = 288.6$ K

The noise power is:

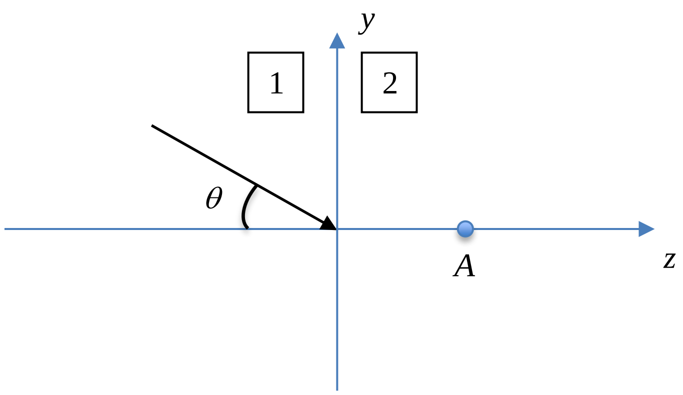
$P_N = kT_{sys}B = 7.96 \times 10^{-14}$ W

Problem 3

A plane sinusoidal EM wave (f = 9 GHz) propagates from free space into a medium with electric permittivity ε_{r2} = 4 (assume μ_r = 1 for both media). The incident electric field is:

$$\vec{E}_i(z,y) = \vec{\mu}_x e^{-j\beta\cos\theta z} e^{j\beta\sin\theta y} \text{ V/m}$$

1) Determine the polarization of the incident field $\vec{E}_i$.
2) Determine the value of θ to minimize the power received in A(z = 1 m, y = 0 m, x = 10 m), where an isotropic antenna is located.
3) Calculate the power received by the antenna in A for the θ value determined at point 2).
4) Calculate the electric field at the origin of the coordinate system (0,0).



Solution

1) The wave has just one component, specifically the TE one, so the polarization is linear along $\vec{\mu}_x$.

2) As ε_{r1} = 1 < ε_{r2} = 4, there is no chance to have an evanescent wave in the second medium. However, the absolute value of the reflection coefficient will increase as the incident angle increases. First, let us consider Snell's law to determine the transmission angle:

$$\theta_t = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\sin\theta\right) = 30°$$

For θ = 90°, the power transmitted will be minimized. In fact:

$$\eta_1^{TE} = \frac{\frac{\eta_0}{\sqrt{\varepsilon_{r1}}}}{\cos\theta} = \infty$$

$$\eta_2^{TE} = \frac{\frac{\eta_0}{\sqrt{\varepsilon_{r2}}}}{\cos\theta_t} = 217.7\ \Omega$$

As a result:

$$\Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -1$$

3) Based on the result obtained at point 1, the power reaching A will be zero.

4) Based on the result obtained at point 2) and considering that, with a TE polarization, the electric field is fully parallel to the discontinuity, the electric field in (0,0) can be easily calculated as:

$\vec{E}(0,0)=\vec{E_i}(0,0)+\vec{E_r}(0,0)=\vec{E}(0,0)+\overrightarrow{\Gamma E}(0,0)=0\,\frac{\mathrm{V}}{\mathrm{m}}$

Problem 4

We need to design a LEO satellite → ground station link operating at f = 15 GHz.

1) What is the best polarization to be used (justify your answer)?

The ground station is equipped with a steerable antenna to track the LEO satellite, and suited to receive the polarization determined at point 1):

2) Calculate the minimum elevation angle θ that can be used to track the LEO satellite to guarantee that the system noise temperature is lower than 520 K for 99.99% of the time in a year. To this aim, assume:

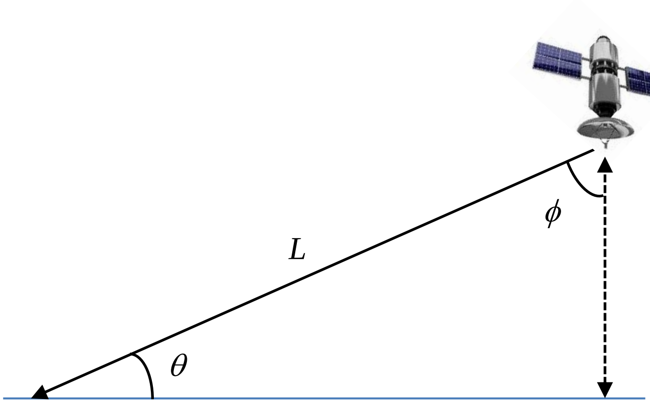
- that the LNA noise temperature is T_R = 240 K;
- to neglect the cosmic background temperature;
- that the mean radiating temperature of is T_{mr} = 10 °C;
- to use a parabolic antenna with Cassegrain configuration with gain G_R = 40 dB;
- that the CCDF of the zenithal atmospheric attenuation A_T is modelled by:

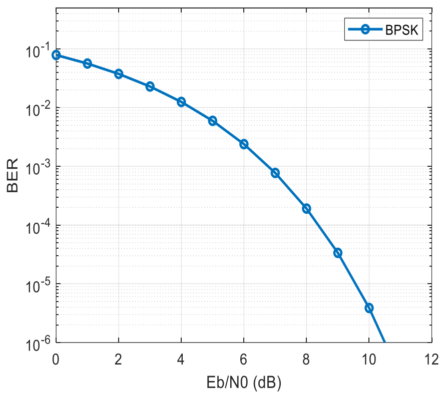
$$P\left(A_T^{dB}\right) = 100 e^{-1.15 A_T^{dB}} \quad (A_T \text{ in dB and } P \text{ in \%})$$

3) In the conditions of point 2), determine the LEO antenna gain G_T (circular parabolic reflector) to guarantee that the power received at the ground station is P_R = 1 pW. To this aim, assume that (make reference to the figure below):

- the transmit power is P_T = 20 W;
- the LEO antenna always points to the centre of the Earth;
- the radiation pattern of the transmit antenna is $f_T = (\cos\phi)^2$;
- the distance between the LEO satellite and the ground station is L = 1000 km.

4) Considering BPSK modulation, assuming that the data rate is R = 12 Mb/s, calculate the BER that can be guaranteed using the link conditions determined in the previous points. Assume that the bandwidth B is equal to R.





Solution

1) The best polarization to be used for non-geosynchronous satellites is the circular one (LHCP or RHCP), mainly for geometrical reasons.

2) The system noise temperature is (for the Cassegrain configuration, the waveguide is very short and its effect on the noise can be neglected):

$$T_{sys} = T_R + T_A = T_R + T_{mr}\left(1 - 10^{-\frac{A_T^{dB}}{10\sin(\theta)}}\right)$$

where A_T^{dB} is zenithal attenuation to be determined using the CCDF model. 99.99% availability corresponds to P = 0.01% exceedance. Inverting the CCDF formula:

$$A_T^{dB} = -\frac{1}{1.15}\ln\left(\frac{0.01}{100}\right) = 8 \text{ dB}$$

Setting T_{sys} = 520 K and inverting the first equation above to solve for θ:

$$\theta = \sin^{-1}\left\{-\frac{A_T^{dB}}{10\log10\left[1-\frac{T_{sys}-T_R}{T_{mr}}\right]}\right\} = 24.2°$$

3) Let us consider the link budget equation:

$$P_R = P_T G_T f_T (\lambda/4\pi L)^2 G_R f_R A_T$$

where f_R = 1 and $f_T = [\cos(90-\theta)]^2 = 0.168$. Also:

$$A_T = 10^{-\frac{A_T^{dB}}{10\sin(\theta)}} = 0.011$$

Solving for G_T:

$$G_T \approx 1056 = 30.23 \text{ dB}$$

4) The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B} = \frac{E_b R}{N_0 B} = \frac{E_b}{N_0} = 11.61 = 10.65 \text{ dB}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). Therefore, considering the BER curve above, the link can guarantee a BER lower than 4×10^{-6}.