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SURNAME AND NAME ______________________________

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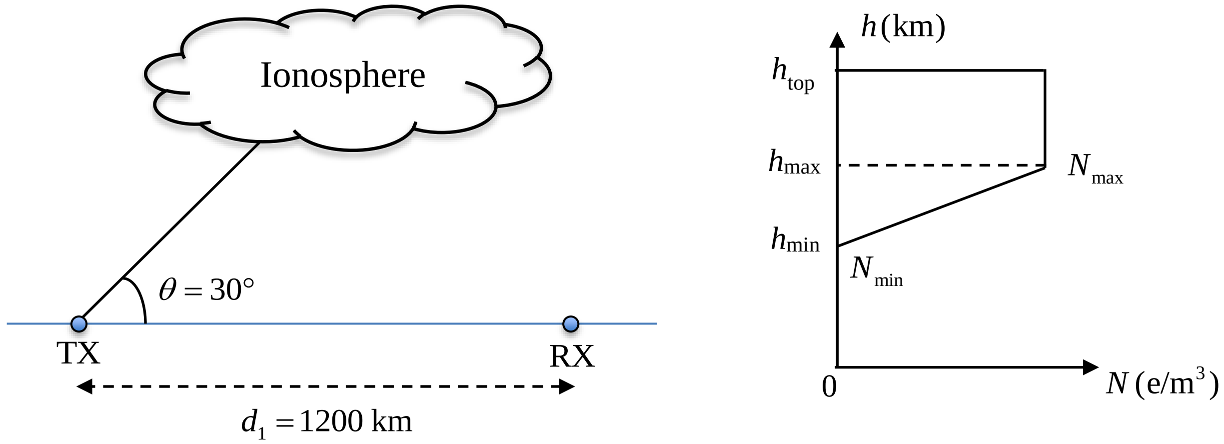
Problem 1

Making reference to the figure below, the transmitter TX needs to reach the user RX, which is at distance d_1 = 1200 km, by exploiting the effects induced by the ionosphere. The elevation angle of TX is $\theta = 30°$. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where $N_{min} = 4\times10^2$ e/m^3, $N_{max} = 4\times10^{12}$ e/m^3, h_{min} = 100 km, h_{max} = 400 km and h_{top} = 700 km.

1) Calculate the link frequency f_1 to reach RX.
2) Assuming now that RX can move further from TX (keeping the same elevation angle), calculate the maximum distance d_2 at which RX can be reached, as well as the corresponding link frequency f_2.
3) What happens if the link frequency increases beyond f_2?

Assume: the virtual reflection height h_V is 1.2 of h_R, the height at which the wave is actually reflected.



Solution:

1) Considering the figure below, given the distance between the TX and RX and the elevation angle, the virtual reflection height h_V is given by:

$h_{V1} = {d_1}/{2} \tan\theta = 346.4 \text{ km}$

The actual reflection occurs at $h_{R1} = h_{V1}/1.2 = 288.7$ km. The value of electron density N associated to h_{R1} can be obtained from the following linear expression (easily derivable from the profile sketch):

$$N = \frac{N_{\max} - N_{\min}}{h_{\max} - h_{\min}}\left(h - h_{\min}\right) + N_{\min}$$

Thus, for $h = h_{R1}$, we obtain:

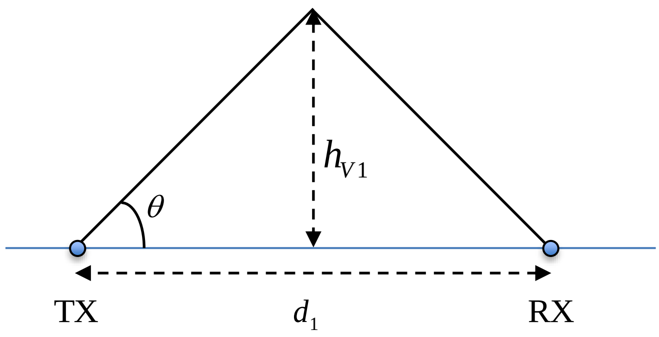
$N_1 = 2.5\times10^{12}$ e/m^3

The elevation angle and the frequency are linked by the following expression:

$$\cos\theta = \sqrt{1-\left(\frac{f_P}{f_1}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_1}}{f_1}\right)^2}$$

Solving for the frequency f_1, we obtain:

$$f_1 = \sqrt{\frac{81N_1}{1-\left[\cos(\theta)\right]^2}} \approx 28.6 \text{ MHz}$$



2) The maximum distance d_2 between TX and RX corresponds to the point of maximum reflection along the profile, which corresponds to h_{max}, i.e. the height where the maximum electron density N_{max} is found. The maximum distance can be easily calculated as:

$$d_2 = {2h_{V2}}/{\tan\theta} = {2\cdot1.2\cdot h_{\max}}/{\tan\theta} = 1662.3 \text{ km}$$

The correspondent link frequency needed to achieve reflection at such height is:

$$f_2 = \sqrt{\frac{81N_{\max}}{1-\left[\cos(\theta)\right]^2}} \approx 36 \text{ MHz}$$

3) If the operational frequency exceeds f_2, the wave crosses the ionosphere.

Problem 2

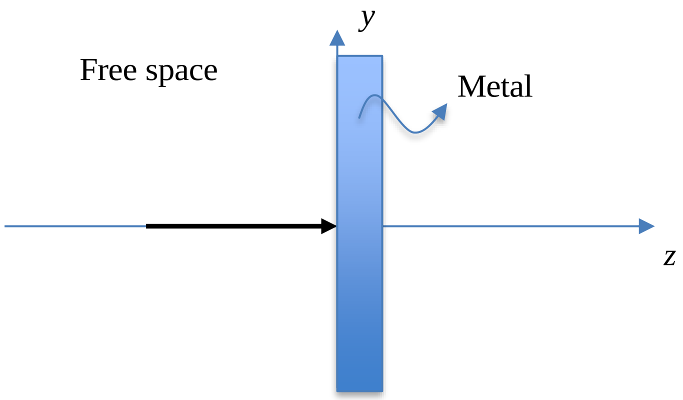
A plane sinusoidal EM wave propagates from vacuum into a perfect metallic surface (perfect electric conductor) with orthogonal incidence (assume both ε_r = 1 and μ_r = 1 for this material). The expression for the electric field is:

$$\vec{E} = \left(j\vec{\mu}_y + \vec{\mu}_x \right) e^{-j188.62z} \ \text{V/m}$$

1) Determine the frequency of the EM wave.

2) Determine the polarization of the incident EM wave.

3) Determine the polarization of the reflected wave.



Solution:

1) The frequency of the incident EM wave can be derived from the phase constant β = 188.62 rad/m:

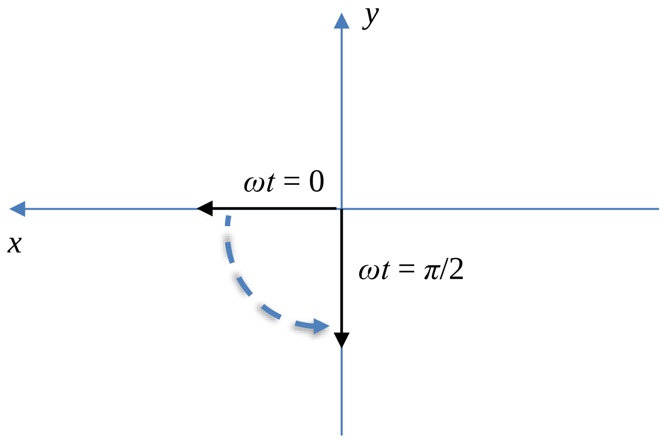
$$\beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{2\pi f}{c} \quad \Rightarrow \quad f = \frac{c\beta}{2\pi} = 9 \ \text{GHz}$$

2) The polarization of the incident wave is circular because the two TE and TM components have the same amplitude (1 V/m) and a phase shift of $\pi/2$. Setting z to 0, and expressing the dependence on time, we can easily understand the electric field rotation direction:

$$\vec{E}(t) = \text{Re}\left\{ \left[j\vec{\mu}_y + \vec{\mu}_x \right] e^{j\omega t} \right\} = \cos\left(\omega t + \frac{\pi}{2} \right) \vec{\mu}_y + \cos(\omega t)\vec{\mu}_x \ \text{V/m}$$

Thus, for t = 0 → $\left.\vec{E}\right|_{\omega t=0} = \vec{\mu}_x$ V/m

Thus, for $\omega t = \pi/2$ → $\left.\vec{E}\right|_{\omega t=\pi/2} = -\vec{\mu}_y$ V/m



The polarization of the incident wave is LHCP.

3) When a plane wave hits a perfect electric conductor (PEC), the reflection coefficient is always Γ = -1. Therefore, both components of the electric field will change their sign, and the wave plane direction will also change. This is shown in the equation of the reflected wave:

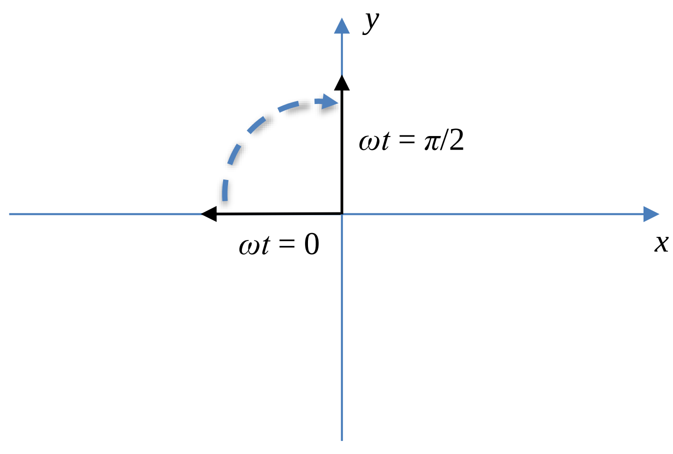
$$\vec{E}_R = \left(-j\vec{\mu}_y - \vec{\mu}_x\right)e^{j188.62z} \text{ V/m}$$

The polarization of the reflected wave is still circular; also, repeating the same reasoning above, we obtain:

$$\vec{E}_R(t) = \text{Re}\left\{\left[-j\vec{\mu}_y - \vec{\mu}_x\right]e^{j\omega t}\right\} = -\cos\left(\omega t + \frac{\pi}{2}\right)\vec{\mu}_y - \cos(\omega t)\vec{\mu}_x \text{ V/m}$$

Thus, for $t = 0$ → $\left.\vec{E}_R\right|_{\omega t=0} = -\vec{\mu}_x$ V/m

Thus, for $\omega t = \pi/2$ → $\left.\vec{E}_R\right|_{\omega t=\pi/2} = \vec{\mu}_y$ V/m

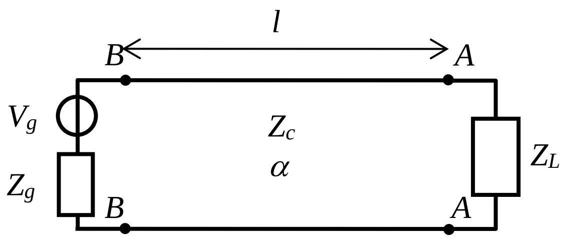


The polarization of the incident wave is RHCP. Note that, as the direction of the reflected plane wave changes (direction = –*z*), so does also the view of the axes, with the *x*-axis pointing to the right side this time.

Problem 3

A transmitter with voltage V_g = 100 V (sinusoidal regime) and internal impedance Z_g = 50 Ω is connected to a load Z_L = -j10 Ω by a transmission line with characteristic impedance Z_C = 100 Ω and attenuation coefficient α = 30 dB/km . The line length is l = 30 m and the frequency is f = 300 MHz.
Calculate:

1) The power absorbed by the load.
2) The value of Z_L to maximize P_L, the power transferred to the load.
3) The value of P_L and the power absorbed by the line, for the conditions at point 2).



Solution

1) As the load is imaginary (it corresponds to a capacitor), no power will be absorbed by Z_L.

2) The value of Z_L maximizing the power absorbed by the load is:

$$Z_L = 100\ \Omega = Z_C$$

In fact, in this case, the load will guarantee at least a partial match of the load with the transmission line.

3) In the conditions at point 2), the power absorbed by the load is given by:

$$P_L = P_{AV}\left(1-\left|\Gamma_g\right|^2\right)e^{-2\alpha l}\ \text{W}$$

$$P_{AV} = \frac{\left|V_g\right|^2}{8\,\text{Re}\left\{Z_g\right\}} = 25\ \text{W}$$

The attenuation coefficient must be first converted to Np/m:

$$\alpha = \frac{30}{8.686\cdot 1000} = 0.0035\ \text{Np/m}$$

Finally:

$$\Gamma_g = \frac{Z_{BB}-Z_g}{Z_{BB}+Z_g} = \frac{Z_L-Z_g}{Z_L+Z_g} = 0.334$$

Note that in this case $Z_{BB} = Z_L$ as the load is matched to the line.
Therefore:

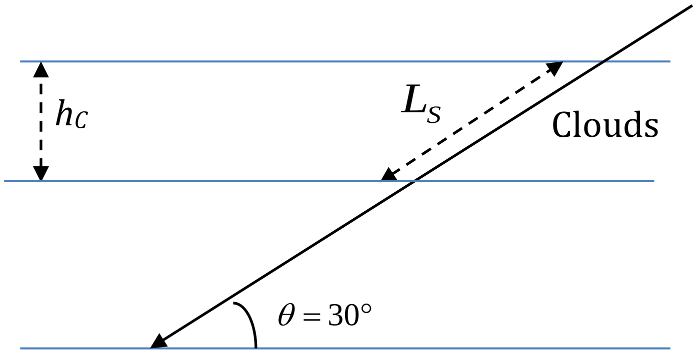
$$P_L = 18.1\ \text{W}$$

Finally, the power absorbed by the line is:

$$P_{line} = P_{AV}\left(1-\left|\Gamma_g\right|^2\right) - P_L = 4.1\ \text{W}$$

Problem 4

Consider a link from a GEO satellite to a ground station (elevation angle $\theta = 30°$) and assume that, initially, there is no atmospheric attenuation: associate this condition to the reference signal-to-noise ratio SNR_0. Evaluate the decrease in the SNR (as a function of SNR_0) when clouds begin to affect the link. Consider: no cosmic background noise, link frequency f = 50 GHz, cloud thickness h_C = 2 km, T_{mr} for clouds = 0 °C, specific cloud attenuation (constant vertically and horizontally) γ = 1.46 dB/km, receiver internal noise temperature T_R = 300 K.



Solution:

The SNR is given by (no attenuation case):

$$SNR_0 = \frac{P_R}{P_N} = \frac{P_T G_T f_T (\lambda/4\pi D)^2 G_R f_R}{k T_R B}$$

where D is the distance between the satellite and the ground station and B is the RX bandwidth.

With cloud attenuation, the SNR changes to:

$$SNR_1 = \frac{P_R A_C}{P_N^C} = \frac{P_T G_T f_T (\lambda/4\pi D)^2 G_R f_R A_C}{k T_C B}$$

where A_C is the rain induced attenuation and T_C is the RX noise temperature in case of clouds.

As a matter of fact, the SNR decreases both because of the additional attenuation induced by clouds and because of the increase in the noise received by the antenna.

As for A_C:

$$(A_C)_{dB} = \gamma L_S = \gamma \frac{h_C}{\sin\theta} \approx 5.84 \text{ dB} \rightarrow A_C = 10^{-(5.84/10)} = 0.26$$

As for T_C:

$$T_C = T_R + T_A = T_R + (1 - A_C) T_{mr} \approx 502 \text{ K}$$

In cloudy conditions, the RX noise increases by a factor 1.67 if compared to the initial conditions; i.e.:

$$\frac{T_C}{T_R} = \frac{T_R + (1 - A_C) T_{mr}}{T_R} \approx 1.67 \quad \rightarrow \quad T_C = 1.67 T_R$$

As a result:

$$SNR_1 = \frac{P_R A_C}{P_N^C} = \frac{A_C}{1.67} \frac{P_T G_T f_T \left(\lambda / 4\pi D\right)^2 G_R f_R}{k T_R B} = \frac{A_C}{1.67} SNR_0 = 0.16 SNR_0$$