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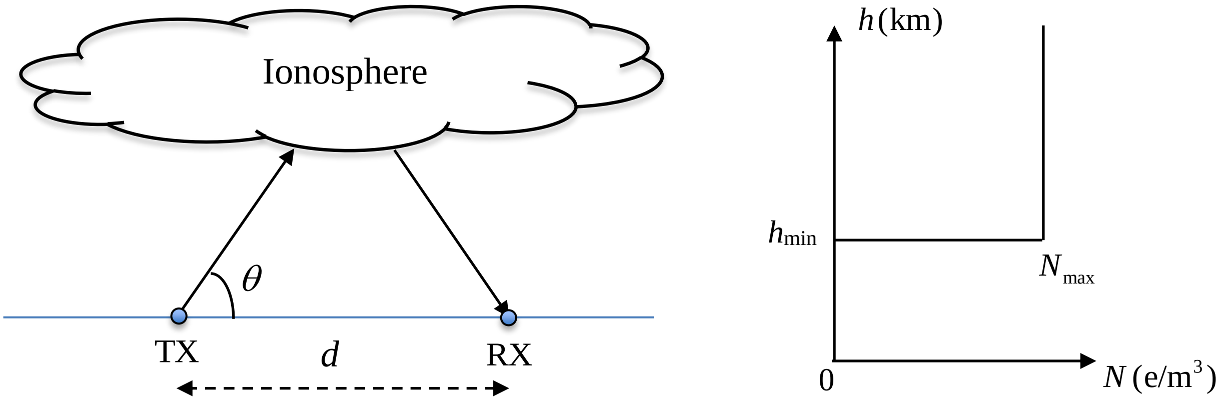
Problem 1

Making reference to the figure below, the transmitter TX wants to reach the user RX by exploiting the ionosphere. The ionosphere can be modelled with the electron density profile sketched in the figure (right side), where $N_{\max} = 9\times10^{12}$ e/m^3, $h_{\min} = 100$ km. The elevation angle is $\theta = 30°$.

1) Calculate the power density reaching the bottom of the ionosphere, assuming that TX features an isotropic antenna with efficiency $\eta = 1$ and that the transmit power is $P_T = 100$ W.
2) Calculate the distance d for TX to reach RX, assuming that the link operational frequency is $f_1 = 30$ MHz.
3) Repeat the calculation at point 2) if the link operational frequency changes to $f_2 = 70$ MHz.

Assume: the virtual reflection height $h_V = h_R$, being h_R the height at which the wave is actually reflected.



Solution

1) The power density reaching the bottom of the ionosphere is:

$$S = \frac{P_T}{4\pi R^2} G_T f_T = 2 \times 10^{-10}\ \mathrm{W/m^2}$$

where both G_T and f_T are equal to 1 and $R = h_V/\sin(\theta) = 200$ km.

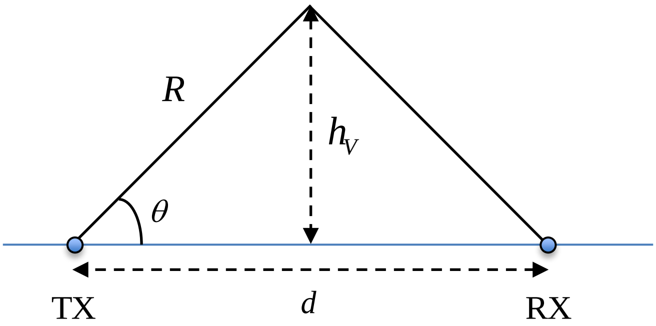
2) The elevation angle, the electron content and the link operational frequency are linked by the following relationship:

$$\cos\theta = \sqrt{1-\left(\frac{f_C}{f}\right)^2} = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f}\right)^2}$$

Solving for the frequency *f*, we obtain:

$$f = \sqrt{\frac{81N_{\max}}{1-[\cos(\theta)]^2}} = 54\text{ MHz}$$

For any frequency lower than 54 MHz, the wave will be totally reflected at the bottom of the ionosphere.



Therefore:

$$d = \frac{2h_V}{\text{tg}\,(\theta)} = 346.4\text{ km}$$

3) As $f_2 > f$, the wave crosses the ionosphere and the RX cannot be reached.

Problem 2

A plane sinusoidal EM wave (frequency f = 10 GHz) propagates from a perfect dielectric medium (ε_{r1} = 4, μ_{r1} = 4) into free space with orthogonal incidence. The expression for the electric field is:

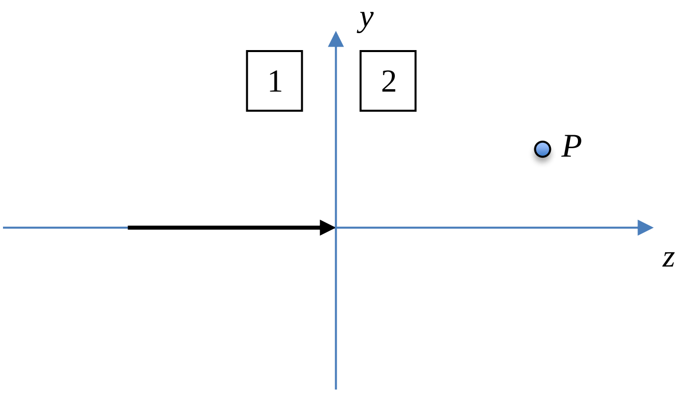
$$\vec{E}(z) = \left[-\vec{\mu}_x + \frac{j}{2}\vec{\mu}_y\right] e^{-j\beta_1 z} \text{ V/m}$$

1) Calculate β_1.

2) Determine the polarization of the incident EM wave.

3) Write the expression of the reflected wave.

4) Calculate the power received by an antenna located in point $P(x = 1$ m, $y = 1$ m, $z = 10$ m) with effective area $A_E = 1$ m^2.



Solution

1) The phase constant in the first medium can be easily calculated as:

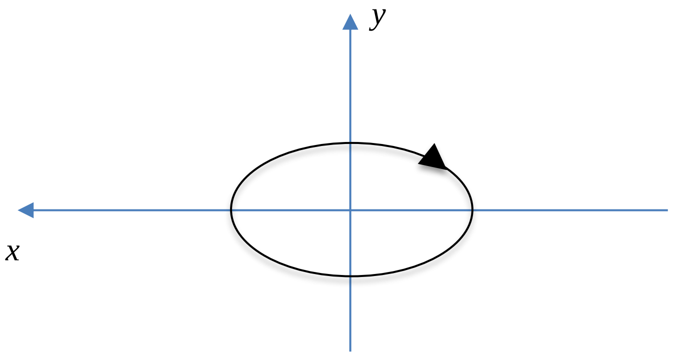
$$\beta_1 = \frac{2\pi f}{c}\sqrt{\varepsilon_{r1}\mu_{r1}} = 837.8 \text{ rad/m}$$

2) The incident electric field consists of two orthogonal components which are phase shifted and have different absolute values. As a result, the polarization of the incident wave is elliptical. To determine the rotation direction, we need to investigate the variation in time of the two components:

$$\vec{E}(0,t) = Re\left\{\left[-\vec{\mu}_x + \frac{j}{2}\vec{\mu}_y\right] e^{-j\beta_1 z}\right\} = -\vec{\mu}_x \cos(\omega t) + \frac{1}{2}\vec{\mu}_y \cos\left(\omega t + \frac{\pi}{2}\right) \text{ V/m}$$

Thus, for $t = 0$ → $\vec{E} = -\vec{\mu}_x$ V/m
Thus, for $\omega t = \pi/2$ → $\vec{E} = -\frac{1}{2}\vec{\mu}_y$ V/m

The incident wave polarization is right-hand elliptical.



3) For both components, the reflection coefficient is 0, so the wave is totally transmitted into the second medium (no reflection). In fact:

$$\eta = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0$$

as:

$$\eta_1 = \eta_0 \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = 377\ \Omega \text{ and } \eta_2 = \eta_0$$

4) As the wave is totally transmitted into medium 2, the power received at *P* will simply be:

$$P = \frac{1}{2} \frac{|\vec{E}|^2}{\eta_2} A_E = 0.0017\ \mathrm{W/m^2}$$

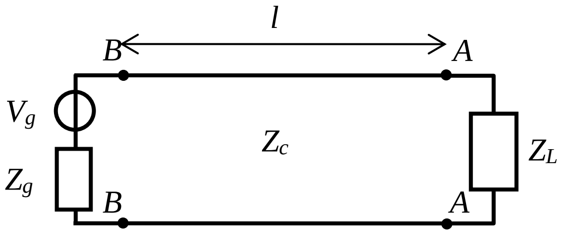
with:

$$|\vec{E}|^2 = 1^2 + 0.5^2 = 1.25\ \ \mathrm{V^2/m^2}$$

Problem 3

A source with voltage V_g = 10 V and internal impedance Z_g = 125 Ω is connected to a load Z_L = 10+j10 Ω by a transmission line with characteristic impedance Z_C = 50 Ω, the frequency is f = 300 MHz and the length of the line is l = 4.75 m.
Calculate:

a) The power absorbed by the load
b) The temporal trend of the voltage at the beginning of the line (section BB below), V_B



Solution

a) The wavelength is:
$\lambda = \lambda_0 = c/f = 1$ m

As a consequence, the length of the line normalized to the wave length is:
$\boldsymbol{l/\lambda = 4.75 = 4.5 + 0.25}$ m

In other terms, the line is a $\lambda/4$.

As a result, the load at section BB can be easily calculated as:
$Z_{BB} = \frac{Z_C^2}{Z_L}$ = 125-j125 Ω

The reflection coefficient for the source is:
$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = 0.2 - j0.4$$

Therefore, the power absorbed by the load is:
$$P_L = P_{AV}\left(1 - |\Gamma_g|^2\right) = \frac{|V_g|^2}{8\mathrm{Re}[Z_g]}\left(1 - |\Gamma_g|^2\right) = 0.08 \text{ W}$$

b) The voltage at the beginning of the line is:
$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB}+Z_g} = 6 - j2$ V → $V_{BB} = 6.32e^{-j0.3218}$ V

The trend of V_{BB} in time is given by:
$v_{BB}(t) = \mathrm{Re}\left[V_{BB}\, e^{j\omega t}\right] = 6.32\cos(\omega t - 0.3218)$ V

Problem 4

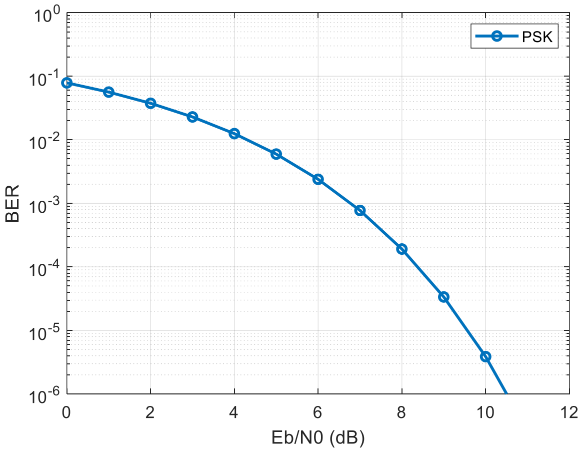
Consider a zenithal link (elevation angle θ = 90°) from a MEO satellite to a ground station, operating at f = 30 GHz. The link is impaired by rain, with constant rain rate R = 2 mm/h (both horizontally and vertically) and height h_R = 2 km. The transmitted wave is circularly polarized.

1) Assuming that all rain drops are oriented horizontally (minor axis aligned with the zenithal direction), what is the wave polarization at the ground station?

For system design purpose, assume that the specific rain attenuation is given by $A_{spec} = aR^b$, with $a = 0.2291$ and $b = 0.9129$.

2) Calculate the transmit power to guarantee that, under the rainy conditions given above, the power received at the ground station is P_R = 1 pW. To this aim, assume that:

- antennas are optimally pointed
- the gain of the antennas on board the satellite is G_T = 10 dB
- the ground antenna has efficiency $\eta = 0.8$
- the ground antenna is parabolic, with diameter D = 1.25 m
- the altitude of the MEO satellite is H = 8000 km



3) Calculate the maximum bandwidth that can be used to guarantee a BER lower than 0.0002, given the P_R value above and using the BPSK modulation (see figure). To this aim, assume that:

- the internal received noise temperature is T_R = 300 K
- the mean radiating temperature of is T_{rain} = 10 °C
- the data rate is equal to the system bandwidth

Solution

1) As rain drops are oriented horizontally, their section from below will appear to be circular: any polarization will undergo the same attenuation, so the wave will not be depolarized.

2) Considering rain attenuation, the link budget is given by:

$$P_R = P_T G_T f_T \left(\lambda/4\pi H\right)^2 G_R f_R A_{rain} = P_T G_T \left(\lambda/4\pi H\right)^2 G_R A_{rain}$$

where f_T and f_R have been set to 1, as the two antennas are optimally pointed, and A_{rain} is the total rain attenuation along the path (linear scale). The latter is calculated as:

$$\left(A_{rain}\right)_{dB} = aR^b h_R = 0.8627 \quad \rightarrow \quad A_{rain} = 10^{-\frac{(A_{rain})_{dB}}{10}} = 0.82$$

Converting also G_T to linear scale → G_T = 10. Also, the receiver antenna gain is:

$$G_R = \frac{4\pi}{\lambda^2}\eta\left(\frac{D}{2}\right)^2\pi = 123370 \rightarrow \left(G_R\right)_{dB} = 50.9 \text{ dB}$$

Solving for P_T:

$$P_T = \frac{P_R}{G_T G_R (\lambda/4\pi H)^2 A_{rain}} \approx 100 \text{ W}$$

3) The total noise power in the RX is affected by the antenna noise temperature, that can be calculated as:

$T_A = T_{rain}(1-A_{rain}) + A_{rain} T_C = 53.3$ K

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{k(T_R + T_A)B}$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K). A BER < 0.0002 is guaranteed by E_b/N0 > 8 dB (see figure). Assuming that the data rate is equal to the system bandwidth, E_b/N0 = SNR. Thus, by imposing that the minimum SNR is 8 dB (6.3), we obtain:

$\frac{P_R}{k(T_R+T_A)B} > 6.3$ → $B < 32.5$ MHz