Telecommunication Systems
February 4th, 2020

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SURNAME AND NAME ______________________________

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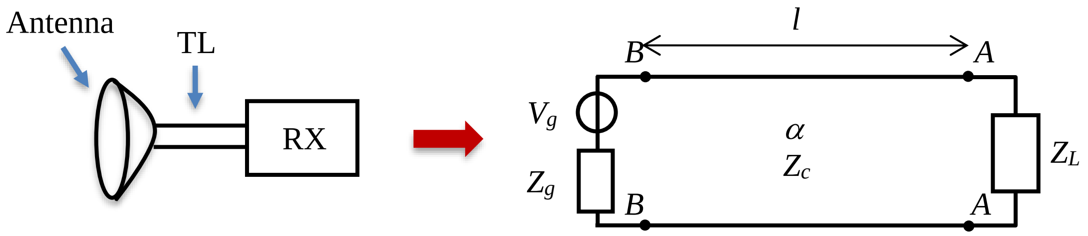
Problem 1

Consider a deep space scenario, where a probe orbiting around Mars transmits data of the planet to a ground station on the Earth.

1) Select the best antenna type for the ground station (justify your answer)

Taking as reference the simple block diagram reported below, the antenna can be modeled as a voltage generator V_g = 100 V (with internal impedance Z_g = 50 Ω), which is connected to the receiver RX through a lossy transmission line (TL) with characteristic impedance Z_C and attenuation constant α = 40 dB/km. The input impedance of the receiver is Z_L = 120 Ω, the line length is l = 5 m and the frequency is f = 1 GHz.

2) Determine the value of Z_C to maximize the power absorbed by the load Z_L



Solution

1) The best antenna to be used in this scenario is a double reflector antenna (Cassegrain configuration), which allows reducing as much as possible the length of the transmission line connecting the antenna to the receiver: this will also minimize both the attenuation and the additional noise introduced by the TL.

2) Given the value of Z_L and Z_g, it is not possible to achieve complete matching. However, it is possible to choose Z_C so as to match it to either of the two impedances. Therefore, we have two options:

a) $Z_C = Z_g = 50\ \Omega$

In this case there is no discontinuity at section BB, but a mismatch at section AA; therefore, the power absorbed by the load is:

$$P_L = P_{AV} e^{-2\alpha l} \left[1 - \left| \Gamma_{AA} \right|^2 \right]$$

where:

$$P_{AV} = \frac{\left| V_g \right|^2}{8 \operatorname{Re}\left[Z_g \right]} = 25\ \text{W}$$

$\alpha = 4.6\times10^{-3}$ Np/m

$$\Gamma_{AA} = \frac{Z_L - Z_C}{Z_L + Z_C} = 0.412$$

Therefore → $P_L = 19.82$ W

b) $Z_C = Z_L = 120\ \Omega$

In this case there is no discontinuity at section AA, but a mismatch at section BB; therefore, the power absorbed by the load is:

$$P_L = P_{AV} \left[1 - \left| \Gamma_g \right|^2 \right] e^{-2\alpha l}$$

where:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = \frac{Z_L - Z_g}{Z_L + Z_g} = 0.412$$

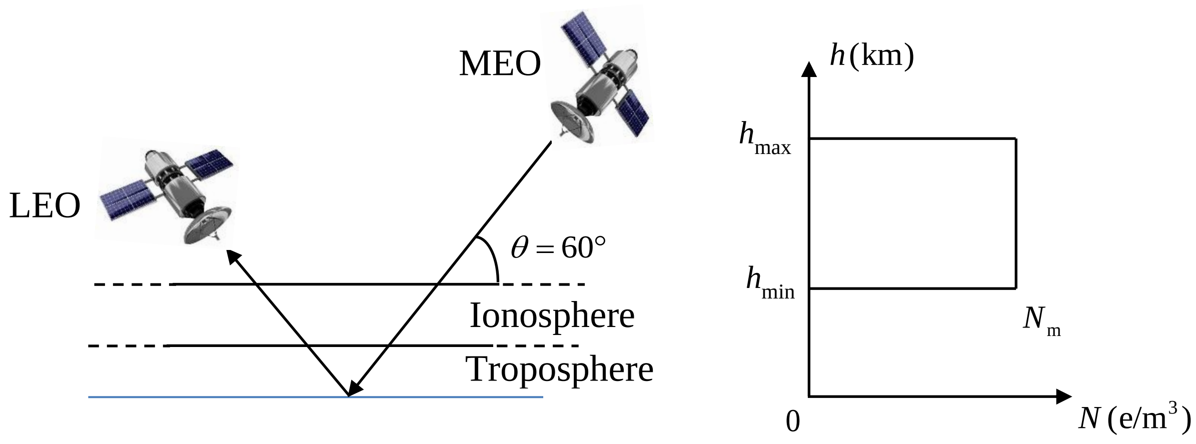
Therefore → $P_L = 19.82$ W

The power transferred to the load is the same in both cases.

Problem 2

As shown in the figure below, a LEO satellite aims at exploiting the signal emitted by a MEO satellite. Such signal reaches the LEO satellite directly and by reflection on the Earth surface (e.g. GNSS reflectometry): the aim is to measure the altitude of the Earth surface by comparing the time it takes for the direct and reflected signals to reach the LEO satellite. Making reference to the electron content profile (h_{min} = 100 km, h_{max} = 400 km, N_m = 2×10^{12} e/m^3, N homogeneous horizontally) and to the geometry (left side) reported below:

1) Determine the frequency range for the system to work properly
2) Determine the additional delay introduced by the ionosphere on the reflected wave (to this aim, select the minimum frequency of the range determined at point 1)
3) Select a specific frequency, falling within the range determined at point 1), to improve the system accuracy, while guaranteeing that the system still works properly



Solution

1) For the wave to avoid total reflection due to the ionosphere, the operational frequency needs to be higher than the following cut-off frequency:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_m}}{f_{min}}\right)^2} \quad \Rightarrow \quad f_{min} = \frac{9\sqrt{N_m}}{\sin\theta} \approx 14.7 \text{ MHz}$$

Therefore any operational frequency $f > f_{min}$ will allow the proper operation of the system (the wave is reflected by the Earth surface and not by the ionosphere).

2) The additional delay ΔT introduced by the ionosphere is associated to the total electron content (TEC). Specifically, considering that the reflected wave will cross the ionosphere twice:

$$\Delta T = 2\left(\frac{1}{2c}\frac{81}{f^2}\text{TEC}\right)$$

where:

$$\text{TEC} = \frac{N_m\left(h_{max} - h_{min}\right)}{\sin\theta} \approx 6.93\times10^7 \text{ e/m}^2$$

The term at the denominator takes into account the slant path, while the integration at the numerator is valid for zenithal pointing.

Selecting $f = f_{min}$:

$$\Delta T = 2\left(\frac{1}{2c}\frac{81}{f^2}TEC\right) \approx 0.866 \text{ ms}$$

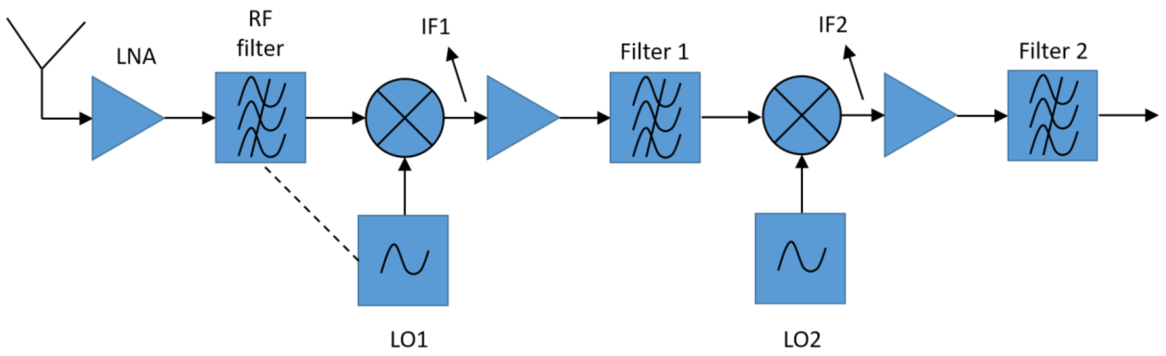
3) Increasing the operational frequency will decrease the value of ΔT, i.e. it will improve the system accuracy. However, if the frequency increases too much, the wave will be affected by the troposphere. Specifically, if $f > 10$ GHz, rain will strongly attenuate the signal. Therefore, a good choice can be $f = 10$ GHz. In this case:

$$\Delta T = 2\left(\frac{1}{2c}\frac{81}{f^2}TEC\right) \approx 1.871 \text{ ns}$$

Problem 3

Consider the typical superheterodyne receiver depicted below, which aims at receiving an AM signal whose carrier frequency is f_{RF} = 10 GHz and whose bandwidth is B = 10 MHz. The received voltage just before the LNA is V_{in} = 1 nV and the receiver needs to amplify the signal such that the output voltage after filter 2 is V_{out} = 1 μV. Also, the frequency of the first local oscillator is f_{LO1} = 9.9 GHz and that of the second local oscillator is f_{LO2} = 110 MHz. For this receiver:

1) Propose values of the gains of the three amplifiers to achieve the desired amplification level (assume that no losses are introduced by the other circuital elements)
2) Determine suitable values for the minimum ($f_{\min}$) and the maximum ($f_{\max}$) frequencies of both band-pass filters after the mixers (Filter 1 and Filter 2)



Solution

1) The total amplification gain, for the whole chain, is:

$$G = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) = 60 \text{ dB}$$

A good choice could be G = 30 dB for the LNA and G = 15 dB for the other two amplifiers. In fact, the LNA needs to have a high gain to reduce as much as possible the overall receiver equivalent noise temperature, while the other amplifiers have a lower gain and are less expensive/critical.

2) After the first down-conversion stage, the intermediate frequency will be:

$f_{IF1} = f_{RF} - f_{LO1}$ = 100 MHz

This is the also the central frequency of Filter 1, while its bandwidth (ideally) is B = 10 MHz. Therefore $f_{\min1} = f_{IF1} - B/2$ = 95 MHz and $f_{\max1} = f_{IF1} + B/2$ = 105 MHz.

After the second down-conversion stage, the intermediate frequency will be:

$f_{IF2} = f_{LO2} - f_{IF1}$ = 10 MHz

This is the also the central frequency of Filter 2, while its bandwidth (ideally) is B = 10 MHz. Therefore $f_{\min2} = f_{IF2} - B/2$ = 5 MHz and $f_{\max2} = f_{IF2} + B/2$ = 15 MHz.

Problem 4

Consider the downlink from a spacecraft to a ground station (zenithal pointing). The link operating frequency is f = 20 GHz. Calculate the signal-to-noise ratio (SNR) in rain-free conditions using the following data:

- the gain of the both antennas is G = 40 dB and their efficiency is 0.6
- the ground antenna is optimally pointed to the satellite and its pointing accuracy is P^A = 0.5°
- the satellite antenna is optimally pointed to the ground station (assume a perfect pointing system)
- the power transmitted by the satellite is P_T = 100 W
- the distance between the ground station and the satellite is H = 36000 km
- the receiver LNA equivalent noise temperature is T_{LNA} = 200 K
- the antenna equivalent noise temperature T_A = 85 K and the mean radiating temperature of the medium is T_{mr} = 120 K
- the system bandwidth is B = 40 MHz

Assume now that rain starts to affect the link by introducing an additional attenuation of A_R = 5 dB (assume same value for T_{mr} = 120 K). In this case:

- calculate the decrease in the maximum data rate achievable by the link

Solution

1) The wavelength is $\lambda = c/f$ = 0.015 m. The gain of the two antennas is:

$$G_T = G_R = 40 \text{ dB} = 10000$$

The tropospheric attenuation, in rain free conditions, can be inferred from T_A and T_{mr} as:

$$A_{RF} = 1 - \frac{T_A}{T_{mr}} = 0.2917 \rightarrow A_{RF} = 5.35 \text{ dB}$$

Considering all the terms, the received power is:

$$P_R = P_T G_T f_T \left(\frac{\lambda}{4\pi H} \right)^2 G_R f_R L^P A_{RF}$$

where f_T = 1 and f_R = 1. The term L^P indicates the pointing loss due to the limited accuracy of the ground antenna pointing system:

$$L^P = 12 \left(\frac{P^{ACC}}{\theta_{-3dB}} \right)^2 = 12 \left(\frac{DP^{ACC}}{70\lambda} \right)^2 \quad \text{(dB)}$$

where D is the antenna diameter. This value can be inferred from the gain and the efficiency:

$$D = \sqrt{\frac{G\lambda^2}{\eta\pi^2}} = 0.616 \text{ m}$$

Therefore:

$$L^P = 1.03 \text{ dB} \rightarrow L^P = 0.788$$

and:

$$P_R \approx 2.53 \times 10^{-12} \text{ W}$$

The noise power depends on the total system equivalent noise temperature:

$$T_{sys} \approx T_A + T_{LNA} = 285\,\text{K}$$

The SNR is:

$$SNR = \frac{P_R}{P_N} = \frac{P_R}{kT_{sys}B} = 16.1$$

where k is the Boltzmann's constant (1.38×10^{-23} J/K).

In case of rain, the total tropospheric attenuation becomes $A = A_{RF}+A_R = 10.35$ dB → $A = 0.922$. Also the antenna noise temperature will change:

$$T_A' = T_{mr}(1-A) = 108.9\ \text{K} \rightarrow T_{sys}' \approx T_A' + T_{LNA} = 308.9\,\text{K}$$

The SNR becomes:

$$SNR' = \frac{P_R A}{A_{RF}} \frac{1}{kT_{sys}' B} = 4.7$$

The maximum achievable data rate R is dictated by the Shannon theorem:

$$R = B\log_2\left(1+SNR\right)$$

Therefore, the decrease in the maximum achievable data rate is:

$$\Delta R = B\log_2\left(1+SNR\right) - B\log_2\left(1+SNR'\right) \approx 63\ \text{Mbit/s}$$