Telecommunication Systems – Prof. L. Luini,
September 7th, 2022

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SURNAME AND NAME ______________________________

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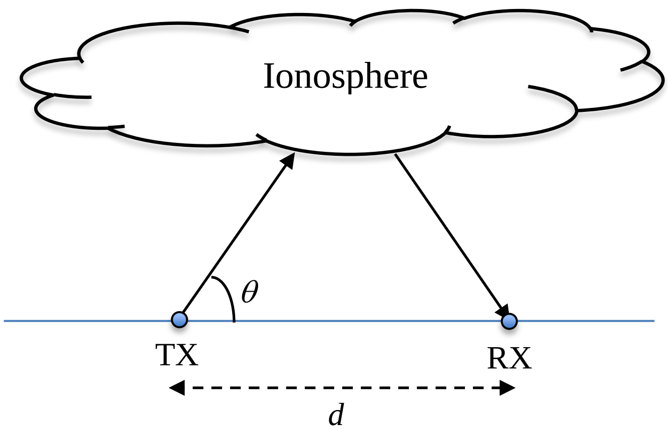
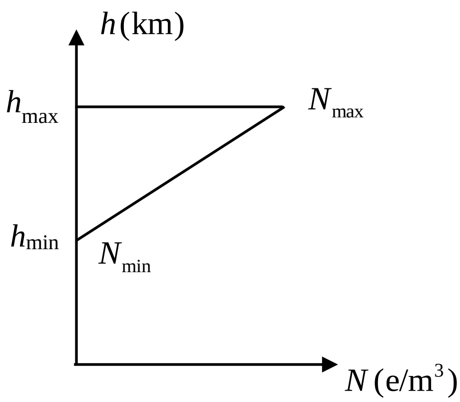
Problem 1

Making reference to the figure below, we want the transmitter TX to reach the user RX at distance *d* by exploiting the ionosphere (elevation angle θ = 40°). The ionosphere is modelled with the electron density profile sketched in the figure (right side), where $N_{max} = 6\times10^{12}$ e/m^3, $N_{min} = 4\times10^{10}$ e/m^3, h_{min} = 50 km and h_{max} = 400 km.

1) Determine the distance *d* to minimize the effects due to the ionosphere on the TX → RX link.
2) Determine the operational frequency *f* to achieve the conditions at point 1) and to maximize, at the same time, the link data rate.

Assume: the virtual reflection height h_V is 1.2 times h_R, the height at which the wave is actually reflected; the Earth is flat.

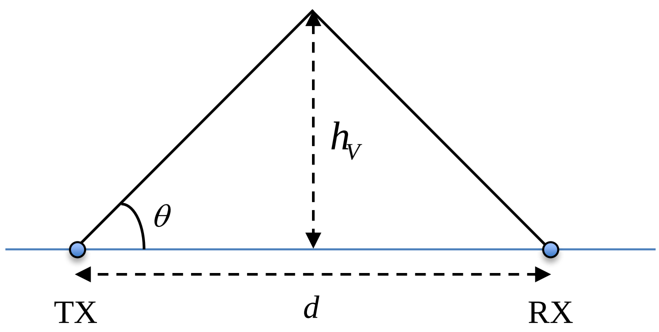




Solution

1) The effects of the ionosphere (delay, attenuation and depolarization) are minimized if the reflection occurs at N_{min}. Considering the figure below, the distance can be found by inverting the following expression:

$h_V = 1.2\, h_{\min} = {}^{d}/_{2} \tan\theta \rightarrow d = \frac{2.4\, h_{\min}}{\tan\theta} = 143 \text{ km}$



2) The link operational frequency f can be determined by inverting the following equation:

$$\cos(\theta) = \sqrt{1 - \left(\frac{f_P}{f_m}\right)^2} = \sqrt{1 - \left(\frac{9\sqrt{N_{\min}}}{f_m}\right)^2}$$

Solving for the frequency f_m, we obtain:

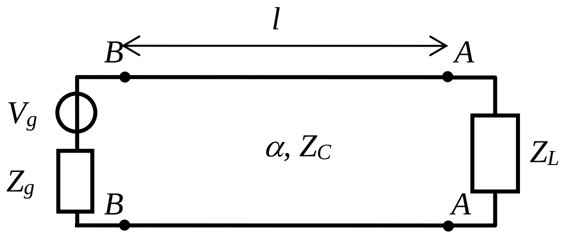
$$f_m = \sqrt{\frac{81 N_{\min}}{1 - [\cos(\theta)]^2}} = 2.8 \text{ MHz}$$

Any frequency lower than f_m would induce total reflection, so there is a degree of freedom in choosing the link frequency. On the other hand, the higher the frequency, the higher the data rate. Therefore → $f = f_m$.

Problem 2

A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a transmission line with characteristic impedance Z_C = 50 Ω, which terminates on a load Z_L = 50 Ω. The frequency is f = 300 MHz and the length of the line is l = 10 m.

1) Calculate the power absorbed by Z_L.
2) Calculate the power absorbed by the line if the line becomes lossy (α = 50 dB/km).



Solution

1) As there is complete match among the circuit components, the power absorbed by Z_L is the available power. Therefore:

$$P_L = P_{AV} = \frac{|V_g|^2}{8\mathrm{Re}[Z_g]} = 0.25 \text{ W}$$

2) When the line becomes lossy, the power absorbed by the load is:

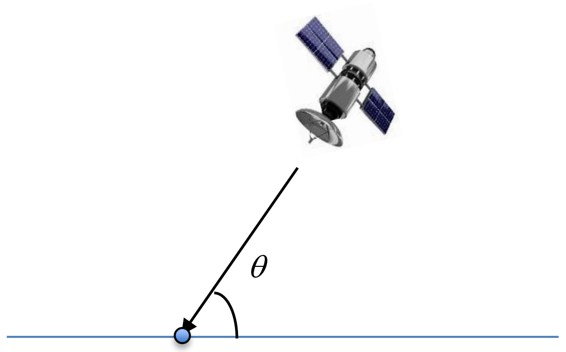
$$P_L = P_{AV} e^{-2\alpha l} = 0.22 \text{ W}$$

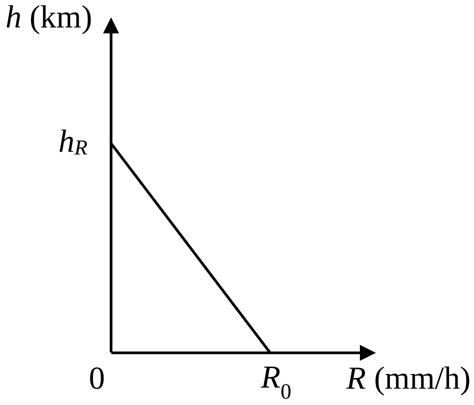
As the power crossing section BB is P_{AV}, the power absorbed by the line is:

$$P_l = P_{AV} - P_L = 0.0272 \text{ W}$$

Problem 3

Consider a ground station pointing with elevation angle θ = 50° to a GEO satellite (distance d = 38000 km). The satellite transmits at f = 15 GHz and it is under rainy conditions; the rain rate, whose vertical profile is given below (h_R = 5 km, R_0 = 30 mm/h), is constant horizontally.





Determine the power to be transmitted from the satellite to guarantee a minimum signal-to-noise ration SNR = 15 dB at the ground station. Assume both antennas has gain G = 38 dB and that they are optimally pointed. Also assume for the calculation of the specific rain attenuation γ → k = 0.0485 and α = 1. Disregard the other contributions to tropospheric attenuation. Also assume: equivalent noise temperature of the LNA, T_R = 300 K; waveguide loss A_{WG} = 2 dB; physical temperature of the waveguide T_{WG} = 280 K; mean radiating temperature T_{mr} = 290 K, receiver bandwidth B = 20 MHz.

Solution

First, let us calculated the total zenithal rain attenuation:

$$A = \int_0^{h_R} \gamma(h)\, dh = \int_0^{h_R} kR(h)^\alpha\, dh = \int_0^{h_R} k\left(R_0 - \frac{R_0}{h_R}h\right)^\alpha dh\Bigg|_{\alpha=1} = \int_0^{h_R} k\left(R_0 - \frac{R_0}{h_R}h\right)\ \ dh =$$

$$= kR_0\int_0^{h_R} dh - k\frac{R_0}{h_R}\int_0^{h_R} h\, dh = kR_0h_R - \frac{kR_0h_R}{2} = \frac{kR_0h_R}{2} = 3.64 \text{ dB}$$

The attenuation, scaled to the link elevation, and in linear scale is:

$$A_{lin} = 10^{-\frac{(A/\sin(\theta))}{10}} = 0.3351$$

The transmit power can be derived from inverting the SNR equation:

$$SNR = \frac{P_R}{P_N} = \frac{P_T \mathrm{G} f_T (\lambda/4\pi d)^2 \mathrm{G} f_R A_{lin}}{k_B T_{sys} B}$$

where k_B is the Boltzmann constant and the system equivalent noise temperature is:

$$T_{sys} = T_A + T_T + T_R = \left(2.73A_{lin} + T_{mr}(1 - A_{lin})\right) + T_{WG}\left(1 - A_{lin}^{WG}\right) + T_R = 597 \text{ K}$$

Inverting the equation above, and setting SNR = 15 dB → $P_T \approx$ 223 W.

Problem 4

A uniform sinusoidal plane wave propagates in a medium characterized by relative electric permittivity $\varepsilon_r = 1$, magnetic permeability $\mu_r = 9$ and conductivity $\sigma = 0.1$ S/m. The expression of the electric field is ($E_0 = 1$ V/m):

$$\vec{E}(z,t) = E_0\ e^{-\alpha z}\cos(2\pi 10^9 - \beta z)\vec{\mu}_y \text{V/m}$$

For such a wave, calculate the power received by an isotropic antenna located at P(0.1λ,0.1λ,0.1λ), which has efficiency $\eta_A = 0.9$.

Solution

Let us first check the loss tangent for the wave:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon} \approx 1.8$$

No approximations can be applied; therefore, the propagation constant is calculated as:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 45.7\ + j77.7\ 1/\text{m}$$

The wavelength is given by:

$$\lambda = \frac{2\pi}{\beta} = 0.0808\ \text{m}$$

Therefore P is in (0.00808 m, 0.00808 m, 0.00808 m).

The power received at P by the antenna is:

$$P_R = SA_e = \frac{1}{2}\frac{\left|\vec{E}(P)\right|^2}{|\eta|}\cos(\measuredangle\eta)\ A_e = \frac{1}{2}\frac{|E_0|^2}{|\eta|}e^{-2\alpha z_P}\cos(\measuredangle\eta)\ \frac{\lambda^2}{4\pi}D\eta_A$$

where:

$D = 1$ (isotropic antenna with directivity 1)

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = 679\ + j\ 399\ \Omega$$

Therefore:

$P_R = 0.12\ \mu$W