Telecommunication Systems
September 9th, 2019

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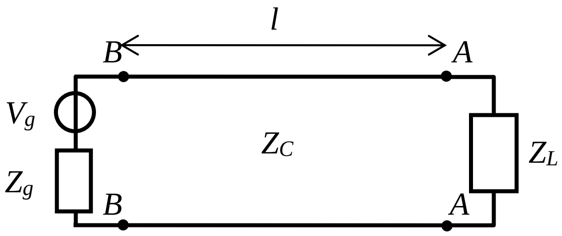
Problem 1

A source with voltage V_g = 15 V and internal impedance Z_g = 75 Ω is connected to a transmission line with characteristic impedance Z_C = 75 Ω, which terminates on a load Z_L = 75 Ω. The frequency is f = 300 MHz and the length of the line is l = 10 m.

a) Calculate the power absorbed by Z_L.
b) Calculate the power absorbed by Z_L if the line length becomes l = 12.4 m.



Solution

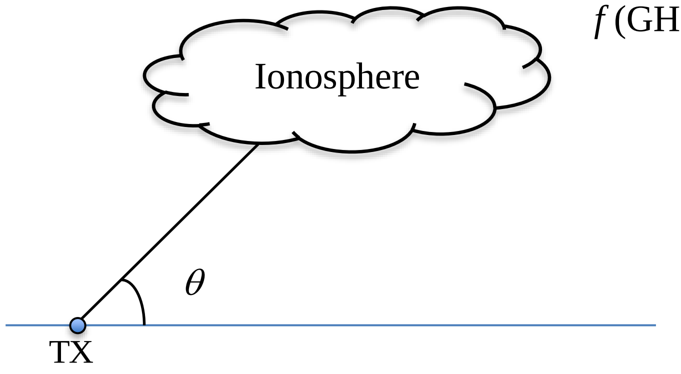
a) As there is complete match among the circuit components, the power absorbed by Z_L is the available power. Therefore:

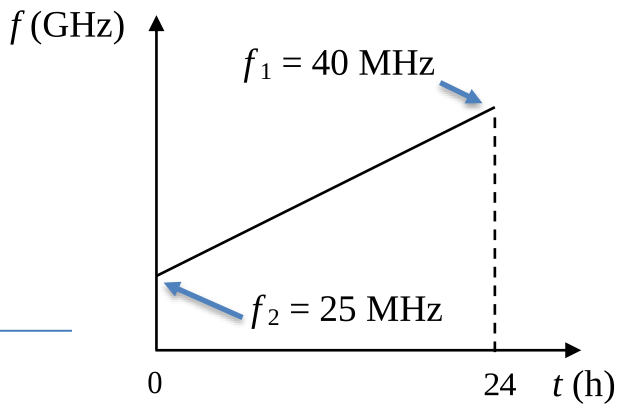
$$P_L = P_{AV} = \frac{\left|V_g\right|^2}{8\,\mathrm{Re}\left[Z_g\right]} = 0.375 \text{ W}$$

b) As there is perfect match along the line and no losses, Z_L still absorb the same amount of power.

Problem 2

Making reference to the figure below, a ground station points to a spacecraft with variable frequency *f*, according to the graph reported below (right side). The elevation angle is fixed to $\theta = 20°$ and the maximum electron content along the profile is $N_{max} = 2\times10^{12}$ e/m^3. Calculate the percentage of time during the day for which the link to the spacecraft cannot be established.





Solution

To determine if the ionosphere can be crossed, we need to use the following equation to determine the frequency *f*, given θ and N_{max}:

$$\cos\theta = \sqrt{1-\left(\frac{9\sqrt{N_{\max}}}{f^*}\right)^2} \quad \Rightarrow \quad f^* = \frac{9\sqrt{N_{\max}}}{\sin\theta} = 37.2\ \text{MHz}$$

If $f > f^*$, the wave crosses the ionosphere, otherwise it is completely reflected.

The trend of *f* in the figure is given by (*t* expressed in hours):

$$f = \frac{f_1 - f_2}{24} t + f_2$$

By imposing that $f > f^*$:

$$\frac{f_1 - f_2}{24} t^* + f_2 > f^* \quad \Rightarrow \quad t^* > \frac{24\left(f^* - f_2\right)}{f_1 - f_2} \approx 19.52\ \text{h}$$

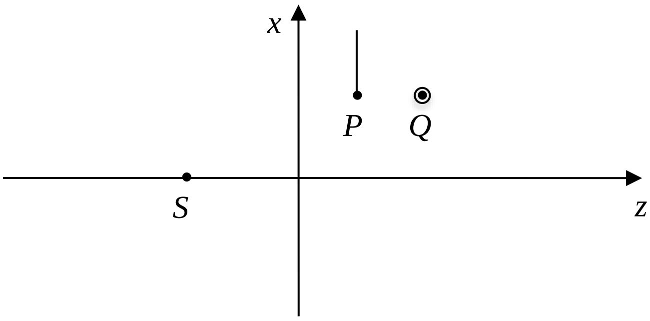
The time percentage of the day for which the link cannot be established is therefore $P = 81.33$ %.

Problem 3

A uniform plane wave (frequency f = 100 MHz) propagates in a perfect dielectric with $\varepsilon_{r1} = 9$ and $\mu_{r1} = 1$, and hits the surface of a medium characterized by conductivity $\sigma = 5\times10^{-2}$ S/m, $\varepsilon_{r2} = 9$ and $\mu_{r2} = 1$. The incident electric field is polarized along y and its value in (0,0,0) is E_0 = 10 V/m (see figure below).
Calculate:

a) The electric field at point S(0, 0, -3 m)
b) The power absorbed by the linear antenna located in P(0, 0, 1.5 m) whose direction is x
c) The power absorbed by the linear antenna located in Q(3 m, 0, 3 m) whose direction is y

Assume that both linear antennas have an effective $A_E = 2$ m^2.



Solution

a) For the second medium, the loss tangent is:

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_{r2}\varepsilon_0} = 1$$

The reflected and transmitted waves can be calculates through the reflection coefficient:

$$\eta_1 = \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = 125.7 \quad \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma + j\omega\varepsilon_2}} = 97.6 + j40.4 \quad \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \text{-}0.090 + j0.197$$

Thus the electric field in S is the combination of the incident and the reflected waves:

$$\vec{E}(S) = E_0\vec{\mu}_y e^{-j\beta_1 z_S} + \Gamma E_0\vec{\mu}_y e^{j\beta_1 z_S} = 9.1 + j1.97 \ \text{V/m}$$

b) As the electric field has a linear polarization along y and the antenna is directed as x, no power will be received by the antenna in P.

c) In this case, the wave polarization and the antenna have the same direction, so the antenna will receive the maximum power from the wave; the power density of the transmitted wave reaching Q is:

$$S(Q) = S_i(1-|\Gamma|^2)\mathrm{e}^{-2\alpha_2 z_Q} = \frac{1}{2}\frac{|E_0|^2}{\eta_1}(1-|\Gamma|^2)\mathrm{e}^{-2\alpha_2 z_Q} \ \text{W/m}^2$$

The attenuation constant can be calculated as:

$$\gamma_2 = \sqrt{j\omega\mu_2(\sigma + j\omega\varepsilon_2)} = \alpha_2 + j\beta_2 = 2.858 + j6.906 \ \ 1/\text{m}$$

Therefore:

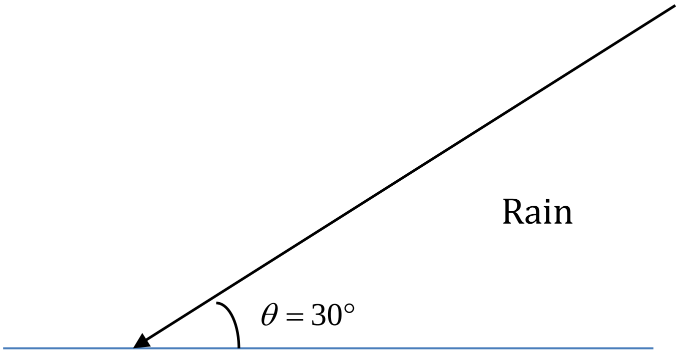
$$S(Q) = 1.35\times10^{-8} \ \text{W/m}^2$$

Finally, the power received by the antenna is:

$$P(Q) = S(Q)A_E = 2.7\times10^{-8} \ \text{W}$$

Problem 4

Consider a link from a GEO satellite to a ground station (elevation angle $\theta = 30°$) and assume that, initially, there is no atmospheric attenuation: associate this condition to the reference signal-to-noise ratio SNR_0. Evaluate the decrease in the SNR (as a function of SNR_0) when rain begins to affect the link.
Consider: no cosmic background noise, rain attenuation along the zenith A_R = 5 dB, constant rain rate both horizontally and vertically, physical rain temperature T_r = 10 °C, LNA equivalent noise temperature T_{LNA} = 280 K.



Solution:

The SNR is given by (no attenuation):

$$SNR_0 = \frac{P_R}{P_N} = \frac{P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R}{k T_{LNA} B}$$

where D is the distance between the satellite and the ground station and B is the RX bandwidth.

With rain attenuation, the SNR changes to:

$$SNR_1 = \frac{P_R A^{rain}}{P_N^{rain}} = \frac{P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R A^{rain}}{k T^{rain} B}$$

where A^{rain} is the rain induced attenuation (along the slant path with elevation angle $\theta = 30°$) and T^{rain} is the RX noise temperature in case of rain. As for A^{rain}, it is necessary to derive the slant path value from the zenithal one; given the assumption of constant rain rate:

$$A^{rain} = \frac{A_R}{\sin\theta} = 10 \text{ dB} = 0.1$$

As for T^{rain}:

$$T^{rain} = T_{LNA} + (1 - A^{rain}) T_r \approx 562.9 \text{ K}$$

Since:

$$\frac{T^{rain}}{T_{LNA}} \approx 2 \quad \rightarrow \quad T^{rain} = 2T_{LNA}$$

As a result:

$$SNR_1 = \frac{P_R A^{rain}}{P_N^{rain}} = \frac{A^{rain}}{2} \frac{P_T G_T f_T \left(\lambda/4\pi D\right)^2 G_R f_R}{kT_{LNA}B} = \frac{A^{rain}}{2} SNR_0 = 0.05SNR_0$$

Problem 5

A deep space probe collects scientific data about Saturn, which are continuously transmitted to a ground station on the Earth. Given this scenario:

1. Discuss the choice of the most suitable carrier frequency for the link: to this aim, it is enough to define a frequency range, but the choice must be justified
2. Discuss the most critical elements of the ground station having the highest impact on the link quality and describe the key choices to increase the SNR (approximate figures are welcome)

Solution

In a deep space scenario, the critical point is the large distance between the spacecraft and the ground station, which makes the link budget very critical. As a result, the choice of the frequency is critical. First the frequency needs to be higher than the typical critical frequency of the ionosphere, i.e. higher than 50 MHz (as a rule of thumb), to guarantee that no reflection occurs due to the ionosphere regardless of the elevation angle. In principle, the higher the frequency, the higher the data rate, but if the frequency exceeds 10 GHz, again as a rule of thumb, rain has a strong detrimental effect, that needs to be avoided. Overall, a frequency between 100 MHz and 10 GHz would be a good choice.

As for the receiver, a critical element is the antenna, which needs to be a parabolic one to guarantee a very high gain (orders of tends of dBs) so as to counteract the large distance to be covered in the link budget. The Cassegrain configuration is preferred (double reflector), as the feed can be positioned in the centre of the main reflector and thus the front-end hardware can be placed as close as possible to the feed itself. This minimize the attenuation introduced by the waveguide connecting the feed to the front-end hardware, which also minimizes the additional noise entering the receiver.

The second key element is the low noise amplifier, which needs to have a very high gain, such as 25/30 dB (so as to reduce significantly the noise introduced by the hardware components following the LNA in the receiver chain) and a very low equivalent noise temperature (noise figure of 0.5 dB in the best receivers), which can be achieved by reducing the temperature of the front-end hardware.